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Perspective

Potential Flow through a Cascade of Aerofoils

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PERSPECTIVE

The potential flow across an infinite cascade of aerofoils is treated as both a direct and inverse problem. In each case, a perturbation expansion about a background uniform flow is claimed, with the size of the perturbation comparable to the aspect ratio of the aerofoils. This perturbation must decay far upstream while also satisfying specific edge conditions, such as the Kutta condition at each trailing edge. The flow field through a cascade of aerofoils of known geometry is calculated in the direct problem.

Analytically, this is solved by recasting the situation as a Riemann-Hilbert problem with only imaginary values prescribed on the chords. Analytic expressions for surface velocity, lift, and deflection angle as functions of aerofoil geometry, angle of attack, and stagger angle are presented, and they agree well with numerical results. However, analytic solutions for uniform flow past periodic arrays are severely lacking. Through the use of asymptotic analysis, early research considers the uniform flow past an array of cylinders with small diameters compared to their separation distance. Crowdy was able to extend Balsa's work to allow for arbitrary diameter-to-separation-distance ratios thanks to the development of a novel transform method known formally as the Unified Transform Method (but more commonly as the Fokas method). In order to solve the problem of uniform flow around cylinders, a conformal mapping from a periodic array of non-cylindrical structures to the cylindrical array described by Crowdy could be used. Although there are several methods for calculating the potential flow through a cascade of aerofoils, analytical solutions that elucidate the underlying physics are uncommon.

Potential flow well beyond a periodic array of bodies occurs frequently in a wide variety of fluid mechanical problems. For example, in aerodynamics, the flow through a rotor cascade, the flow through structured porous materials, and the flow around large schools of fish. In these applications, it is necessary to calculate not only the potential flow through the structure, but also the complicated interactions between unsteady perturbations to the flow (such as turbulence) and the structures themselves. It is critical to understand the background steady flow because it can convect and distort the unsteady perturbations when approaching any of these complex unsteady interactions. Through the use of asymptotic analysis, early research considers the uniform flow past an array of cylinders with small diameters compared to their separation distance. Crowdy was able to extend Balsa's work to allow for arbitrary diameter-to-separation-distance ratios thanks to the development of a novel transform method known formally as the unified transform method (but more commonly as the Fokas method). In order to solve the problem of uniform flow around cylinders, a conformal mapping from a periodic array of non-cylindrical structures to the cylindrical array described by Crowdy could be used. Unfortunately, these mappings are frequently impossible to invert analytically, as a result, a numerical scheme must be implemented. Although there are several methods for calculating the potential flow through a cascade of aerofoils, analytic solutions that reveal the underlying physics are uncommon.

The potential flow past an isolated airfoil is well known to be easily solved using high order vortex panel methods, in which the airfoil is discretized into panels, each panel consisting of a continuous vortex sheet whose strength must be determined. As explained by Hess, the no-penetration conditions on each panel, as well as the Kutta condition at the trailing edge, provide a complete set of linear equations with which the strength of the vortex sheet on each panel can be determined. Constructing a conformal map from a canonical circular domain to the cascade yields one well-known solution for flat plates at angle of attack. The method of singularities is another common method in thin-aerofoil theory: the rigid aerofoil surface is modelled as a distribution of mass sources and vortices on the chord (for thin aerofoils) or the surface (for thick aerofoils). Surface singularity methods, which are simple to programme, take a different approach to solving the potential flow problem in cascades by distributing singularities like vortices and/or sources on the blade contour. Because of the infinite cascade's geometry, every singularity on a portion of one cascade blade must be repeated an infinite number of times, one for each of the cascade blades.

It should be noted that the induced velocity at any arbitrary point on a panel can be obtained in closed form analytical expressions, so no numerical integrations across the panel are required. The current method works by first using a conformal mapping to transform the infinite cascade to a single closed contour before using the standard higher order vortex panel method to solve the cascade problem. We avoid the series of transformations that are typically used in pure conformal transformation methods to reduce this single closed contour to a regular circle by doing so. We can also solve the cascade problem using the panel code of the

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isolated airfoil problem, which eliminates the need for numerical integrations, which is required in higher order surface singularity methods such as McFarland's.

To solve the potential flow in cascades, a simple method is presented that combines conformal mapping and a simple higher order panel method. The potential flow in linear and circular cascades can be easily calculated using this method, which combines a simple higher order panel method, as used for isolated airfoils, with a single conformal transformation. The proposed simple method has the advantage of allowing the use of higher order vortex panel methods, which are considered more accurate, for the cascade problem without the need for numerical integrations. Comparisons of experimental and analytical results are presented, demonstrating that the current method is effective. The methodology may be useful for quickly calculating forces and moments on linear and circular cascade blades, as well as designing cascade blade geometries. The potential flow solutions for circular cascades may also provide some insight into the flow past guide vanes in centrifugal compressors and turbines. Because the current potential flow code is very fast when compared to a full Navier–Stokes solution, it may be used in inverse design problems of cascades using a heuristic method such as the genetic algorithm. It may also be useful in the simulation of unsteady cascade problems.