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Research Article

Study and Analysis of Visualizations of Vortex Structures Developed on the Upper Surface of the Cone of Revolution and the Prophile view of Vortex Shards.

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ABSTRACT

Visualizations and analysis of the flows, developed on the upper face of delta or gothic wings and cones, were carried out in the wind tunnel of the aerodynamics and hydrodynamics laboratory of the Hautes de France Polytechnic University of Valenciennes. These analyzes made it possible to better understand the behavior of the evolution and the positioning of the vortex structures and made it possible in particular to determine the preferential character of the interortex angles thus defined. Such a notable angular characterization revealed, in the case of delta and Gothic wings, the existence of a simple law, that of filiation, which is expressed in an angular correspondence between the main vortex couple and the leading edges. of the wing. The study of the vortical structures developed on the upper face of the cones resulted in an equally simple definition, also called the law of filiation, which, by analogy, is applicable to an angular correspondence between the main and secondary vortex pairs. This article, which is limited to discussing the case of a cone having an included angle of 68.6 °, provides a detailed description of the phenomenon. However, no current theory seems to be able to provide a simple explanation for these vortex behavioral properties.

INTRODUCTION

The concept of a preferential angle was introduced for the first time in 1972 by M. LE RAY and his colleagues and stems from their studies of liquid helium [1 to 4]. These angles, which have the following relatively simple analytic formula :

$$\cos \theta_{lm} = m / (l (l + 1))^{1/2}$$

where m > 0 and l > m

can be classified by their most frequent values into two groupings :

1st grouping

(l = m) : $\theta_{11} = 45°$; $\theta_{22} = 35.3°$, $\theta_{33} = 30°$, $\theta_{44} = 26.6°$, $\theta_{55} = 24.1°$, $\theta_{66} = 22.2°$, $\theta_{77} = 20.7°$, $\theta_{88} = 19.4°$, $\theta_{99} = 18.4°$

2nd grouping

(m = 2) : $\theta_{22} = 35.3°$, $\theta_{32} = 54.7°$, $\theta_{42} = 63.4°$, $\theta_{52} = 68.6°$, $\theta_{62} = 72°$, $\theta_{72} = 74.5°$, $\theta_{82} = 76.4°$, $\theta_{92} = 77.8°$

A particularly high number of studies have been carried out to date into delta or gothic wings and into some combinations of such components that form more or less simple slender bodies; they have dealt as much with the development of approximate theories {especially those studies by R.T. JONES} as with the definition of models specifying vortex lift per unit area.

Visualisations of hyperlifting vortex structures, mainly those carried out by H. WERLE [5 to 13], the analysis of pressure fields and of speeds created by these vortices, with or without their bursting, notably J.L. SOLIGNAC's analysis [14 to 17], also provide quite outstanding studies that are the standard works in their field.

Already described fully in such papers as those by WERLE, SOLIGNAC & STAHL [18 to 22], these studies offer today in

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their entirety, a thorough knowledge of the properties of various types of slender bodies.

However, given that the character of most of the aspects referred to remain empirical and limited to this or that scope of incidence [5 to 27] or to a numeric field [28], the way lies open, starting out from experimental data and various factors of analysis [29 to 31], for new attempts to be undertaken to examine the fundamental problems related to the positioning of vortices created by such slender bodies.

A large number of photographic and videographic visualisations concerning vortex flows developed on the upper-surface of delta or gothic wings and of cones [32, 57, 58,59,60,61,62] have been carried out within the Valenciennes University laboratory with a view to providing a better understanding of the development and the positioning of vortex structures under the influence of apex angles and angles of incidence.

These visualisations have enabled priority to be accorded to the study of examples of the most elementary shaped bodies, i.e. delta and gothic wings.

The results obtained, and already fully described in previous papers [29, 30, 31, 33 and 34], have been expressed by some very simple mathematical formulae; that very simplicity perhaps calls for some essential explanations.

The angles between the vortices have, in fact, been found, on a wide range of Reynolds, to have a preferential nature, thereby underlining a simple angular characterisation of the positioning relative to the vortex torques.

This new behavioural property, equally called the law of filiation, without doubt concerns cones as a whole. Limited as it is to the case of a cone having an apex angle of 68.6°, our present study can consequently only be taken as an example, albeit a detailed one, of this phenomenon.

MODELS USED IN THE EXPERIMENT

The various models subjected to trials in the wind tunnel are of the circular-base cone-shaped type.

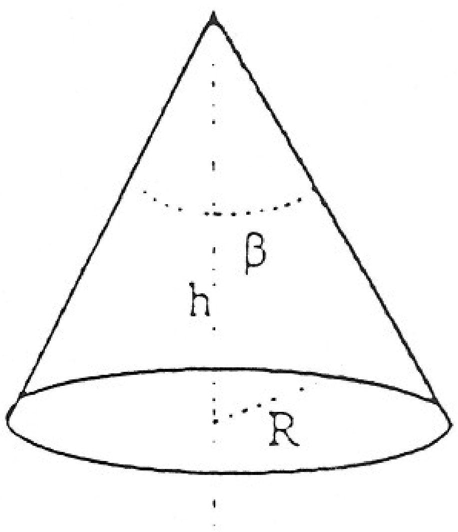


Diagram n° 1 The principles of constructing cones

β : summit (apex) angle

h : height of the cone

R : radius of circular base

VISUALISATION TECHNIQUES

The visualisations in their entirety were carried out within the vein of the wind tunnel {a 45 x 45 cm² section} at the Valenciennes University aerodynamic and hydrodynamic laboratory.

The vortex structures developed on the upper surface were made visible by providing for emissions of white smoke at the cone tip; this was obtained by injecting oil under pressure through a tube of small diameter and then, by means of an integrated electrical system, vaporising the oil immediately as it left the probe.

The cones were fixed onto an axis relayed to a cursor graduated from 0° to 360° which enabled the incidence of cone rotation to be varied. The visualisations were captured on photographs - using an automatic camera with flashgun incorporated.

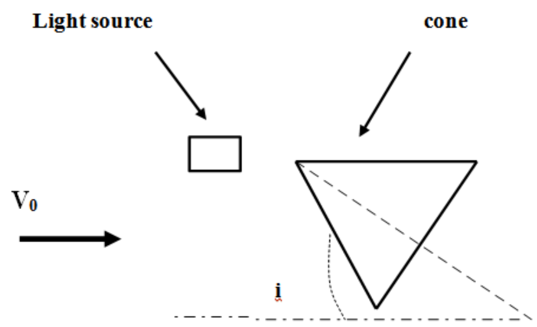


Diagram n° 2 Diagrammatic representation of the experimental device

i : incidence α

vo : speed of flow to infinity upstream

CONDITIONS UNDER WHICH THE EXPERIMENTS WERE CARRIED OUT

Visualisations, employing smoke, were carried out at low speed (v = 3 to 5 m/s) which gives the flow a Reynolds' number ranging from 19000 to 80000 {h : length of reference}. The height h and the apex angle β of the cone are respectively 110 mm and 68.6°. Variations in the incidence i range from 25° to 65°.

RESULTS OF THE EXPERIMENTS

In order to avoid rendering this paper unduly lengthy, we summarise the findings by stating simply that visualisations at increasing incidence enabled the progressive birth of vortex structures to be carefully monitored; at mean incidence, these structures become concentrated and stable and thus pass from a structure of flow with raised edges to the standard vortex tube structures.

Those vortices, derived from the apex and perfectly described by H. WERLE, result in fact from the "cornet-like" spiral coiling of the flow which detaches itself from each side of the streamlined body. Their axes are rectilinear and are cut near the apex depending on the angles {respectively interior and exterior ω1 and ω2} while the pace of the pseudo-flow in a transversal plane is probably close to that which figures in diagram n° 3 where the external vortices are closer to the wall than are the interior ones.

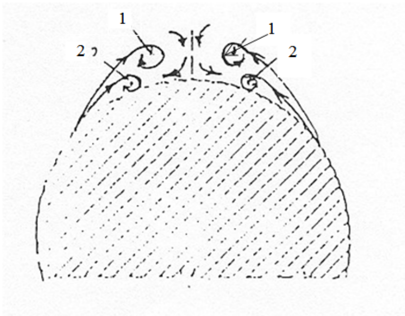
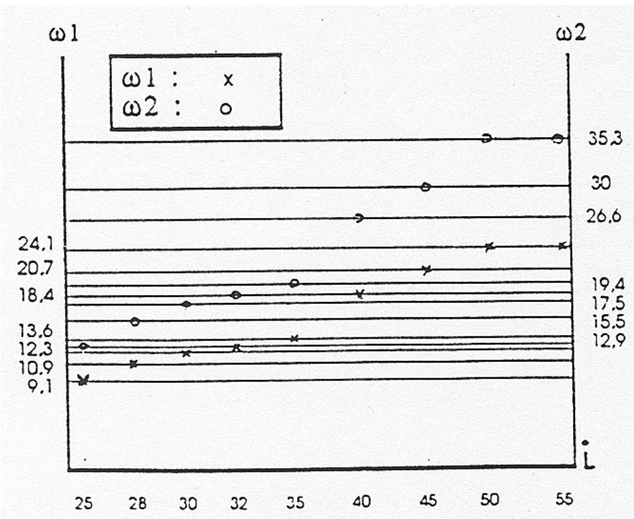


Diagram n° 3, 1: interior vortex, 2: exterior vortex

Whatever the incidence, these intervortex angles have shown themselves to have a preferential nature and have proved to be especially stable since they do not depend, in the conditions applied to the experiment, on speed {or on Reynolds}.

However, it is worth noting that such angles, contrary to what takes place on the upper surface of delta or gothic wings, even so depend on the incidence, a fact which bestows on them a discontinuous evolution since they have a tendency to conserve their preferential aspect {see graph n° 1}.



Graph n° 1 The evolution of interior and exterior intervortex angles in relation to the incidence 19000<Re<80000

Moreover, we noticed that there existed, in every case, a notable correspondence between both the interior and exterior intervortex angles.

It has in fact been confirmed that the intervortex angles ω2, in their entirety, belong to the first grouping of preferential angles {those with the notation θ11} and consequently infer, in accordance with the law of filiation, that the intervortex angles ω1 are also of the same first grouping {see Table n° 1} but with the notation

$$\theta_{2l+1,2l+1}$$

$$\omega_2=\theta_{ll}\rightarrow\omega_1=\theta_{2l+1,2l+1}$$

i	w2		w1	
25	12.9	19.19	9.1	39.39
28	15.5	13.13	10.9	27.27
30	17.5	10.10	12.3	21.21
32	18.4	9.9	12.9	19.19
35	19.4	8.8	13.6	17.17
40	26.6	4.4	18.4	9.9
45	30	3.3	20.7	7.7
50	35.3	2.2	24.1	5.5
55	35.3	2.2	24.1	5.5

Table n 1 Angular relationship between interior and exterior intervortex angles (in degrees) 19000<Re<80000

This law, established in the particular case of the cone, inevitably refers back to the quite analogous results we observed [33 and 34] from our studies of delta and gothic wings, even if, in the latter, the notable angular correspondence was described between the apex angles β of the first grouping and the associated intervortex angle α (see Table n° 2).

$$\beta = \theta_{ll} \rightarrow \alpha = \theta_{2l+1,\,2l+1}$$

▯		▯	
		14.5	
20.7	▯7.7		▯15.15
22.2		15.5	
	▯6.6		▯13.13
26.6			
	▯4.4	18.4	▯9.9
30	▯3.3	20.7	▯7.7
35.3		24.1	
	▯2.2		▯5.5
45		30	
	▯1.1		▯3.3

Table n° 2 Angular correspondence between preferential angles of the first grouping and intervortex angles inferred in the case of delta or gothic wings (in degrees) 41000<Re<88000

Be that as it may, the description of the behavioural law may be taken a little further if we make a point of referring again to the first definition of preferential angles, namely :

$$\cos\theta_{ll} = l \,/\, (l(l + 1))^{1/2}$$

where l > θ

In these conditions,

$$\sin^2\theta_{ll} = 1 + \cos^2\,\theta_{ll} = 1 \,/\, (l + 1)$$
$$\sin^2\theta_{2l+1\;2l+1} = 1 \,/\, (2\,(l + 1))$$

which can be simplified to :

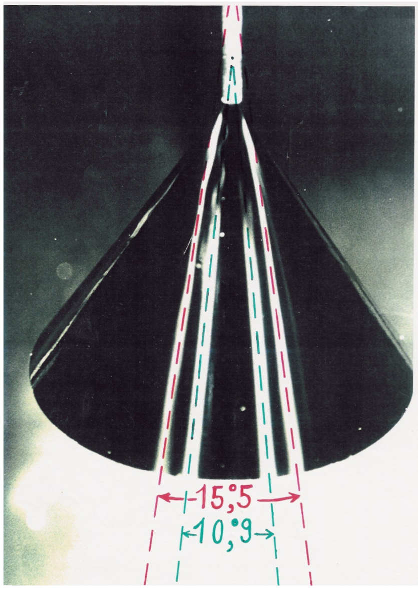
$$\sin^2\theta_{2l+1,\,2l+1} = (\sin\,^2\theta_{ll})\,/\,2$$

and thus :

$$\sin^2\omega_1 = (\sin^2\omega_2)\,/\,2$$

Some of the visualisations obtained, where incidences were increasing from 25▯ to 55▯, provide a better appreciation of the evolution of vortex structures developed on the upper surface of a cone having an included angle of 68.6▯ {see Views n▯ 1, 2, 3, 4, 5 and 6}.

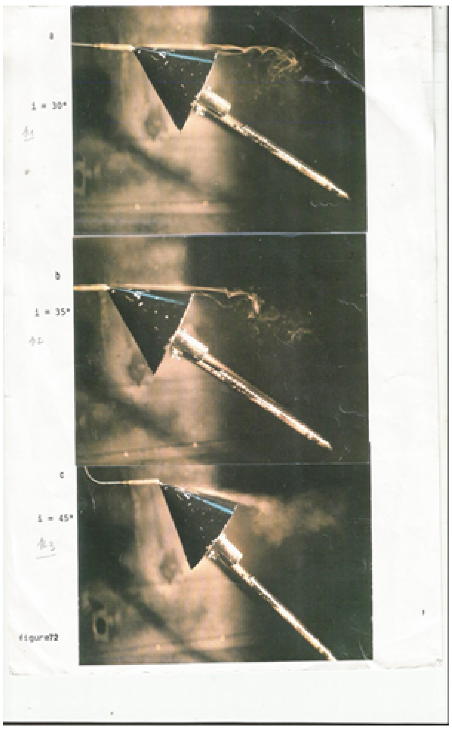
However, it is worth noting that even at a higher incidence of 65▯, we can notice the existence of two flows separated by an interval not fed by smoke and thus constituting a non vortex passage {see View n▯ 7}.



View n▯ 1[33-54-57] β = 68.6▯ i = 28▯ ω1 = 10.9▯ ω2 = 15.5▯ 19000<Re<80000

View n▯ 2 [33-54-57] β = 68.6▯ i = 45▯ ω1 = 20.7▯ ω2 = 30▯ 19000<Re<80000

View n° 3 [33-54-57] β = 68.6° i = 45° 19000<Re<80000



Profile view °4 β = 68.6° i = 30° i =35 i=45 [33-54-57]

19000<Re<80000

The whole of these two air flows and of this passage is contained in a global angle of 45° whereas the angle formed between the left- and right-hand edges of each of the two air flows is 30°.

At higher incidences, the turbulence causes considerable diffusion of the smoke and therefore does not enable clear visualisations to be made of the vortex structures; the latter subsequently deteriorate by bursting into torch forms that are initially stationary and then are subjected to alternate movements before those movements become thereafter chaotic [32,57].

CONCLUSION

A wide range of Reynolds, the preferential nature of the intervortex angles present on the upper face of delta and gothic wings and cones would seem to be entirely cataloged, the very existence of the law of descent relating to these slender bodies expresses a certain universality behavior and reveals the fundamental characteristic of our study.

At present, no complete theoretical approach seems capable of providing a direct explanation for the simplicity of these results.

The progressive evolution of the elementary vortices of the pure flow before take-off towards a particularly stable vortex system, seen [1-2-3-4] where the spatial positioning reveals an original organization, still remains an enigma today. [33-54- 57].

It is, of course, difficult to prejudge the lines along which one or more future studies may follow, studies which could lead to a theoretical explanation.

However, perhaps we may be permitted to note that the phenomena, in which the sine squared of an angle also plays a part, are created by the simple structures of stationary and unsteady fluid mechanics.

It is in this way that the flow - emitted by a cone having a demi-span of θ at its summit on a tridimensional dipole with the same axis as that of the cone, or with its summit at the centre of a vortex ring equivalent to the dipole - is proportional to $Sin^2θ$ [35].

This is how energy - emitted by an oscillating electromagnetic dipole (with properties analogous to those of the oscillating fluid dipole that plays a part in aerodynamics or in hydrodynamics) in a θ direction with regard to the axis of the dipole - is, energy too, proportional to $Sin^2θ$ [36] or to $cos^2θ = 1-sin^2θ$ in the case of the acoustic dipole [33,37, 38 and 39].

It is also a law in $Sin^2θ$ that gives the dependence, with regard to the angle of attenuation of the second sound thermal waves in liquid helium, by rectilinear vortices which form the θ angle with the direction of the propagation of this wave [40].

It is, moreover, by coupling this law with the concept of that preferential angle, formed by helicoidal vortices with their axes, that the authors of the papers referred to in [1] and [4] interpreted the discontinuous angular behaviour of these vortex systems in liquid helium [1, 2, 3, 4 and 29].

Finally, and this ultimate remark is probably not the least important one, the suction force, to which a profile - an infinitely thin and localised plane, let us remember - is subjected in the immediate vicinity of its leading edge, is, that force, too, proportional to the sine squared of an angle, in this case the angle of incidence [41].

As concerns the possible links, of the phenomena we have described, with the properties of an emission or of an absorption of a flow or of a wave - whose source may be dipolar or multipolar - it is perhaps interesting to note that the range of speeds of a tridimensional dipole {characterised by the angle between the radius vector of a point of the fluid and the speed of this fluid} is linked to the angular positioning of this point, with regard to the dipole {characterised by the polar angle between the axis of the dipole and the radius vector}, by a series of striking correspondences between the most simple preferential angles. Moreover, if we now consider the force of interaction between two dipoles of relatively simple orientation - the most simple is one with two parallel dipoles, but numerous other layouts give equally curious results - the range of interaction forces {characterised by the angle between the radius vector joining the two dipoles with this interaction force}, possesses, in its turn, together with the two other ranges of relative positionings and of speeds {characterised as described above}, two new entireties of quite striking correspondences [33,35, 42, 43, 44 and 57].

Where the notable orientations of interaction forces between two parallel dipoles are concerned, and with regard to the common direction of the axes of these dipoles, we have extracted a few particular cases from the general calculations made by KONIG [46] and from his final result given in RAYLEIGH's very famous book on acoustics [47].

These references contain expressions of : the components of the force exerted by one sphere on another in the presence of a uniform wind pattern to infinity; where the fluid flow is perfect; the line joining the centres of spheres forming the θ angle with the direction of the wind to infinity - this force is the same as that exerted by one sphere on another when those spheres are moving parallel to each other and at the same speed in an immobile fluid to infinity - but it is known that each of these spheres is equivalent to a tridimensional dipole.

The sole particular cases commented on by KONIG and RAYLEIGH are those where the centres of the spheres are aligned, {i.e. in the direction of the wind}, and where these spheres therefore exert on each other a repulsion force {i.e. perpendicularly to the wind} with, in this case, a gravitational interaction force which explains the formation of very fine powder ridges, perpendicular to the axis of a sound tube, within its antinodes (ventral segments) of vibration.

But a whole series of other consequences from the general formulae found in references [46] and [47] seem, they too, to be very significant. We have attempted, from those found in references [43], [44] and [57], to determine a few of them. One of the most important of these particular cases seems to us to be that where the interaction force between two parallel dipoles is itself parallel to them. Formula n° 4 on page 47 of reference [47] immediately shows that this case corresponds to the θ angle, cancelling the Legendre polynomial 1-5cos²θ, i.e. at

$$\cos\theta = 1/(5)^{1/2} = 2/[4(4+1)]^{1/2} = \cos\theta_{42}$$

according to the defining formula of preferential angles given at the beginning of this paper.

This angle θ42=63.4° is, moreover, the angle between the diagonals of the famous "Golden Rectangle" discovered by architects and employed by them from time immemorial [43, 44, 45 and 31].

In this same train of thought, it is striking to note, in references [37] and [49 to 51] the role played systematically by the angle θ32=54.7° (cancelling the Legendre polynomial 1-3cos²θ) in the sound emission of an axisymetric jet and of two interaction forcing vortex rings or of one forcing vortex ring in the presence of a sphere.

In short, many other well-known hydrodynamic and aerodynamic phenomena are rich in preferential angles, the theory of which has at some or other been fully elaborated. This is the case found in the very subtle and elegant theory proposed by KELVIN and FROUDE concerning the wake of ships, described in particular in the works of LAMB [55] and of LIGHTHILL [55]. In the wake, the crests of waves, in a curvilinear triangle form, will in fact each disappear at two counterflow points, the alignment of which, along two right-hand sides, determines a total span of the wake at twice 19.4° here and there of the axis of this wake, axis with which the counterflow tangents, associated with the crests, form an angle of 54.7° while also forming with each corresponding edge of the wake an angle of 35.3° {i.e. 54.7° - 19.4°}.

It is there where the following relation is to be found, never interpreted before now, in terms of preferential angles :

$$\theta_{32} = \theta_{22} + \theta_{88}$$
$$54.7° = 35.3° + 19.4°$$

The link between the wake of a ship, being the result of the combination of bidimensional surface waves shed in various directions, and the phenomena described above may appear at first sight to be very mysterious. We may, however, be permitted to reason that the paper by E. LEVI [56] under the title "An oscillating approach to turbulence", so suggestively illustrated by figure n° 1 on page 352 of his study {an illustration which represents the frontier of a wake or of a maximum layer as a swell induced by the emission of vortices} perhaps provides the starting point of a profitable line of further research which could lead to a better understanding of the omnipresence of preferential angles and of their filiations in tridimensional flows, and in particular in those developed around slenderbodies.

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