

Initial Instability of Rear-Wheel Braking on Cars

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Abstract

The problem of initial instability of rear-wheel braking on cars is presented in this paper. The analysis of the braking transient process was performed, with computer simulation of the two different cases of wheel suspension: an existing one and a new proposed concept. Presented below is the behaviour of the rear wheels during braking, where the force of the wheel pressing the ground surface appears as a key parameter. The conclusion is that the existing concept, with backward oriented oscillating suspension arm, has a natural imperfection as regards a quality of adhesion of the wheel with the ground surface. The new proposed concept of the wheel suspension is given. The result of analysis shows that the natural stability of braking could be improved with the new concept, independently of existence of ABS (anti-lock braking system) on the car.

Keywords: Car; Suspension arm; Brake; Braking reliability; Down pressing force

Introduction

In the car movement dynamics, the driving safety and quality are determined by three basic characteristics: drive, steering in the direction of travel and braking. These three characteristics are generally not to be considered separately, but the specific problems can be considered separately, in order to optimize the characteristics of the individual systems and cars in general. This paper refers to the cars, but this matter is applicable to all wheeled vehicles in general, also including aircraft.

Braking is a delicate process composed of the interaction of the driver and the car with the surface, in which the driver can brake ranging from "soft" to a very abrupt braking. The problem that is analyzed here is not directly correlated with advanced technical systems of active braking control (e.g. ABS), or combined systems of active control on the vehicle. This paper presents the analysis of the impact of the way of wheel suspension on braking characteristics, especially on the rear wheels.

There are various ways of rear wheels suspension on the cars. Classical ones, with leaf flexion springs, are rare today, but the suspension with oscillating arm is much spread. The reason is probably that majority of modern cars have front drive, and smaller cars are shortened at the rear end. So, having in mind that many cars, that exist today, have such suspension system, this analysis has dealt with the braking process on rear wheels and its quality, and tried to propose a way of improving the natural braking reliability.

Problem Background

When a car moves uniformly, the wheel rolling resistance force and the air resistance force, are balanced by the drive force. When driver presses the brake pedal, the braking process begins. In this process, the friction force between the wheel and the ground surface appears, as a reaction to the slowing down of the wheel because the brakes are on. The cumulative friction force of the wheels with surface causes the inertia force of the car, as a result of slowing down. At that point, the weight of the car is redistributed to the front and rear wheels (Figure 1), which has the effect of raising the rear end and lowering the front end of the car, while driving forward. This phenomenon is universal and does not depend on the suspension and braking systems. This is the reason why all brake systems have smaller brakes on the rear wheels than on the front ones.

In the braking process, local occurrence of vertical movement of the rear wheels also takes place, enabled by the wheel suspension system, and caused by the braking force. Figure 2 presents the forces acting on the wheel during braking. The force of the weight G_B and its reaction from the ground F_N are balanced before braking. When braking starts a friction force between the calliper and disc appears, and, as a consequence, friction force with the ground and force of inertia because of slowing down the vehicle. The forces acting on the isolated wheel are shown in Figures 3 and 4 (disc brake, drum brake). The friction force F_{fr} at disc makes a couple with its reaction in the bearing of the wheel (F_{fr} - F_{fr}), thus creating the braking moment M that causes slowing down of the wheel, and the car too. Similar is in the case of drum brake. The counter couple (F_{dr} - F_{dr}) acts on the suspension arm tending to raise it (Figure 5), and thus reduce the adhesion force of the wheel with the ground. This is natural imperfection of this kind of suspension, and takes place in transient process while braking.

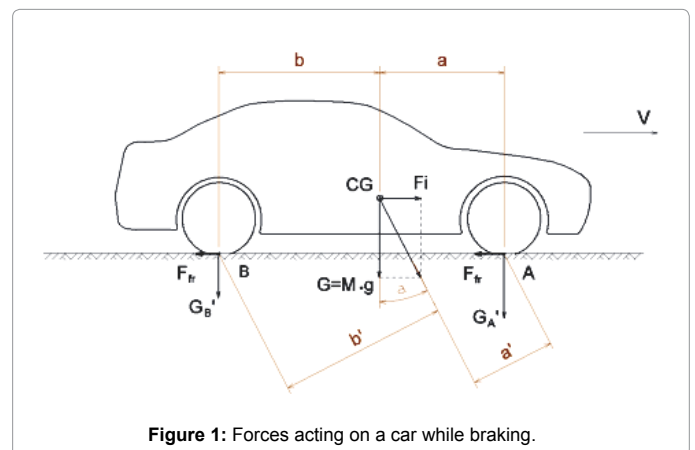


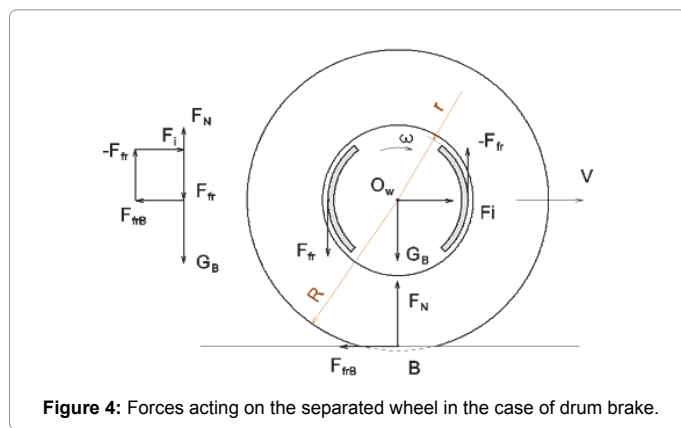
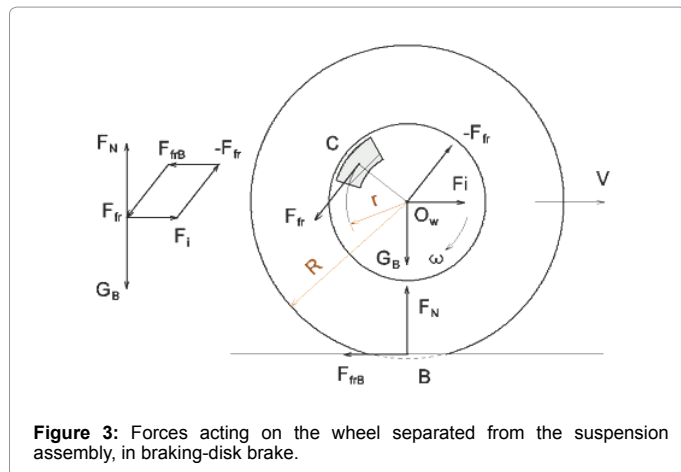
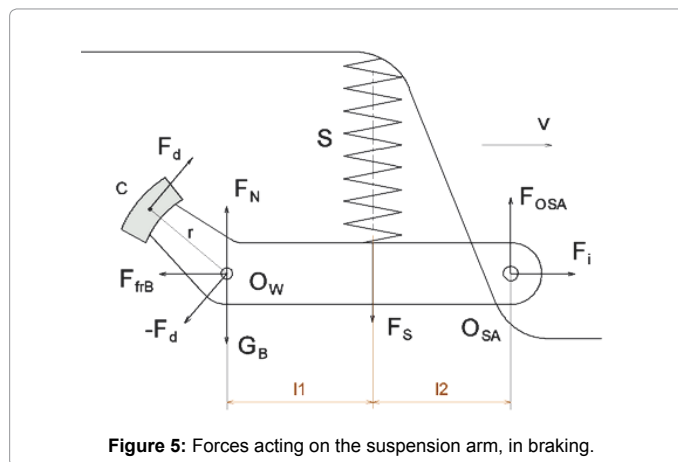
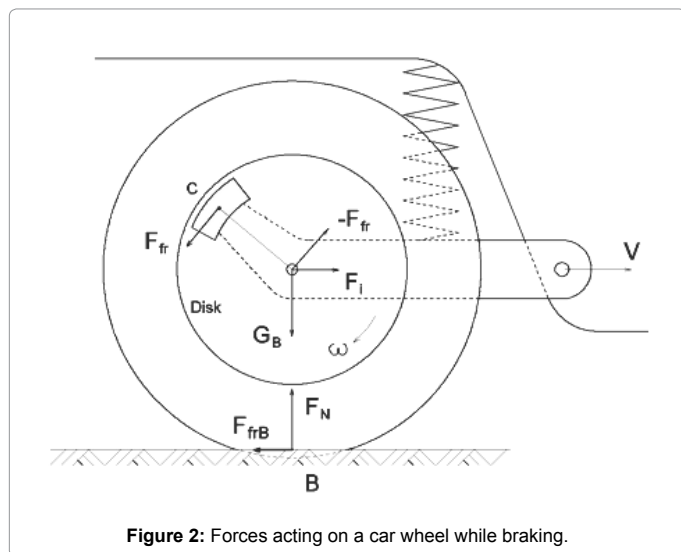
Figure 1: Forces acting on a car while braking.

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When driving, namely when braking, the key requirement, besides stability of the vehicle, is that there is no skidding. During the braking process, in addition to the manner in which the driver brakes, the most important parameter is the pressing force of the wheel on the ground surface. The pressing force of the rear wheels (axle) on the surface is reduced in comparison with its value before braking, due to the previously mentioned phenomena (redistribution of weight and

influence of the suspension). So, this analysis will deal with influence of the two appearances on braking process, particularly through the down pressing force of the rear wheels on the ground.

It is necessary to note here that ABS on the vehicle works independently of these problems (weight redistribution and influence of the suspension), i.e. it ensures that there is no skidding regardless of magnitude of the forces applied to the ground. In some cases characteristics of the ABS are dependent on active suspension control [1]. So, the ABS is not directly correlated with this analysis.

Unlike active methods, this analysis is directed at the manner of *natural* increase of braking reliability by increasing normal pressing force of the rear wheels on the surface. The analysis was made in order to establish the difference in the braking character in case of existing ways of the wheel suspension, and a new one possible way of suspension.

Dynamics

The analysis presented here has been made upon some suppositions:

- Braking is symmetrical (left and right wheels are loaded with equal forces), so the movement of the rear axle with wheels is considered as translator.
- Movement of the axle with rear wheels is considered as vertical in limited amount, because simplicity of the model.
- The point of the analysis is to determine the difference in behaviour of the rear wheels, while braking, in two cases of suspension, so the linearization of the equations is reasonable.
- All loads on the wheels are reduced to the axis of the wheel: weight (transferred via spring), disk (drum) force and reaction force from the ground.
- Numerical analysis of the rear wheels (axle) behaviour in transient process has been done in two cases: partial influence of the suspension system, and cumulative influence of the suspension and weight redistribution. These two cases differ in the weight of the rear end of the vehicle.

Existing suspension

Figure 2 shows the scheme of the wheel suspension system with backward oriented suspension arm, with forces acting on the wheel. At uniform movement, the wheel rotates at constant relative angular velocity with respect to its axle. Then the external points of the wheel

move along the cycloid in relation to the surface, or, in the absolute coordinate system, the wheel has a current rotation centre at point B. At the start of the braking process, brake friction pads are pressed on the disc with normal force, and as a result, the friction force F_{fr} appears at point C, together with its reaction $-F_{fr}$ at wheel axis (bearing). The couple $(F_{fr}, -F_{fr})$ represents the braking moment $M = F_{fr} \times r$. The force F_{frB} (Figures 3 and 4) is then: $F_{frB} = M/R$. The reaction to the friction force is the force F_d , namely the force by which the disc acts on the calliper on the suspension arm. Reaction to the force $-F_{fr}$ is force $-F_d$. There appear two couples of forces: the couple $(F_{fr}, -F_{fr})$, and the counter couple $(F_d, -F_d)$.

Figure 5 shows the scheme of the same wheel suspension, with forces acting on the suspension arm. The couple $(F_d, -F_d)$ has such direction that disturbs the balance of the suspension arm and tends to raise it and to compress the spring. A transient process is initiated, which ends by balancing the spring bias force with the sum of the forces at the point O_w . The new balanced state (under the upward impact of the couple $(F_d, -F_d)$) is achieved in the new position of the spring, where the rear axle is raised to Δh . This means that the rear end of the vehicle will then be lowered by that amount, because of gravity. Pressure force on the surface is reduced in transient process, while after completion of the transient process it is re-balanced with the weight of the G_1+G_2 . G_1 is the weight of the rear axle with wheels and suspension parts, and G_2 is the weight of the car body that loads rear axle. This is a local problem, and not directly connected to the general problem of raising the rear end due to weight redistribution, but these two occurrences are simultaneous and give the summary result.

Scheme of the forces acting on the suspension arm is shown on the Figure 6. The couple of forces $(F_d, -F_d)$ can be replaced by force F_1 and reaction in the support O_{SA} (articulated joint). The assumption involved here is that O_{SA} is the reference point for movement of the suspension arm (physical centre of rotation), so the force F_1 will be: $F_1 = F_d \times r / (l_1 + l_2)$. The force F_1 here appears as disturbing force, and will be an input force in dynamical model. Actually, the gravity centre of the vehicle should be the reference point, but here the problem is treated as a local one, limited on the suspension elements.

This local effect of raising the suspension arm (axle with the wheels), thus reducing the adhesion force with the ground surface is, of course, very undesirable, because it additionally contributes to the reduction of adhesion force that is once already reduced by the weight redistribution. So, since the effect of the weight redistribution cannot be avoided, there rests a possibility to reduce this local (and global) negative effect in some other way.

New suspension

In order to compensate for a loss in the down pressing force while braking, a new concept of the rear wheels suspension is introduced,

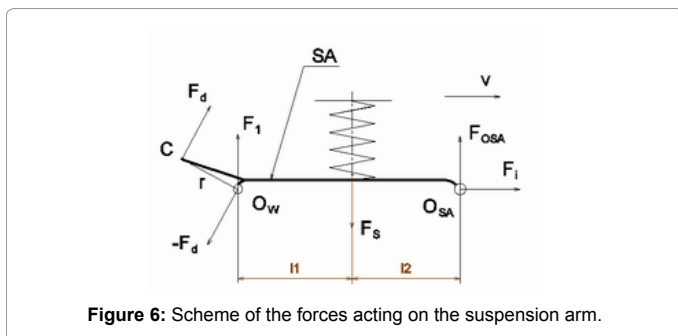


Figure 6: Scheme of the forces acting on the suspension arm.

which has quite different dynamical behaviour, regarding forces acting on the rear axle. In the new concept (Figures 7 and 8) the intermediate element of suspension is involved (auxiliary arm), oriented forward and tending to prevent raising of the suspension arm, and to lower it. The auxiliary lever (AA) is connected to the wheel axle by articulated joint in O_w , with axis coinciding with wheel axis, so the relative rotation of the auxiliary arm and wheel axle is enabled when wheel goes up and down. The auxiliary lever is also connected at its rear end to the support lever by a spherical joint in O_{AA} . Support lever (SL) is connected to the chasis of the car at fixed point (spherical joint). The calliper with disc pads (or drum base with brake shoes) is tightly connected to the auxiliary arm, and this is essential characteristic. So, the AA and the SL are kinematically fitted as to enable the wheel to go up and down in full range, without any objection from AA and SL. In this disposition the force couple $(F_d, -F_d)$ acts on the AA so that lowers both the auxiliary arm and the suspension arm, thanking to the support at the point O_{AA} . The weight G_b is transferred via the spring to the axle O_w . The scheme of forces for that case is given in Figures 9 and 10. The dynamical model is described based on those forces.

The forces acting on the AA and SA are presented on the Figure 9. Only the forces induced by braking are shown, but not the weight and reaction to the weigh in O_{SA} . The weight actually is transferred through the spring and has its reaction in the wheel, but in calculation this force will be considered as if it acts in the vertical plane through the wheel axis O_w .

It is already stated that the couple $M = F_d \times r$ is acting on the auxiliary arm AA. This couple could be replaced by the couple $(F_1', -F_1')$, having

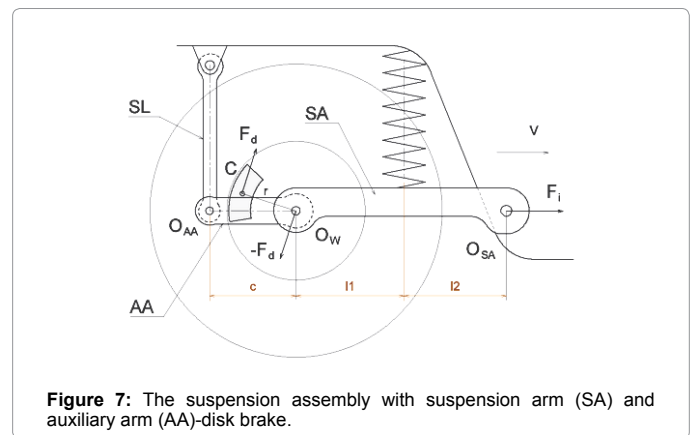


Figure 7: The suspension assembly with suspension arm (SA) and auxiliary arm (AA)-disc brake.

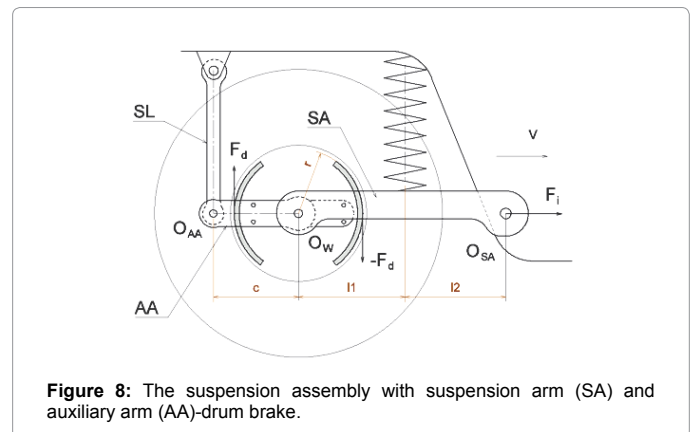


Figure 8: The suspension assembly with suspension arm (SA) and auxiliary arm (AA)-drum brake.

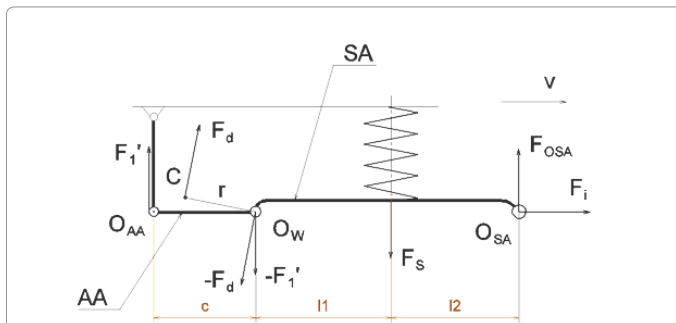


Figure 9: Forces acting on the suspension arm and the auxiliary arm-disk brake.

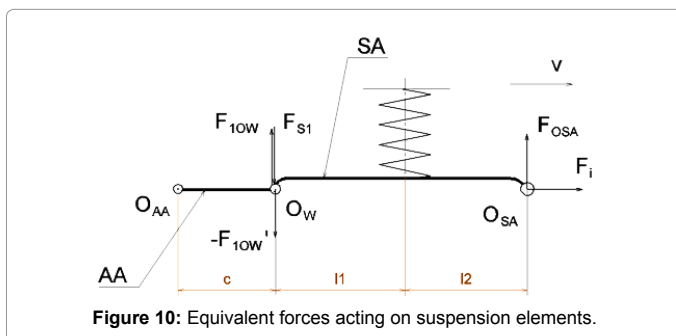


Figure 10: Equivalent forces acting on suspension elements.

the same moment $M = F_1' \times c$, and acting at the ends of the AA. The force F_1' is determined by: $F_1' = M/c = F_d \times r/c$. The couple $(F_1', -F_1')$ tends to lower the auxiliary arm AA and suspension arm SA, so that the force $-F_1'$ acts on the AA and on the SA downwards. On the other side the car body will be raised in the corresponding extent by acting of the force F_1' upwards to the car body, via SL. It is achieved in that way that the auxiliary arm AA will rotate relatively about the wheel axle O_W . Lowering of the axle O_W will thus compensate the reduction of adhesion force of the wheel with the ground surface. A magnitude of lowering of SA and AA is determined by the intensity of braking i.e. by force F_d determined as $F_d = -F_{fr} = F_N \times \mu$, where μ is a friction coefficient and $F_N = f(p)$ is normal force with which the calliper acts on the disc. Pressure of the working fluid “p” is the basic parameter of the braking process, and it is determined by a strength of the driver’s pressing to the brake pedal in simple version, or by action of ABS (anti-lock braking system) in modern systems. It is necessary to note here that the ABS is working independently of this matter, it only means that the ABS will work at a higher level of the down pressing force to the ground surface, and further ensuring that skid will not happen. That means that the braking will be more efficient and the braking distance will be less in total. So, this analysis should be considered as a natural way of increasing the braking efficiency and independently of existence of the ABS. All the forces mentioned are steady, they relate to the steady braking process, but this analysis will determine the character of behaviour of the suspension system, or precisely the down pressing force, in transient process of braking (starting and ending of the process).

In modeling the dynamic process, the force acting on the suspension arm will be assumed that acts in vertical plane through the wheel axle O_W , and will be: $F_1 = F_1' \times (c+l)/l$, where $l = l_1 + l_2$ (Figure 9). Since the point O_{SA} is used as a reference point, then the forces F_1 and $-F_1'$ can be replaced by new forces: F_{10W} and $-F_{10W}'$ (Figure 10),

which will be: $F_{10W} = F_1 \times c/l$, and $-F_{10W}' = -F_1' \times c/l$. Also, the stiffness of the spring is transferred (reduced) to the point O_W , thus being: $k_{S1} = k_S \times l_2/l$, and $F_{S1} = -k_{S1} \times d_{S1}$. This simplification is reasonable, and does not influence the character of process calculation. So the new forces scheme will be as it is shown in the Figure 10. Another simplification will also be introduced. It is obvious from the figures that the motion of the arms (SA and AA) is rotational, but the rotation can be replaced by vertical movement in limited amount, thus simplifying the model. This neglecting of small difference in character of movement, and moments of inertia, is justified by a fact that the main goal of this analysis is to determine the difference between the present systems of the suspension and the new proposed system.

The effect of change in down pressing force and behaviour of the rear end of the vehicle can be tested by placing the vehicle on rollers (rear wheels), thus showing the difference between an existing suspension and the new concept. The rollers should be of smooth surface, in order of less interference with wheels behaviour.

Dynamical Model

In order to perform a numerical analysis of the braking system behaviour, the dynamical model of the wheel suspension system has been defined, which represents two degrees of freedom (DOF) movement (movement of the wheel axle and the car body movement), and includes the basic parameters of the system. There are, of course, various full car models recently developed [2], but no one treats this particular problem of braking. Therefore, instead of adapting some model, a simplified half-car model is used, with supposition that there is no rotation of the car body. This is justified with the character of this analysis-main goal is to determine the difference between the two ways of suspension and to emphasize the advantages of the new system. Certain inaccuracy of the model does not change the result of the analysis. Figure 11 represents such a dynamic model (existing suspension), where the wheel has an elastic connection with the surface (tyre) and the car body (spring). It is presented the damping too, and the linear hydraulic model has been adopted as a damping model. Quantities that are included in the dynamic analysis are:

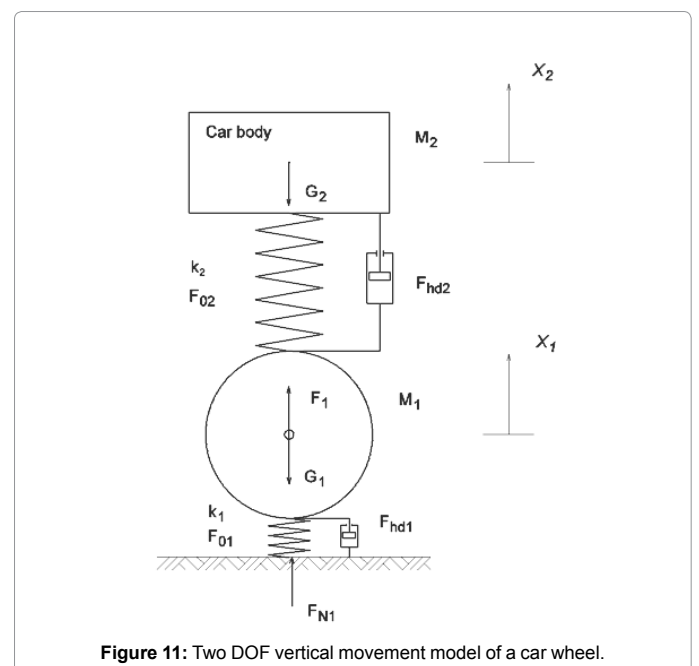


Figure 11: Two DOF vertical movement model of a car wheel.

M_1, M_2	The wheel axle mass, car body rear end mass
G_1, G_2	The wheel axle weight, car body rear end weight
k_1	Deformation stiffness of the tyre
F_{01}	Balance force of the elastic deformation of the tyre
F_{hd1}	The damping force at tyre deformation
X_1, X_2	Wheel movement coordinate, car body rear end movement coordinate
V_1, V_2	Wheel velocity, car body rear end velocity (vertical)
k_2	Spring stiffness
F_{02}	The equilibrium force of the spring elastic deformation
F_{hd2}	Damping force of the relative wheel and car body movement
F_1	Component of the force couple reduced to the point OW, for existing suspension system
F_{10W}	Force acting to the suspension arm upwards, for a new suspension system
$-F_{10W}'$	Force acting to the auxiliary arm downwards, for a new suspension system
F_{N1}	Normal pressure force of the wheel on the ground surface

Notes:

The forces G_1+G_2 and F_{01} , as well as G_2 and F_{02} , are balanced and therefore can be omitted from the equations, but here have been intentionally retained.

The effect of weight redistribution (in both cases of suspension) has been taken into account through reduction of the weight of the rear end of the vehicle, since only vertical loads are considered in this analysis.

The output parameters of the numerical model of movement are vertical movement of the wheel and the car body rear end and the normal pressing force of the wheel on the ground surface.

Existing suspension

The dynamic model of the system is described by four linear differential equations (1-4) and the corresponding algebraic equations.

$$dV_1/dt = (F_1+F_{01}-(G_1+G_2)-(k_1 \cdot X_1 + k_2 \cdot (X_1-X_2)) + F_{hd1} + F_{hd2})/M_1 - Grav_1 \tag{1}$$

$$dX_1/dt = V_1 \tag{2}$$

$$dV_2/dt = (F_{02}-G_2+k_2 \cdot (X_1-X_2) - F_{hd2} + G_d)/M_2 - Grav_2 \tag{3}$$

$$dX_2/dt = V_2 \tag{4}$$

Where:

$$F_{hd1} = -V_1 \cdot k_{hd1} \tag{5}$$

$$F_{hd2} = -(V_1-V_2) \cdot k_{hd2} \tag{6}$$

k_{hd1} Damping coefficient of tyres

k_{hd2}

Damping coefficient of the wheel damper, different for directions of shrinking and expansion

$$Grav_1=0, \text{ for } X_1 \leq 0, Grav_1=g \cdot (F_{10} - k_1 \cdot X_1)/(M_1+M_2), \text{ for } X_1 > 0 \tag{7}$$

$$Grav_2=Grav_1 \tag{8}$$

$g=9.806 \text{ m/s}^2$ Earth gravity acceleration

$G_d = G_B' - G_B$ Amount of rear end weight reduction, due to effect of weight redistribution.

G_B Weight of the rear end of the vehicle

G_B' Reduced weight of the rear end of the vehicle in the process of braking

$G_d=0$ In local problem analysis (partial influence of the suspension).

The pressure force on the surface generally is determined by the relation:

$$F_{N1} = k_1 \cdot X_1 - F_{01} \tag{9}$$

New suspension

For a new suspension adequate equations have been obtained by replacing the equations 1,3,7 and 8 by equations 10,11,12 and 13:

$$dV_1/dt = (-F_{10W}' + F_{01} - (G_1+G_2) - (k_1 \cdot X_1 + k_2 \cdot (-X_2)) + F_{hd1} + F_{hd2})/M_1 - Grav_1 \tag{10}$$

$$dV_2/dt = (F_{10W} + F_{02} - G_2 + k_2 \cdot (X_1 - X_2) - F_{hd2} + G_d)/M_2 - Grav_2 \tag{11}$$

$$Grav_1=0 \tag{12}$$

$$Grav_2=0, \text{ for } X_1 \geq X_2, Grav_2=g \cdot (F_{20} - k_2 \cdot (X_2 - X_1))/M_2, \text{ for } X_1 < X_2 \tag{13}$$

Figure 12 represents a dynamical model for the new suspension system. Force F_1 i.e. forces F_{10W} and $-F_{10W}'$ occur as input parameters, which are modeled according to Figure 13. In addition, for the force F_1 (first case-existing suspension), two cases of braking intensity have been processed (mild and more severe), and one case for the case 2 (new suspension concept).

A Runge-Kutta 4th order numerical method was applied in program solution, in solving differential equations.

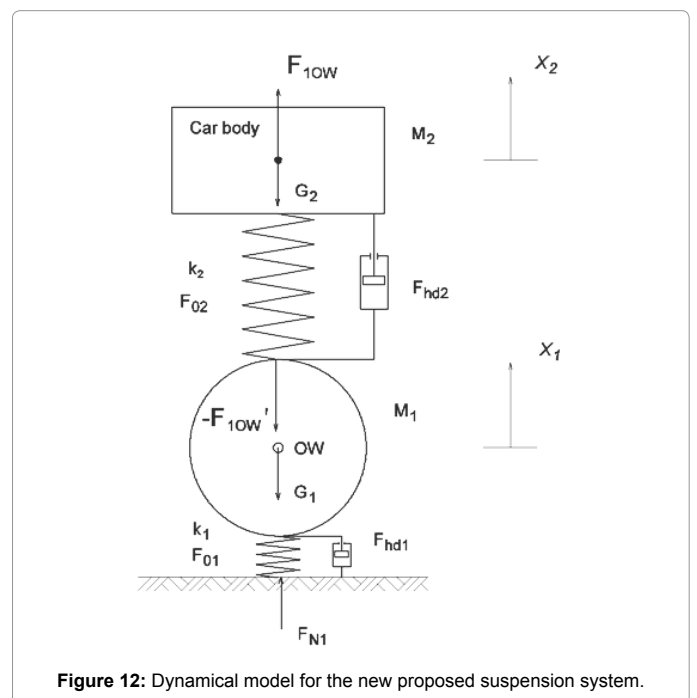


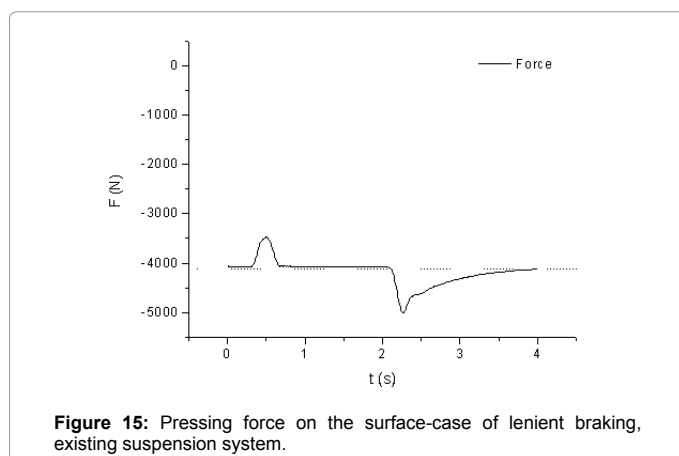
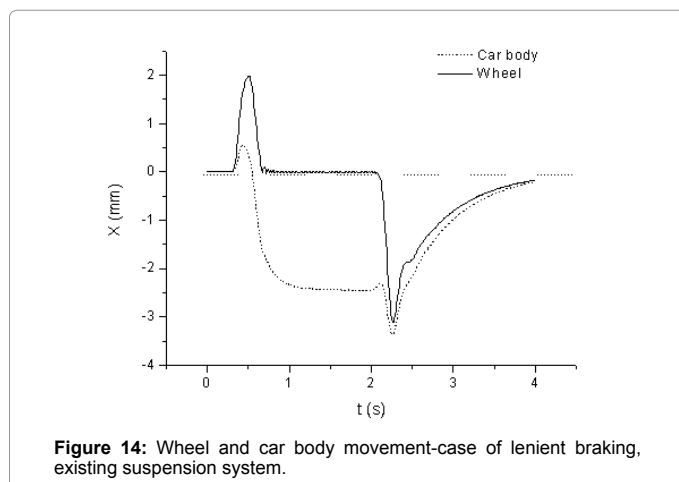
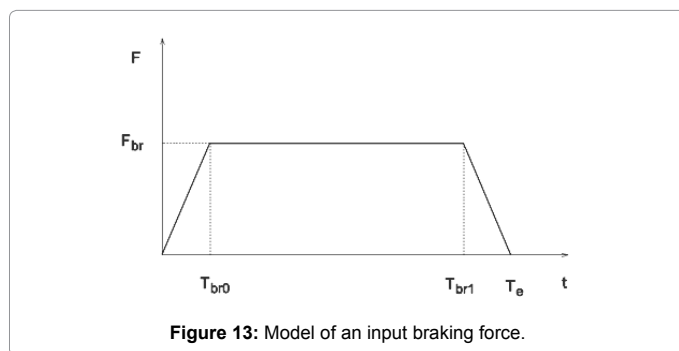
Figure 12: Dynamical model for the new proposed suspension system.

Results and Discussion

As an example for analysis, a vehicle with the mass of 1600 kg has been taken with the corresponding parameters ($M_1, k_1, k_{ind1}, M_2, k_2, k_{ind2}$). The input force has been modeled for two cases: lenient and more severe braking, which has been expressed through the difference in time T_{br0} and $(T_e - T_{br1})$, as well as through the force value F_{br} (Figure 13). Total time of braking was 2.1 s and 4.5 s. In both cases of the way of suspension, diagrams of the partial influence of the suspension and diagrams of total braking effect (with weight redistribution effect) were presented.

Existing suspension

Figures 14-17 refer to the case of partial influence of the suspension,



and Figures 18 and 19 refer to the case of actual driving, taking into account weight redistribution effect.

Figure 14 shows a diagram of the wheel and the bodywork movement in case of a more lenient braking. Figure 15 shows a diagram of the pressing force of the wheel on the surface for the same case. One can see that immediately after the commencement of the braking process, there is a reduction of the force applied to the surface, which is later normalized.

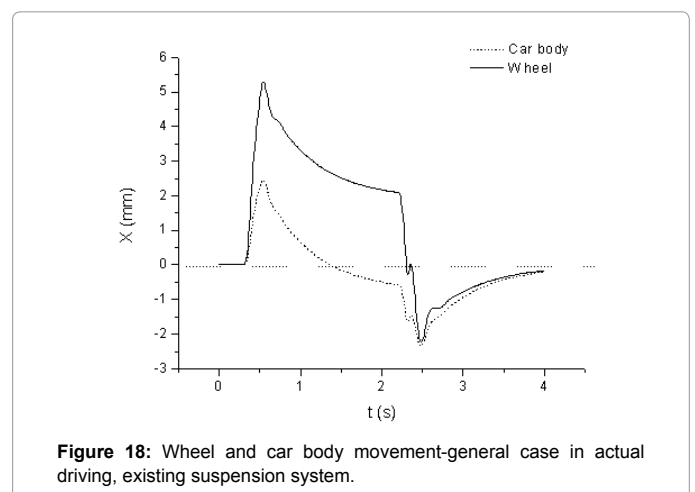
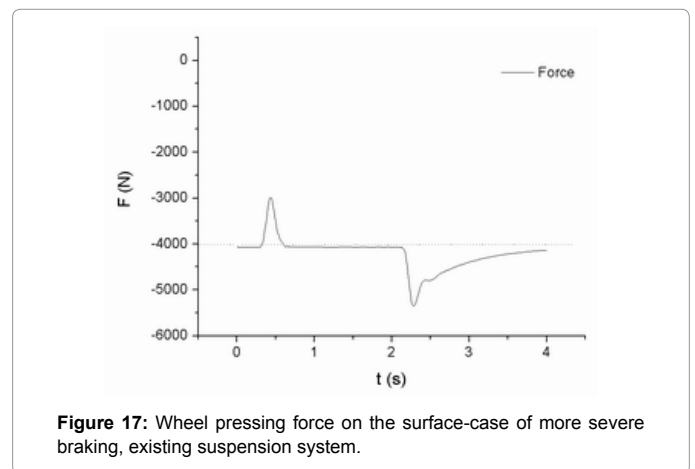
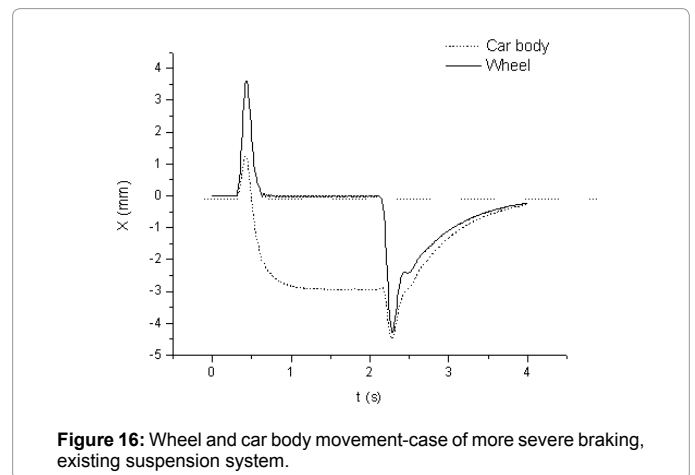


Figure 16 shows a diagram of the wheel and the bodywork movement in case of more severe braking. Figure 17 shows a diagram of the wheel down pressing force for the same case. The character of the wheel and the car body movement and the changes of the down pressing force is the same as in the previous case.

Figure 18 shows a diagram of the wheel and the bodywork movement in case of a more lenient braking and taking into account the redistribution of weight during braking, which corresponds to the actual driving. Figure 19 shows a diagram of the down pressing force of the wheel onto the surface for the same case.

New suspension

Figure 20 shows a diagram of the wheel and the bodywork movement in case of a more lenient braking for the new proposed suspension. Figure 21 shows a diagram of the wheel down pressing force on the surface for the same case. One can see that the down pressing force increases during braking.

Figure 22 shows a diagram of the wheel and the bodywork movement in case of a more lenient braking and taking into account the redistribution of car weight during braking, which corresponds to the actual driving. Figure 23 shows a diagram of the down pressing force of the wheel onto the surface for the same case.

Notes:

- Dotted horizontal line on the diagrams means initial level of the

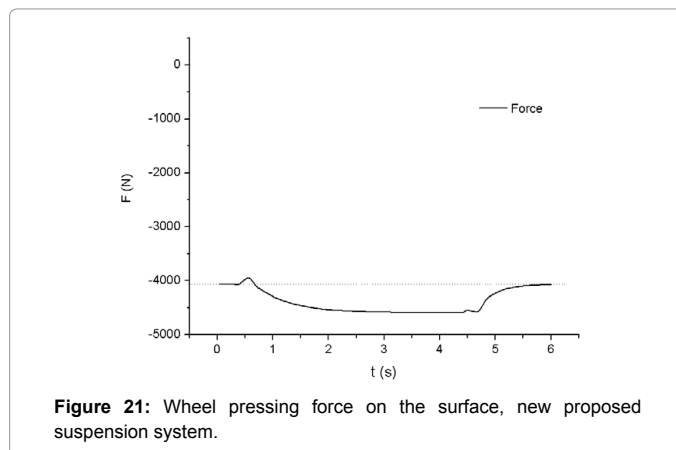


Figure 21: Wheel pressing force on the surface, new proposed suspension system.

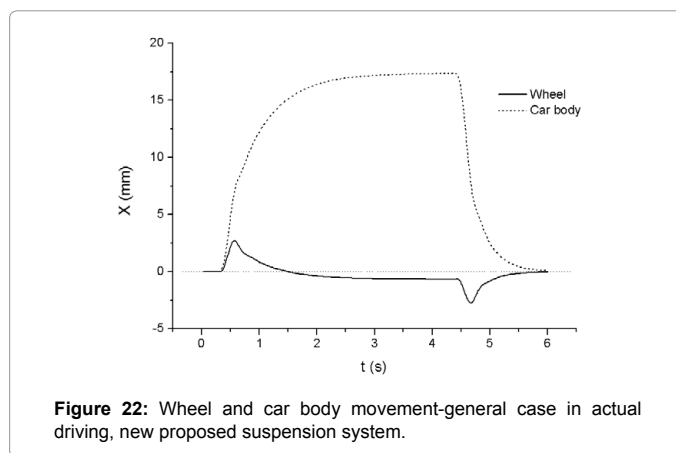


Figure 22: Wheel and car body movement-general case in actual driving, new proposed suspension system.

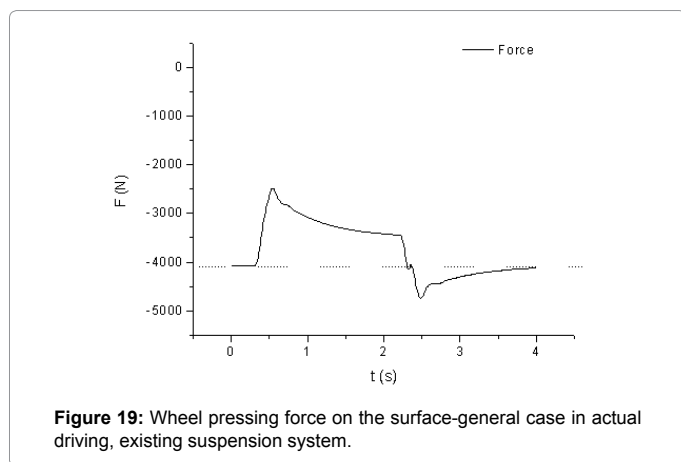


Figure 19: Wheel pressing force on the surface-general case in actual driving, existing suspension system.

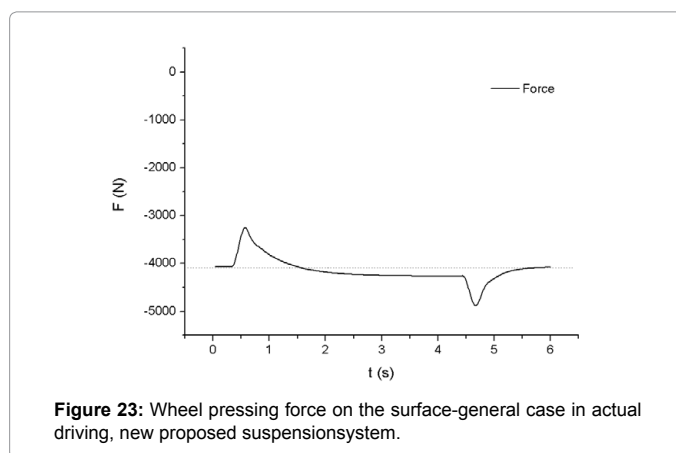


Figure 23: Wheel pressing force on the surface-general case in actual driving, new proposed suspension system.

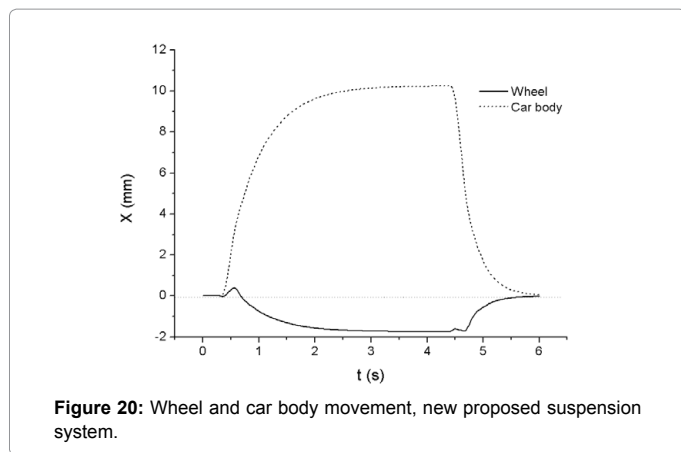


Figure 20: Wheel and car body movement, new proposed suspension system.

variable shown, so the change of the variable during braking process is relative to that level.

- The analysis has been done for the rear wheels axle dynamics, but the diagrams of force and movement show a half value i.e. for one wheel, because of plasticity of insight.

Comparison

Comparing the diagrams in Figures 14 and 15 with Figures 20 and 21, as well as Figures 18 and 19 with Figures 22 and 23, it is obvious that the case of existing suspension system has a natural imperfection that

results in reduction of down pressing force, particularly in the beginning of braking. On the contrary, in the case of new proposed suspension there is significant increase of down pressing force. The difference is greatest just in the first phase of braking, which is most important from the aspect of possible skidding, in case of abrupt braking. In this example that period is about 0.3 s. This is very important for the rear wheels, because the reduction of the down force in them occurs due to the redistribution of weight, so it is important that this force is not additionally diminished, but if possible maintained or increased. In this example, for mild intensity of braking, the difference between down pressing force, for the two cases of suspension, is about 31% in peak (beginning of braking), and about 23% in steady braking, after 2 s from the start of braking. Mean value of the force during 2.1 s of braking is greater for 21%. This comparison is for real driving, where the complete effect of weight redistribution is taken into account. This means that the increased contribution of the rear wheels in reducing the braking distance of a car will correspond to about 21%, and increased contribution in reducing the risk of skidding of rear wheels to 31% of their nominal participation (in existing systems). The cited percentage of the relative increase of down pressing force is much greater for greater forces of braking, because in existing suspension it is getting reduced, and in the new suspension it remains at maximum.

Complete analysis is also valid for drum brakes.

Conclusion

The suspension with backward oriented oscillating arms, on which

the wheel axle is attached, is most common, especially on the light and short cars. This analysis has dealt with that type of suspension and its specifics in the braking process, but it is also applicable to all kinds of rear suspension on vehicles, also including aircraft. The analysis has shown that the existing suspension concept has certain imperfection as regards quality of braking. The reduction of down pressing force of the wheel to the ground is undesirable, but in fact it occurs. The problem was treated as a local one (limited to the suspension) that emphasizes the source of problem, and a real one, taking in account occurrences in actual driving. This is all regardless of working of the ABS. The advantage of the new proposed concept is shown through the diagrams of down pressing force, as well as the diagrams of the wheel motion. The amount of the force increase will depend on design parameters of suspension system. Relative increase of the down pressing force will also depend on the braking force. Therefore, as an unambiguous conclusion of this analysis, comes the stance that the natural reliability of rear wheels braking on cars could be improved by introduction of the new proposed auxiliary suspension system.

References

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