

## Calculation of the Air Displaced by a Wing

Landell-Mills N\*

Department of Environment and Development, Edinburgh University, Edinburgh, UK

### Abstract

This paper provides the theoretical framework to calculate the mass of air displaced downwards by the wings of an airplane in flight; and whether this provides insight into the physics of flight.

Example calculations are used to demonstrate: (i) The key factors that affect the amount of air displaced; including wing shape and aircraft momentum. (ii) The air displaced and lift generated by a wing can be estimated using the equation:  $F=m/dt \times v$ . (iii) An airplane can displace a mass of air downwards equal to its own mass each second; Which is consistent with Archimedes principle of buoyancy being applied each second.

This paper is mostly theoretical. The next step would be to verify this methodology and conclusions by empirical experimentation on a real aircraft in realistic conditions.

**Keywords:** Physics; Buoyancy; Lift; Flight; Mass; Wing; Airfoil

### Introduction

#### Executive summary

In summary, example calculations show that a Harrier with a wingspan of 9.4 m, flying at 222 m/s, will fly through 2,087 m<sup>3</sup> of air each second; which is equal to about 2,500 kg/s of air; given a standard air density of 1.2 kg/m<sup>3</sup>. This includes all the air 0.5 meters above and below the wing. If this 2,500 kg/s of air is displaced down 4 meters in one second, then a total of 10,000 kg m/s<sup>2</sup> of air will be displaced in one second. This is sufficient lift for a 10,000-kg aircraft such as the Harrier (Figures 1 and 2).

These calculations were made using academic reports on the air displaced down by wings, interviews with pilots; as well as videos and photographs obtained from public sources (such as YouTube) of aircraft flying a few meters above the ground.

This paper starts with the observation that all airplanes in flight push air downwards. Therefore, a plane will displace a mass of air equal

to their own mass over some time period (which this paper estimates to be each second). This implies that aircraft achieve buoyancy every second to fly.

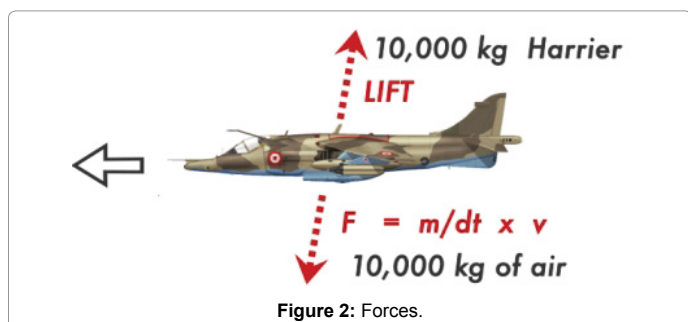
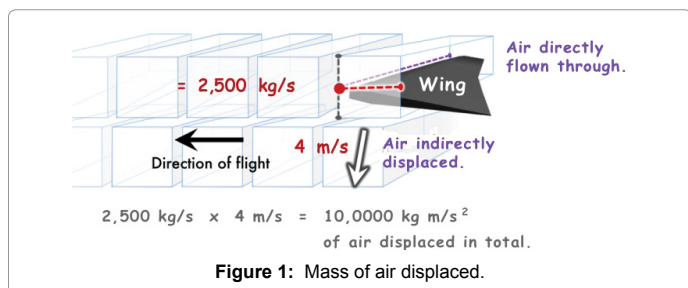
The mass of air displaced includes the air directly and indirectly displaced downwards. (i) Air directly displaced includes: The mass of air directly flown through by the wings, that are affected by the wings, per unit time (seconds). (ii) This air directly displaced then indirectly displaces more air, depending on how far down it is pushed in one second (Figure 1).

An effective method to calculate the mass of air displaced is using the equation:  $F=m/dt \times v$  (which is consistent with Newtons 3<sup>rd</sup> law of motion;  $F=ma$ ). Where the force required to displace the air down is equal to: The mass of air directly displaced per unit time ( $m/dt$ ), times the vertical velocity downwards of this mass of air (i.e. the mass of air indirectly displaced) [1].

Coincidentally, this force used to displace air down, is also equal to the force required to fly. So:  $Lift=F=m/dt \times v$ . This makes sense, as the force from the wings which is used to push the air down, has an equal and opposite force (lift) that pushes the plane up (Figure 2).

The equation:  $F=m/dt \times v$ ; is also the same basic equation that can be used to calculate the upward thrust generated by the gasses pushed down from the engines of a rocket.

The mass of air displaced by a wing each second, is affected by aircraft momentum and wing shapes. For example, gliders and fighter jets displace air very differently. Aircraft momentum determines how much air a plane can displace, through how much energy the plane has available to transfer to the air via the wings. Differences in wings shape (eg. wingspan and wing depth) determine how this air is displaced down [2].



\*Corresponding author: Landell-Mills N, Department of Environment and Development, Edinburgh University, Edinburgh, UK, Tel: 0033-638773940; E-mail: [nicklm@gmx.com](mailto:nicklm@gmx.com)

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### Why is this Important?

Currently there is scant research and analysis in this area. There is no significant research into how much air planes push down in flight or what significance this has.

Historically, research of lift has focused on analyzing how air flows around wings. But for the most part, research has ignored how much air is displaced downwards. This is a critical oversight. By comparison, when engineers design boats they consider how the boat displaces water when it sails, and that the boats displace enough water to stay afloat.

Also, research of lift tends to ignore how energy that is transferred from the wings to the air, to achieve lift. This analysis of the mass of air displaced by the wings, remedies this.

Currently Archimedes principle of buoyancy is selectively applied only to static objects such as boats and hot air balloons. Buoyancy is not applied to moving objects such as planes. This paper updates Archimedes principle to include the dimension of time, which then allows the principle of buoyancy to be applied to moving objects such as airplanes.

If it can be subsequently shown that all airplanes displace a mass of air downwards equal to their own mass, over the same time period (such as one second), then this will add significantly to the understanding of flight and wing design. It will also support the assertion that aircraft need to achieve buoyancy in the air each second in order to remain airborne. In turn, this will alter how pilots are trained and how airplanes are designed; to achieve better aviation safety and efficiency [3].

### Controversial and Diverging Hypotheses

Discussions with over 20 academics working (or retired) in aeronautical engineering at reputable universities (eg. Bristol, Cambridge, Michigan, Sydney, Stanford), confirmed that there is no consensus on the physics of how planes fly. Nor is there any definitive proof for any theory of flight (a proper scientific experiment on a real plane in realistic conditions). Therefore, it is worth questioning current theories of flight. Even NASA [4,5] doesn't offer an explanation for how planes fly.

### Background Information

#### Definitions

**Buoyancy:** Buoyancy is the principle of physics whereby an object displaces a mass of fluid (e.g. water or air) equal to its own mass to float.

**Dynamic buoyancy:** Dynamic Buoyancy is an active form of

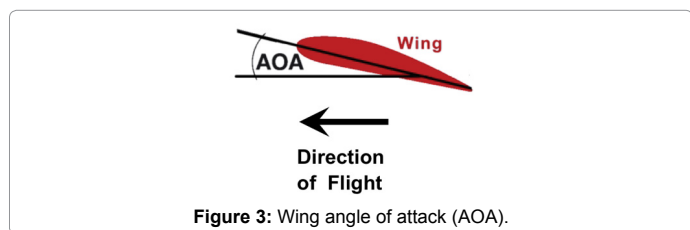


Figure 3: Wing angle of attack (AOA).

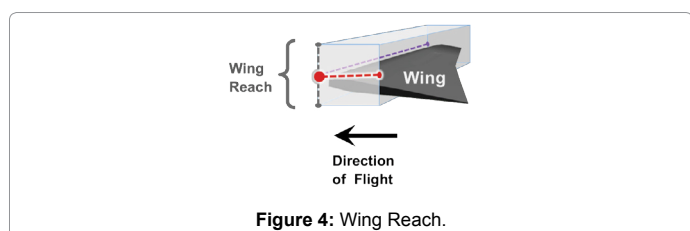


Figure 4: Wing Reach.

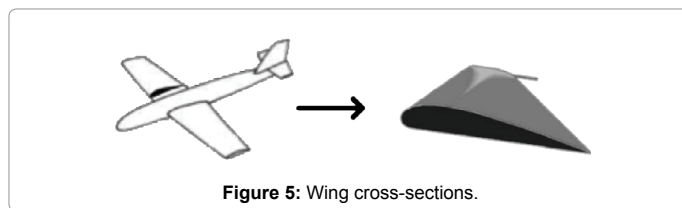


Figure 5: Wing cross-sections.

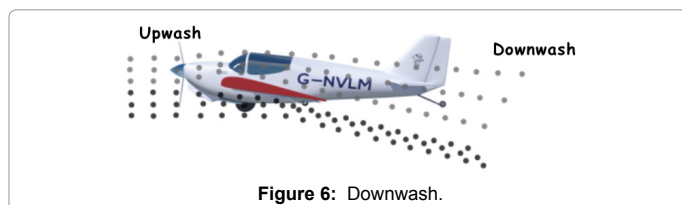


Figure 6: Downwash.

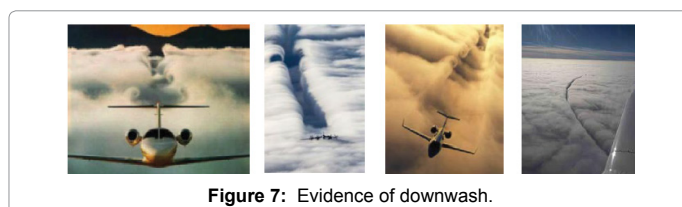


Figure 7: Evidence of downwash.

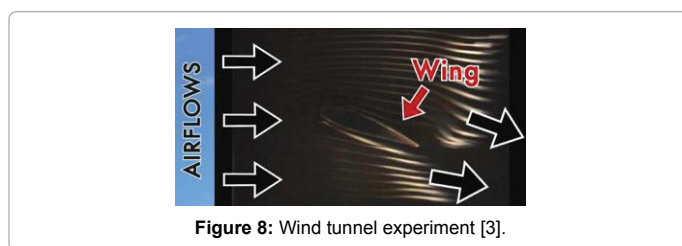


Figure 8: Wind tunnel experiment [3].

buoyancy where a fluid (air or water) needs to be constantly displaced downwards to allow an object to maintain buoyancy. For example, planes achieve 'dynamic' (active) buoyancy as the wings are constantly pushing air down [4-9].

**Wing AOA:** The Wing Angle of Attack (AOA) is the angle between the wings chord line and the direction of flight (Figure 3)

**Wing reach:** Wing reach is the vertical distance away from the wing, that the wing influences the air (Figure 4).

### Wing Diagrams in this Paper

In this paper, the wing diagrams are shown as a cross section of the wing.

#### Downwash- A quick review

Wings with a positive AOA push air slightly upwards in front of their path (upwash), and displace air a long way downwards behind them (downwash).

Note that in general, wings need a positive angle of attack (AOA) towards the direction of flight to generate enough lift to fly. A plane cannot fly without a positive AOA. The greater the wing AOA, the more air that it displaces downwards.

Downwash is evident behind planes flying over clouds as shown in Figures 5-10 (wind tunnel); as well as the on the dust on the ground behind the low flying Harrier in Figures.

Upwash is relatively small compared to downwash. This is evidence

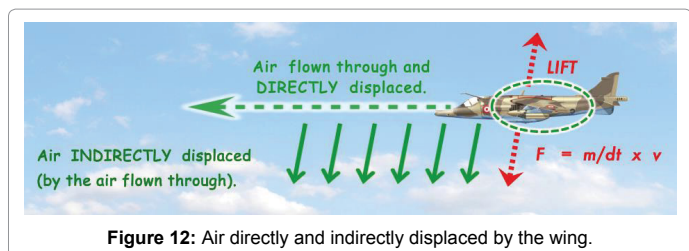
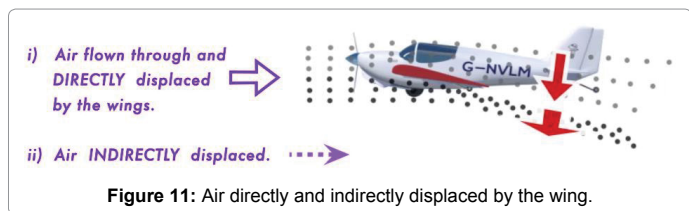
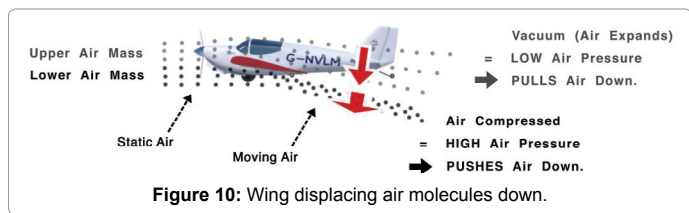
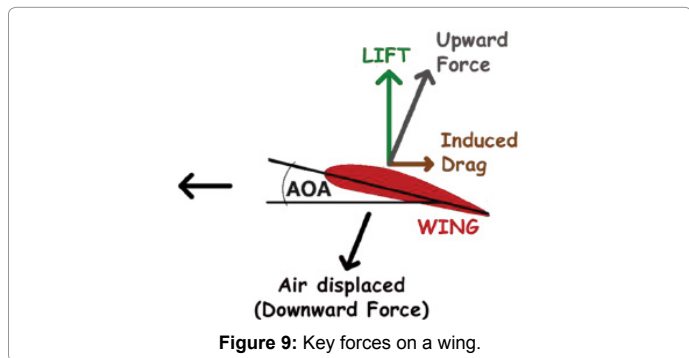
by the lack of displacement in air in front of the plane, compared to the significant amount of air displaced down behind the plane.

### How a wing pushes air down

The key forces on a wing are shown in Figure 9. In short, the wing pushes air downwards and slightly forward, creating an equal and opposite upward force. Most of this upward force results in vertical lift. But a small proportion of this upward force results in induced drag as some of the air is pushed slightly forwards.

It is beyond the scope of this paper to explain in detail how wings displace air down; beyond this brief description: The wing displaces static air downwards and slightly forwards, as shown in Figure 10. The wing splits the air mass into two separate and distinct airflows; above and below the wing.

- (1) The underside of the wing faces the direction of flight and compresses the lower air mass under the wing. This produces high air pressure and pushes the lower air mass down. The equal and opposite force generated under the airflow pushes the plane up.
- (2) The topside of the wing faces away from the direction of flight. As this air expands behind the wing, this produces low air



pressure. This pulls the upper air mass downwards. This is helped by the Coanda Effect; whereby the curved top-side of wing re-directs the upper airflow downwards, along the direction of the top-side of the wing.

### Wings directly and indirectly displace air

The total air displaced by the wing, includes (Figures 11 and 12):

- i) The air directly displaced by the wing, (that the plane flies through); and
- ii) The air indirectly displaced (i.e. displaced by the direct airflows pushed down).

### Key references- downwash

**Stick and Rudder (1944):** The book: “Stick and Rudder” by Wolfgang Langewiesche in 1944; which is famous among pilots for its accurate, practical and common-sense advice on how to fly a plane well. In Chapter 1 the book states: The main fact of heavier-than-air-flight is this: the wing keeps the plane up by pushing air down. It shoves air down with the bottom surface, and it pulls air down with the top surface. But the really important thing to understand is that the wing, in whatever fashion, makes air go down. In exerting a downward force on the air, the wing receives an upward counterforce- by the same principle, known as Newton’s law of action and reaction, as well as: That’s what keeps a plane up. Newton’s law says that if the wing pushes the air down, the air must push the wing up [5].

**Understanding flight:** Extracts from the book Understanding Flight; Chapter 1: ‘The Principles of flight’. The book attributes lift to air being displaced downwards that then causes low air pressure on the top of the wing; which pulls the plane up [6].

- “The air behind the wing is going almost straight down the air has, in fact, a slight forward direction.” i.e. This is the downwash.
- “In the simplest form, lift is generated by the wing diverting air down, creating the downwash.”
- “From Newton’s second law, one can state the relationship between the lift on a wing and its downwash: The lift of a wing is proportional to the amount of air diverted per time times the vertical velocity of that air.”
- The lift of a wing is proportional to the amount of air diverted (down) per time times the vertical velocity of that air.”
- where the air is initially at rest and the wing is moving, the air is moving almost straight down after the wing passes.”
- “The key thing to remember about lift is that it is a reaction force caused by the diversion of air down”.
- “The physics of diverting the air down are expressed by the lowering of the pressure on the top of the wing producing the lifting force. The lift of a wing is proportional to the amount of air diverted times the vertical velocity of that air” i.e.  $LIFT = \text{mass of air diverted down} \times \text{vertical velocity of that air}$ . “Wing Reach”-A new term.

“Wing Reach” is the vertical distance away from the wing that the wing influences the air. It is the volume of air ( $m^3$ ) displaced by each  $1 m^2$  of effective wing area. Wing Reach depends on things such as the wing’s angle of attack and the wing shape. The greater the Wing Reach, then the greater the volume of air displaced by the wings (Figure 13).

For example, a Wing Reach of 1.0 m means that: 1.0 m<sup>2</sup> of wing area can displace at least 1.0 m<sup>3</sup> of air (at least 1.0 m down). A 1.0 m Wing Reach is calculated as: 0.5 m both above and below the wings. Note that 0.5 m is about knee height of an adult. As shown in Figure 4: If a wing has a 1.0 m Wing Reach, and a wing area of 5 m<sup>2</sup>, then it can displace 5 m<sup>3</sup> of air each meter flown, down at least 1 m. (1 m × 5 m<sup>2</sup>=5 m<sup>3</sup>).

The wings affect the air unequally across the wingspan; with the weakest influence being at the wing-tips. Also, the air closest to the wings are affected the most aggressively by the wings. Different types of wings and different wing AOA will produce different Wing Reach. For example, deeper wings will have a greater wing depth. But this is beyond the scope of this paper. Wing Reach can be estimated using smoke in wind tunnel experiments, or condensation on the top of wings in Figure 14 (Wind tunnel) and Figure 15 (Wing condensation).

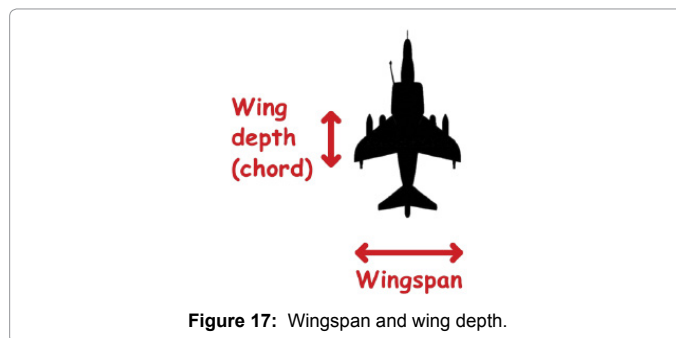


Figure 17: Wingspan and wing depth.

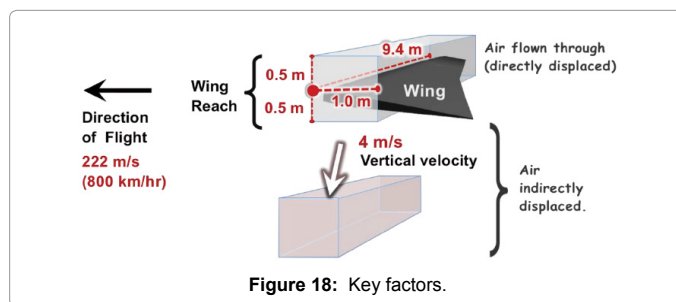


Figure 18: Key factors.

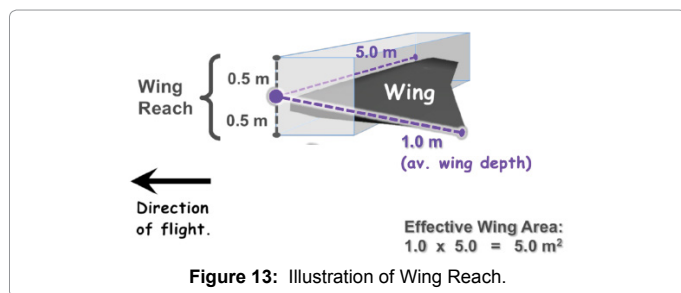


Figure 13: Illustration of Wing Reach.

### Wing Characteristics and Key Functions

Wings essentially have two key functions: To “catch” air, then to displace this air downwards. The wingspan and (wing AOA and wing shape), primarily determines how much air is “caught.” The wing depth (chord), aircraft momentum and wing AOA primarily determine how far down the air is displaced (Figure 16 (Wing characteristics) and Figure 17 (Wingspan and wing depth).)

Note that engineers frequently discuss the difference in wings in terms of the aspect ratio.

Where: Aspect ratio=Wingspan/Wing depth.

The wings displace air down unequally across the wingspan. Air closest to the fuselage will be pushed down the most, as this is where the wings are deepest (from front to back).

### Materials and Methods

These calculations were made using academic reports on the air displaced down by wings, interviews with pilots; as well as videos and photographs obtained from public sources (such as YouTube) of aircraft flying a few meters above the ground.

### Methodology

(i) First, the volume of air that a wing directly flies through and “catches” or directly displaces each meter flown, is estimated. This volume of air is based on the wingspan and Wing Reach (Figure 18) (key factors).

Volume of air flow through (directly displaced) each meter  
 =1 m × Wingspan × Wing Reach

(ii) Multiplying (i) by aircraft speed provides the volume of air displaced each second.

(iii) Then multiplying (ii) by the Air density provides the mass of air directly flown through each second.

Volume of air × Air density=Mass of air displaced

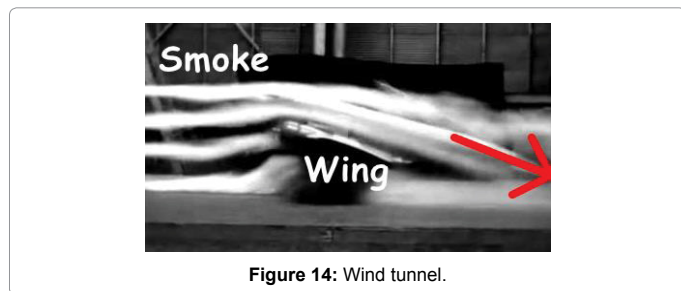


Figure 14: Wind tunnel.



Figure 15: Wing condensation.

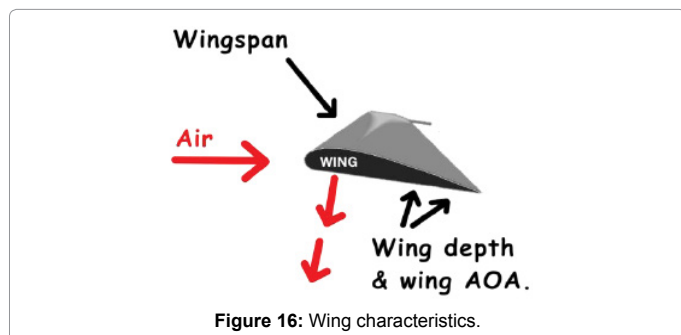


Figure 16: Wing characteristics.

(iv) Multiplying (iii) by the vertical velocity downwards of this mass of air, provides the total mass of air displaced. This relies on the equation:  $F=m/dt \times v$ .

**Example calculation**

Using the example of a Harrier AV8 jet:

(i) First, the volume of air that a wing directly flies through and catches or directly displaces each meter flown, is estimated. This primarily depends of the wingspan and Wing Reach. Wing Reach is the vertical distance away from the wing that the wing influences the air.

A Wing Reach of 1.0 meters is assumed; this is 0.5 meters above and below the wings. This Wing Reach is relatively conservative, as it is well below what empirical evidence suggests; As shown by images of condensation of the wings of fighter jets and commercial airliners (Figures 19 and 20). Note that condensation only appears in the low-pressure areas above the wing, never below the wing.

Note that Wing Reach depends much on the wing shape and wing AOA.

Volume of air flow through (directly displaced) each meter=1 m × Wingspan × Wing Reach



Figure 19: Wing condensation-fighter jets.

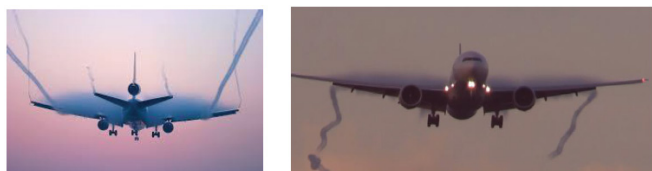


Figure 20: Wing condensation-commercial airliners.

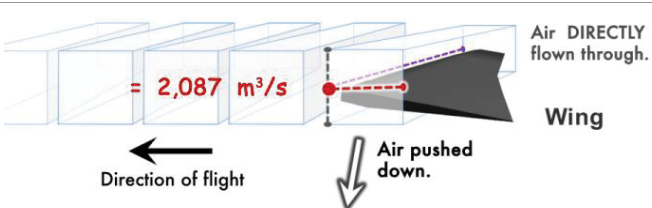


Figure 21: Volume of air directly displaced each second.



Figure 22: Low flying Harrier.

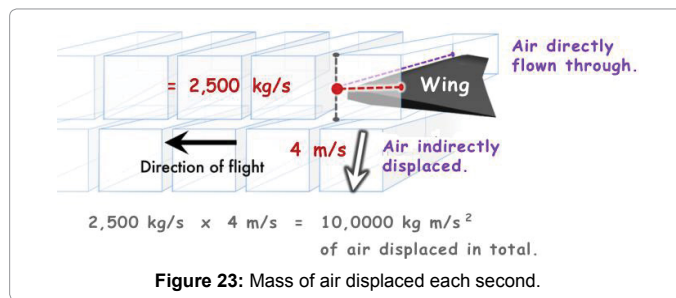


Figure 23: Mass of air displaced each second.

$$9.4 \text{ m}^3/\text{m}=1.0 \text{ m} \times 9.4 \text{ m} \times 1.0 \text{ m}$$

Then this 9.4 m<sup>3</sup> volume of air directly displaced each meter flown, is multiplied by the aircraft speed, to estimate the volume of air directly displaced each second (Figure 21).

Volume of air flow through (directly displaced) each second=Volume of air flow through (directly displaced) each meter × aircraft speed

$$2,087 \text{ m}^3/\text{s} (\text{approx.})=9.4 \text{ m}^3/\text{m} \times 222 \text{ m/s}$$

(ii) This 2,087 m<sup>3</sup>/s volumes of air directly displaced each second, is converted into mass of air directly displaced each second using the standard density of air [7]; of 1.2 kg/m<sup>3</sup>.

Mass of air directly displaced per second=Volume of air directly displaced per second × Air Density

$$2,500 \text{ kg/s} (\text{approx.})=2,087 \text{ m}^3/\text{s} \times 1.2 \text{ kg/m}^3$$

(iii) Then it is estimated how far this 2,500 kg/s mass of air directly displaced is pushed down each second. This is effectively the amount of energy that is transferred from the plane to the air to generate lift.

It is estimated that the Harrier can displace this air down 4 meters in one second; which is an average vertical velocity of 4 m/s (or 14.4 km/hr). This assumption is based on videos of low flying Harrier taken in Afghanistan which show dust on the ground, caused by the aircraft's downwash (Figures 22 and 23).

Each 1 kg/s of air flow through, that is displaced down 1 meter, will indirectly displace 1 kg/s of air. So, if 1 kg/s of air flow through by the wings, is displaced down 4 meters, then the wings will displace 4 kg/s of air in total.

Therefore, as the Wing Reach is 1.0 meters; the mass of air displaced can be written as a multiple of 1.0 m Wing Reach:

Mass of air displaced per second= m/dt × distance air is displaced down

$$10,000 \text{ kg/s}=2,500 \text{ kg/s} \times 4$$

This is an adaptation of the equation:  $F=m/dt \times v$

Where:

F=Force (lift)

m=Mass of air directly displaced by the wings per second

dt=Per unit time (seconds)

v=Vertical velocity down of this mass of air

Now, as discussed in the next section, the equation for lift and the mass of air displaced can also be written as:

$$\text{Force (Lift)}=\text{Mass}/\text{time} \times \text{velocity}$$

$$F = m/dt \times v$$

$$10,000 \text{ kg m/s}^2 = 2,500 \text{ kg/s} \times 4 \text{ m/s}$$

$$\text{Units check: (kg m/s}^2) = (\text{kg/s}) \times (\text{m/s})$$

Note that the units of the total mass of air displaced are also those used for a force. This is the force required to displace the air.

### References- Distance Down that Air is Displaced

As a rule of thumb, most engineers and pilots consider that a plane can push the air that it flies through, downwards up to one wingspan in distance. This distance is significantly greater than the 4 meters assumed in this analysis. The plane pushing air down is particularly noted in ground effect, when a plane flies a short distance above the ground. For example, the quote from the book Understanding Flight [6] (chapter 1, page 36 Ground Effect); (Ground effect is noted on) a wing as it comes to within about a wing’s length of the ground.

### Results

In summary, example calculations show that a Harrier with a wingspan of 9.4 m, flying at 222 m/s, will fly through 2,087 m<sup>3</sup> of air each second; which is equal to about 2,500 kg/s of air; given a standard air density of 1.2 kg/m<sup>3</sup>. This includes all the air 0.5 meters above and below the wing. If this 2,500 kg/s of air is displaced down 4 meters in one second, then a total of 10,000 kg m/s<sup>2</sup> of air will be displaced in one second. This is sufficient lift for a 10,000-kg aircraft such as the Harrier (Figures 23 and 24).

### References- Air Displaced by a Wing

Extracts from the book: ‘Understanding Flight [6]; Chapter 1, Principles of flight. Estimate of air displaced by a plane. This is provided for comparison purposes.

The book provides a back-of-the-envelope calculation to estimate that: “Thus a Cessna 172 at cruise is diverting about five times its own weight in air per second to produce lift.”

The quote in detail: “Take, for example, a Cessna 172 that weighs about 2,300 lb. (1,045 kg). Traveling at a speed of 140 mi/h (220 km/h) and assuming an effective angle of attack of 5 degrees, we get a vertical velocity for the air of about 11.5 mi/h (18 km/h) right at the wing. If we assume that the average vertical velocity of the air diverted is half this value, we calculate from Newton’s second law that the amount of air diverted is on the order of 5 (English) tons per second. Thus, a Cessna 172 at cruise is diverting about five times its own weight in air per second to produce lift.”

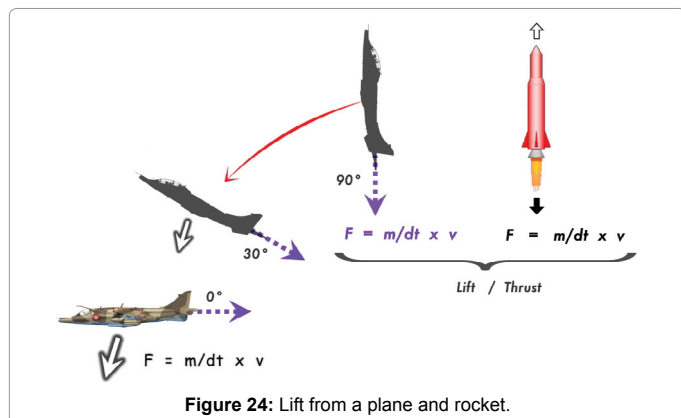


Figure 24: Lift from a plane and rocket.

### Discussion

#### Additional comments and explanations

1. Given that 10,000 kg/s of air is displaced in total; and that 2,500 kg/s of air is directly flown through and displaced. Then it would be accurate to say that 7,500 kg/s of air is indirectly displaced. But note that the 2,500 kg/s air directly flown through is not automatically displaced. As described, it is simply the air flow through by the wing. If the wing has a zero or negative angle of attack, this air flow through might not actually be displaced down.

2. The further each 1 m<sup>3</sup> (or each 1.2 kg) of air is displaced down by the wings, then the more air mass in total that is displaced, and the more energy that is transferred from the wings to the air.

3. The distance that each m<sup>3</sup> of air is displaced down by the wings, depends primarily on: the angle of attack (AOA), the shape of the wing, wing depth, wing vortices, and the aircraft’s momentum (the velocity and mass of the aircraft). The greater the aircraft momentum, the easier it will be for the wings to push air further down; as they will have more energy available to transfer.

4. The presence of descending wing vortices (spiraling air), will also impact the amount of air displaced and how it is displaced. But this is beyond the scope of this paper. Vortices should be included in the calculation of the mass of air displaced. Contrary to the conventional view of aircraft vortices, vortices can have either a negative or positive impact on the mass of air displaced depending on the circumstances. It is noted that birds are known to use vortices to boost lift. [9]

5. This analysis works well when the wing has a 1-meter Wing Reach, when displacing the air. If the Wing Reach is not 1 meter, then the air displaced has to be adjusted.

- In the example provided above, a Wing Reach of 1 m means that every 1 m that 1 kg/s of air is displaced down, will then indirectly displace 1 kg/s of air. 2,500 kg/s displaced down 1 m will indirectly displace 2,500 kg/s off air.

- But, taking the same assumptions in the example above, except the wings have a 2-meter Wing Reach, and the plane flies half as fast (111 m/s), to displace the same amount of air of 2,500 kg/s. Then this air needs to be displaced down 2 meters in order to displace the same 2,500 kg/s of air. In summary: The volume of air flow through by the wings is twice as high at 2 m. So, it needs to be displaced down twice as far as in the previous example; in order to displace the same volume of air as before.

This aspect of the calculating the amount of air displaced down, will need to be adjusted for in the formula for the vertical velocity that is displaced down.

$$v = \text{vertical velocity/Wing Reach}$$

$$\text{Therefore, a more precise equation would be: } F = m/dt \times v$$

Where:

$$F = \text{Force (lift)}$$

$$m = \text{Mass of air directly displaced by the wings per second}$$

$$dt = \text{Per unit time (seconds).}$$

$$v = \text{Vertical velocity down of this mass of air/Wing Reach}$$

This will ensure that the units for measuring vertical velocity and the mass of air displaced are consistent. i.e. That each 1 m/s of vertical velocity, will correspond to 1 kg/s of air being displaced.

1. Energy and the laws of physics. The aircraft's engines generate energy, which is used to create forward movement and push the aircraft forward. The wings then transfer some of this energy to the air; by pushing the air down. The equal-and-opposite force provides upward lift. There is no net loss or gain of momentum, mass or energy.

2. This paper estimates that Archimedes principle of buoyancy acts over a one second time period. i.e. An object must displace a mass of air equal to its own mass each second to maintain buoyancy. This one second time period is just an initial estimate and experimentation needs to be done to verify if this one second estimate is accurate. The actual time frame may be slightly shorter or longer, and not be exactly one second. But it will be close to one second.

The core idea proposed by this paper does not alter if experimentation shows that the time period is different to one second. The theoretical explanation for this one second time period is uncertain. Why one second; rather than 1.2 seconds or 0.8 seconds (for example), is unclear. There does not appear to be anything intrinsically or fundamentally special about this one second time period.

**New equation for lift:  $F=m/dt \times v$**

Using Newton 3<sup>rd</sup> law of motion [1,2]: Force=mass  $\times$  acceleration;  
 $F=ma$

Then assuming:

- Average force;
- Constant mass of air directly displaced down each second;
- Constant vertical velocity of this mass displaced down:  $dv=v$ ;

This means that the formula for acceleration can be re-stated:

Acceleration= $a=dv/dt=v/dt$

Therefore, the equation for Newtons 3<sup>rd</sup> law of motion can be re-written as follows:

$F=ma=m \ v/dt=m/dt \times v$

Therefore,

Lift= $F=m/dt \times v$

For the case of lift of an airplane:

$F=$ Force (lift)

$m=$ Mass of air directly displaced down by the wing.

ie. The mass of air that the wing flies directly through.

$dt=$ Change in time. (ie. Per unit time - seconds)

$v=$ Vertical velocity of this mass of air directly displaced downwards.

$dv=$ Change in velocity.

In summary, this paper applies this equation for lift above; where lift equals the mass of air displaced down per unit time (seconds), times their vertical velocity downwards of this air.

**Rockets and thrust**

Note that this is very similar to the equation for the thrust generated by a rocket [2]:

Thrust= $F=ma=dm \times dv/dt$

Thrust= $dm \times dv/dt$

This equation states that: Thrust equals the (changing) mass of gases ejected by the rocket, times their (changing) velocity, per unit time.

Then, making similar assumptions to those above for lift:

- Average force;
- Constant mass of gases ejected from the rocket; So:  $dm=m$ .
- Constant vertical velocity of the mass ejected. So:  $dv=v$ .

These hypothetical assumptions would hold if the rocket's engines were able to manage the fuel combustion accordingly.

Consequently, the equation changes as follows:

Thrust= $dm \times dv/dt=m \times v/dt=m/dt \times v$

Thrust= $m/dt \times v$

NASA [2] uses the same equation for thrust of a rocket; but call " $m/dt$ " the "mass flow rate".

Therefore, the equation used for lift in this paper, is also the same equation used for the thrust of a rocket. The only difference is that thrust refers to the mass of gasses ejected by the rocket (mostly CO<sub>2</sub> and H<sub>2</sub>O). Whereas the equation for lift, mass refers to air (which is mostly nitrogen and oxygen) displaced down (Figure 5).

This comparison between an airplane and rocket, for the equation for thrust / lift, is highlighted in Figure 5. For a plane in a vertical climb, only the engines produce lift. Here the wings provide no lift. Therefore, the equation for lift/thrust for both the rocket and plane are the same. ie.  $F=m/dt \times v$ .

As the plane's trajectory decreases from the vertical climb to a horizontal flight, the wings will generate an increasing proportion of lift. Also, correspondingly, the engines will provide a decreasing proportion of lift. However, the equation for lift remains the same. The critical change is that lift generation changes from the engines pushing exhaust gases (CO<sub>2</sub> and H<sub>2</sub>O) downwards, to the wings pushing air (nitrogen and oxygen) downwards. In principle, the physics remains the same.

**Four key assumptions**

This section highlights four critical assumptions made in this analysis to calculate the mass of air displaced by a wing. The data on velocity, wingspan, and mass of the aircraft; as well as air density are relative simple to obtain; The two key variables that are harder to estimate accurately, include:

- (i) Wing Reach (as already discussed).
- (ii) Vertical velocity that air is displaced down in one second.

Whereas, two other important assumptions include:

(iii) This analysis assumes that only the wings generate lift along the entire wingspan. This also assumes that the tail does not generate lift; which compensates for the assumption that lift is generate by the body of the aircraft. (Figure 25).

(iv) This analysis is relatively conservative as it only takes into consideration the air indirectly displaced below the wing. The air indirectly displaced downwards above the wing is ignored. This is to use conservative assumptions and to avoid potential criticism of double-counting the amount of air displaced (Figure 26).

### Different wing designs and aircraft momentum

A comparison between a glider and a Harrier, of how differences in wing designs and aircraft momentum affect the amount of air displaced downwards. Taking the example of a glider and Harrier with the same wing area, as shown in Table 1, as well as in Figure 27.

#### Aircraft momentum

$$\text{Aircraft Momentum (kg m/s)} = \text{mass (kg)} \times \text{velocity (m/s)}$$

$$\text{Glider: } 50,000 = 900 \times 55.6$$

$$\text{Harrier: } 2,220,000 = 10,000 \times 222$$

$$\text{Difference: } 44.4 \times 11.1 \times 4.0$$

#### Glider

The glider with long 30-meter wingspan, is 3.2 times that of the Harrier. This will directly displace about 3.2 times more air each meter flown; as the wings will directly fly through more air. But the wings are much thinner (less deep / smaller chord), so unable to displace air down very far.

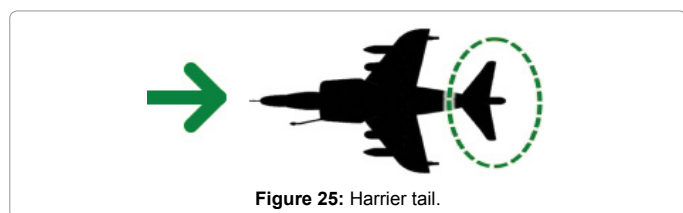


Figure 25: Harrier tail.

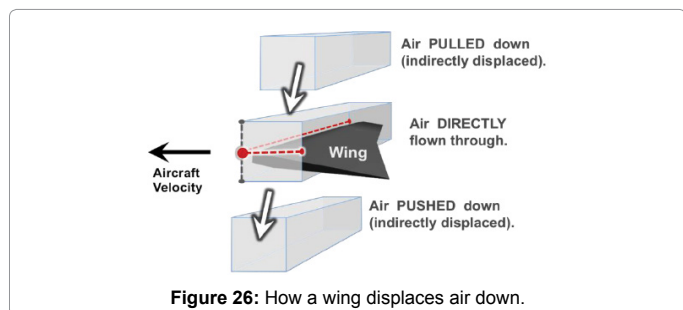


Figure 26: How a wing displaces air down.

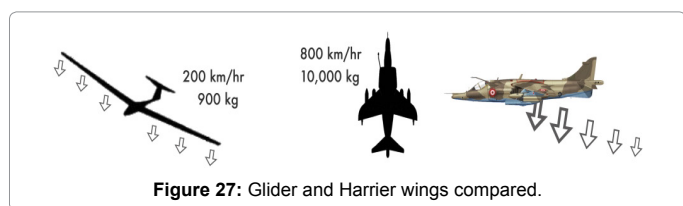


Figure 27: Glider and Harrier wings compared.

Aircraft	Aircraft Mass (kg)	Aircraft Velocity (m/s)	Wingspan (m)	Av. Wing Depth (m)	WingArea (m <sup>2</sup> )
Glider	900	55.6	30	0.75	22.6
Harrier	10,000	222.2	9.4	2.4	22.6

Table 1: Aircraft dimensions.

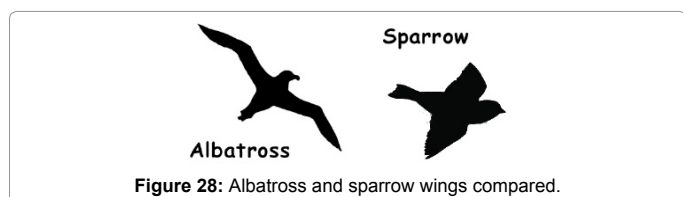


Figure 28: Albatross and sparrow wings compared.

Worse, the glider has a much smaller mass (just 900 kg) and flies relatively slowly at 55.6 m/s (200 km/hr), and therefore it has much less momentum. This lack of momentum that the glider has little energy to transfer to the air, to push the air down. So, even if the glider had deep wings, it would not help it to displace much more air down.

#### Harrier

In this example, the Harrier’s wings have a much shorter wingspan at 9.4 meters, but are about 3.2 times deeper on average (longer chord). So, the Harrier will directly displace less air each meter flown, as it flies through less air with its shorter wingspan. But the deeper wings allow the Harrier the potential to displace this air more aggressively downwards.

The critical point in determining the difference in how much air can be displaced by these aircraft, is that the Harrier has about 44.4 times the momentum of the glider. This is due to the Harrier’s larger 10,000 kg mass and higher speed of 222 m/s (800 km/hr). Consequently, the Harrier will have much more energy available to transfer to the air. In turn, this energy is used to displace more air further down.

In summary, due to the substantial difference in momentum, the Harrier will be able to displace much more air in total than the glider.

Albeit, it is less efficient for the Harrier to generate lift from pushing air further downwards, compared to the glider using a longer wingspan to displace air a short distance down. But this is beyond the scope of this paper. The different wing designs reflect that the Harrier is built for speed and maneuverability. Whereas the glider is built for efficiency.

#### Reference- Burt Rutland, Aircraft Designer

This analysis comparing a glider to a Harrier is consistent with comments made by the famous American aircraft designer, Burt Rutland, in a video documentary “Understanding Flight” by Pamela Caragol in 2009. Note that Burt Rutland is famous for designing: (i) the Voyager (the first aircraft to fly around the world without stopping in 1984); and (ii) SpaceshipOne (the first manned private spaceflight in 2004) [8].

Quote by Rutland [8]: When I’m designing a wing, I think first about what I want to do with the air. If I want the plane to be very efficient, I take a lot of air and move it just a little bit down. You do that with a long-wing airplane (e.g. glider). A short-wing airplane, like a fighter (e.g. Harrier). It has to take less air and move it more energetically down.

#### Comparison to Birds

Birds’ wing designs function on the same basic principles as aircraft; which implies that they are subject to the same forces of physics.

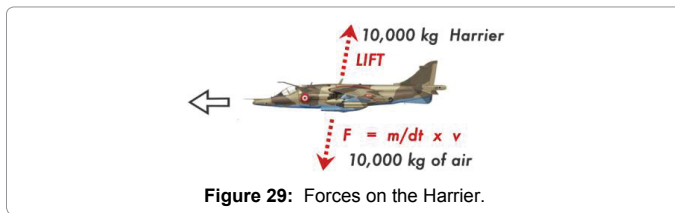
For example, sparrows have short deep wings which function best for their need for speed and maneuverability over short distances; similar to fighter jets. Whereas Albatross have long and narrow wings, which function best for travelling efficiently over long distances, similar to gliders (Figure 28).

#### Conclusion

In the example used above, the Harrier displaces 2,500 kg/s of air down 4 meters in one second, so that a total of 10,000 kg m/s<sup>2</sup> of air is displaced. This is sufficient lift for a 10,000-kg aircraft such as the Harrier.

There are two interpretations of this result; Under the conditions described above:





(i) The wings are able to displace 10,000 kg of air downward each second; which is equal to the Harrier’s mass. This is logical, a force of 10,000 kg m/s<sup>2</sup> will displace 10,000 kg of air each second.

(ii) This downward displacement of 10,000 kg/s of air is equal to a downward force of 10,000 kg m/s<sup>2</sup>; which then generates the corresponding upward force of 10,000 kg m/s<sup>2</sup>; which is sufficient lift for a 10,000 kg Harrier to fly (Figure 29).

It is feasible that a Harrier to displace a mass of air equal to its own mass every second. Therefore, the Harrier can be described as achieving buoyancy in the air every second.

The formula to estimate the lift generated by a wing, based on the mass of air displaced, is:

$$\text{LIFT} = F = m/dt \times v$$

m/dt = Mass of air directly displaced (flown through) by the wing downwards, per unit time (seconds).

v = Vertical velocity of this mass of air downwards.

Analyzing the mass of air displaced by a wing, can explain the differences in lift generated by different types of aircraft with different wing shapes. Aircraft momentum and the energy transferred to the air, is critical to understand how much air an aircraft can displace each meter flown.

## Appendix

### Supplementary materials

Explanatory videos of varying lengths titled: “Buoyancy explains

how planes fly,” and “Calculation of the amount of air displaced by a wing,” are available on YouTube and Vimeo, on channel of ‘N Landell’ (the author of this paper).

### Author Contributions

This paper and the related analysis is entirely the work of the author, Mr. Nicholas Landell-Mills. This work was completed after extensive research, as well as numerous discussions with academics, engineers, pilots and aviation authorities. Extensive flying (experimentation) was done on small, single engine aircraft, to test the validity of the assertions documented in this paper. The findings are consistent with what pilots experience and the observed aerodynamics when flying a plane.

### Conflicts of Interest and Affiliations

None. The author declares that there are no conflicts of interest and no affiliations.

### Acknowledgements

None. I did it all myself. In fact, I experienced considerable resistance and criticism from some engineers, pilots and academics.

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