Mathematica Eterna

OPEN ACCESS Freely available online

Research Article

Large Time Behavior of Multi-dimensional Unipolar Hydrodynamic Model of Semiconductor

Lumin Qiu, Yanqiu Cheng*, Ran Guo

Department of Mathematics, Shandong Normal University, Jinan, 250014, China

ABSTRACT

In this paper, we are concerned with the large time behavior of weak entropy solutions to the multi-dimensional unipolar hydrodynamic model of semiconductor with insulating boundary conditions and non-zero doping profile. For any space dimension, we prove the solutions converge to the stationary solutions exponentially in time. No smallness conditions are assumed.

Keywords: Large time behavior; Unipolar hydrodynamic model; Insulating boundary conditions; Non-zero doping profile

INTRODUCTION

In this paper, we consider the following unipolar hydrodynamic model of semiconductor:

$$\begin{cases} \partial_t n + \nabla\cdot\mathbf{J} = 0, \\ \partial_t \mathbf{J} + \nabla\cdot(\dfrac{\mathbf{J}\otimes\mathbf{J}}{n}) + \nabla p(n) = n\mathbf{E} - \mathbf{J}, \\ \nabla\cdot\mathbf{E} = n - D(x) \end{cases} \quad (1.1)$$

Where $(x,t) \in \Omega \times (0,\infty)$ with \Omega being a dounded open set in R^d, $d \ge 1$. The unknowns $n(x,t) > 0$, $\mathbf{J}(x,t)$ represent the scaled partial density and current density of the electrons. The unknown function $\mathbf{E}$ denotes the electric field, which is generated by the Coulomb force of particles. If we introduce the electrostatic potential ϕ then $\mathbf{E} = \nabla\phi$. In this paper, we consider the isothermal case $p(n) = n$, which is of importance in industry. The symbols $\otimes$ and $(\nabla\cdot)$ denote the Kronecker tensor product and the divergence in R^{d}. D(x)>0 is the doping profile, which means the density of impurities in semiconductor materials. We suppose.

$$D(x) \in C^2(\mathrm{R}),\ D^* = \sup_x D(x) \ge \inf_x D(x) = D_* > 0 \quad (1.2)$$

In this paper, we consider problem (1.1) with the initial conditions

$$n(x,0) = n_0(x) > 0,\ \mathbf{J}(x,0) = \mathbf{J}_0(x), \quad (1.3)$$

And the following insulating boundary conditions

$$\mathbf{J}(x,t)\cdot\mathbf{n}|_{\partial\Omega} = 0,\ \mathbf{E}(x,t)\cdot\mathbf{n}|_{\partial\Omega} = 0 \quad (1.4)$$

Where $\mathbf{n}$ is the outer unit normal vector on $\partial\Omega$.

Now let's recall some known results for the model (1.1). The existence and uniqueness of the subsonic steady solutions was first established by Degond- Markowich, Gamba investigated the stationary transonic solutions [1-5]. For the time dependent model, Hsiao-Yang, Luo-Natalini-Xin and Guo-Strauss proved the existence of global smooth solutions near a given steady state for different kinds of initial or initial-boundary conditions [6-14]. However, Chen proved the existence of the local generalized solutions and gave the blow up phenomenon of this equation [3]. Therefore, it is necessary to study weak solutions. The existence result of weak solutions was given in [7,15-26]. Huang and Yu proved the weak solutions converge to the stationary solutions exponentially in time when space dimension [9,25]. For more results about the unipolar model of semiconductor, we can refer to [1,8,10-12,14,16-20].

In this paper, our main goal is to prove the exponential convergence of multi-dimensional unipolar hydrodynamic model of semiconductor with insulating boundary conditions and non-zero doping profile. That is, all weak entropy solutions of problem (1.1) (1.3) (1.4) converge to the corresponding stationary system.

$$\begin{cases} \nabla\tilde{n} = \tilde{n}\tilde{\mathbf{E}}, \\ \nabla\cdot\tilde{\mathbf{E}} = \tilde{n} - D(x) \\ \tilde{\mathbf{E}}\cdot\mathbf{n}|_{\partial\Omega} = 0, \end{cases} \quad (1.5)$$

With an exponential decay rate, when d=1,2, he existence to smooth solution of problem (1.5) can be proved by variation method [6].

Before stating the main result, we first give the definition of weak entropy solution and some common notations.

Definition 1.1. A For every T>0, the function (n,**J**,**E**) $(x,t) \in (L^2(\Omega\times[0,T]))^{2d+1}$ is said to be a L^2

***Correspondence to:** Yanqiu Cheng, Department of Mathematics, Shandong Normal University, Jinan, 250014, China, E-mail: 734447529@qq.com; 1564290231@qq.com

Received: May 3, 2019; **Accepted:** October 10, 2019; **Published:** October 22, 2019

Citation: Qiu L, Cheng Y, Guo R (2019) Large Time Behavior of Multi-dimensional Unipolar Hydro-dynamic Model of Semiconductor. Mathematica Eterna 9: 102. doi: 10.35248/1314-3344.19.09.102

weak solution of problem (1.1)(1.3)(1.4) if,

$$\begin{cases} \int_0^T\int_\Omega (n\varphi_t + \mathbf{J}\cdot\nabla_x\varphi)dxdt + \int_\Omega n_0\varphi(x,0)dx = 0, \\ \int_0^T\int_\Omega (\mathbf{J}\varphi_t + (\frac{\mathbf{J}\otimes\mathbf{J}}{n} + p(n)\ \nabla_x\varphi)dxdt + \int_0^T\int_\Omega (n\mathbf{E}-\mathbf{J})\varphi dxdt \\ + \int_\Omega \mathbf{J}_0\varphi(x,0)dx = 0, \\ \int_0^T\int_\Omega \mathbf{E}\cdot\nabla_x\varphi dxdt + \int_0^T\int_\Omega (n - D(x)\ \varphi dxdt = 0, \end{cases} \quad (1.6)$$

For all $\varphi \in H_0^1(\Omega\times[0,T)$, with $\varphi(\cdot,t)|_{\partial\Omega}=0$ and $\varphi(\cdot,t)|_{\partial\Omega}=0$, and J, E satisfies (1.4)

In the sense of trace. Furthermore, a weak solution of system (1.1) (1.3) (1.4) is called an entropy solution if the following entropy solution if the following entropy inequality.

$$\frac{\partial\eta}{\partial t}(n,\mathbf{J}) + \partial_r q^r(n,\mathbf{J}) + \frac{\mathbf{J}}{n}\cdot(\mathbf{J}-n\mathbf{E}) \le 0, \quad (1.7)$$

Hold in the distributional sense, where (1.7) use the Einstein's summation symbols (η,q), is entropy flux pair satisfying.

$$\begin{cases} \eta(n,\mathbf{J}) = \frac{|\mathbf{J}|^2}{2n} + n\ln n, \\ q^r(n,\mathbf{J}) = (\frac{|\mathbf{J}|^2}{2n} + n(1+\ln n))\frac{J^r}{n}. \end{cases} \quad (1.8)$$

Let $\mathbf{U}=(n,\mathbf{J})^T$, $\tilde{U}=(\tilde{n},\mathbf{0})^T$ $\|f\|=\|f\|_2=(\int|f(x)|^2dx)^{\frac{1}{2}}$

$\mathbf{J}=(J^1,J^2,\cdots,J^d)$ and we choose $C_i(i=0,1,2,\ldots)$ to represent different positive constants in different places.

The main result of this paper is given below.

Theorem 1.1. Suppose $(\tilde{U},\tilde{\mathbf{E}})(x)=(\tilde{n},\mathbf{0},\tilde{\mathbf{E}})(x)$ is a smooth solution of problem (1.5), $(\mathbf{U},\mathbf{E})(x,t)=(n,\mathbf{J},\mathbf{E})(x,t)$ is any L^2 weak entropy solution of problem (1.1)(1.3)(1.4). If there exist positive constants N^*, N_* and C_0 such that,

$$N_* \le \tilde{n}(x) \le N^*, \quad (1.9)$$

$$0 \le n(x,t) \le C_0, \quad (1.10)$$

For any x ∈ Ω, t > 0, then

$$\int_\Omega((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n}) + |\mathbf{E}-\tilde{\mathbf{E}}|^2)dx \le \alpha e^{-\beta t}\int_\Omega((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n}) + |\mathbf{E}-\tilde{\mathbf{E}}|^2)(\cdot,0)dx$$

Holds for some positive constants α and β.

THE PROOF OF THEOREM 1.1

From equations (1.1) and (1.5), we obtain the following system.

$$\begin{cases} \partial_t(n-\tilde{n}) + \nabla\cdot\mathbf{J} = 0, \\ \partial_t\mathbf{J} + \nabla\cdot(\frac{\mathbf{J}\otimes\mathbf{J}}{n}) + \nabla(n-\tilde{n}) = (n\mathbf{E}-\tilde{n}\tilde{\mathbf{E}}) - \mathbf{J}, \\ \nabla\cdot(\mathbf{E}-\tilde{\mathbf{E}}) = (n-\tilde{n}) = \Delta(\phi-\tilde{\phi}), \end{cases} \quad (2.1)$$

Satisfies in the sense of Definition 1.1.

The proof of Theorem 1.1 is completed in the following two theorem.

Theorem 2.1. Suppose (U,E) (x,t) be a weak entropy solution of (1.1)(1.3)(1.4) in the time interval [0,T], $[\tilde{U},\tilde{E}]$ is a smooth solution of problem (1.5), If (1.9) and (1.10) satisfy for any $x\in\Omega$ and $t>0$, then,

$$\int((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n}) + |\mathbf{E}-\tilde{\mathbf{E}}|^2)dx \le C_1\int((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n} + |\mathbf{E}-\tilde{\mathbf{E}}|^2)(\cdot,0)dx, \quad (2.2)$$

Holds for some positive constants C_1

Proof: Using Einstein's summation convention, we can rewrite the first $d+1$

equations of (1.1) as a hyperbolic system of conservation laws.

$$\partial_t\mathbf{U} + \partial_r\mathbf{w}^r = \mathbf{F}(\mathbf{U},\mathbf{E}) \quad (2.3)$$

Where,

$$\mathbf{w}^1 = \begin{pmatrix} J^1 \\ n+(J^1)^2/n \\ J^1J^2/n \\ \cdots \\ J^1J^d/n \end{pmatrix}, \quad \mathbf{w}^2 = \begin{pmatrix} J^2 \\ J^1J^2/n \\ n+(J^2)^2/n \\ \cdots \\ J^2J^d/n \end{pmatrix}, \ldots, \quad \mathbf{w}^d = \begin{pmatrix} J^d \\ J^1J^d/n \\ J^2J^d/n \\ \cdots \\ n+(J^d)^2/n \end{pmatrix} \quad (2.4)$$

(2.5)

$$\mathbf{F}(\mathbf{U},\mathbf{E}) = (0, n\mathbf{E}-\mathbf{J})^T$$

From (1,7), we obtain

$$\frac{\partial\eta}{\partial t}(\mathbf{U}) + \partial_r q^r(\mathbf{U}) \le g(\mathbf{U},\mathbf{E}) \quad (2.6)$$

With the energy production

$$g(\mathbf{U},\mathbf{E}) = -\frac{|\mathbf{J}|^2}{n} + \mathbf{J}\cdot\mathbf{E}$$

Let,

$$\tilde{\eta} = \eta - Q\ ;\ \tilde{q}^r = q^r - P^r \quad (2.7)$$

Where,

$$Q = \tilde{n}\ln\tilde{n} + (\ln\tilde{n}+1)(n-\tilde{n})\ ;\ P^r = (\ln\tilde{n}+1)J^r \quad (2.8)$$

From(1.5) and (2.6) - (2.8), we obtain

$$\begin{aligned} 0 &\ge \partial_t\eta + \partial_r q^r + \frac{|\mathbf{J}|^2}{n} - \mathbf{J}\cdot\mathbf{E} \\ &= \partial_t\tilde{\eta} + \partial_t Q + \partial_r\tilde{q}^r + \partial_r P^r + \frac{|\mathbf{J}|^2}{n} - \mathbf{J}\cdot\mathbf{E} \\ &= \partial_t\tilde{\eta} + \partial_r\tilde{q}^r + \frac{|\mathbf{J}|^2}{n} - \mathbf{J}\cdot\mathbf{E} + \mathbf{J}\cdot\tilde{\mathbf{E}}. \end{aligned} \quad (2.9)$$

Integrating the last equation in (2.9) over Ω and using the boundary condition (1.4), we get

$$\frac{d}{dt}\int_\Omega\tilde{\eta}dx \le -\int_\Omega\frac{|\mathbf{J}|^2}{n}dx + \int_\Omega(\mathbf{E}-\tilde{\mathbf{E}})\cdot\mathbf{J}dx \quad (2.10)$$

On the other hand, after integrating by parts and using the boundary condition (1.4) for several times, we obtain

$$\begin{aligned} \frac{d}{dt}\int_\Omega\frac{1}{2}|\mathbf{E}-\tilde{\mathbf{E}}|^2dx &= \int_\Omega(\mathbf{E}-\tilde{\mathbf{E}})\cdot\partial_t(\mathbf{E}-\tilde{\mathbf{E}})dx \\ &= -\int_\Omega(\mathbf{E}-\tilde{\mathbf{E}})\cdot\mathbf{J}dx. \end{aligned} \quad (2.11)$$

Combining (2.10) with (2.11), we obtain

$$\frac{}{dt}\int_\Omega(\eta + -|\mathbf{E}-\mathbf{E}|)dx \le -\int_\Omega\frac{||}{n}dx \le \quad (2.12)$$

Moreover, we notice that $\tilde{\eta}$

is the quadratic remainder of the Taylor expansion of the convex function $n\ln n$ around $\tilde{n} > N_* > 0$. Therefore, using (1.9) and (1.10), we obtain there exist positive constants C_2 and C_3 such that,

$$C_2\left((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n}\right) \le \tilde{\eta} \le C_3\left((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n}\right).$$

We finish the proof of Theorem 2.1.

Theorem 2.2. Under the same assumptions as in Theorem 2.1, we further have the exponential decay rate, that is,

$$\int_\Omega \left((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n} + |\mathbf{E}-\tilde{\mathbf{E}}|^2\right)dx \le \alpha e^{-\beta t}\int_\Omega \left((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n} + |\mathbf{E}-\tilde{\mathbf{E}}|^2\right)(\cdot,0)dx,$$

For some positive constants α and β.

Proof: To get the exponential decay rate, we would like to use the Gronwall inequality. To do this we define,

$$W = \tilde{\eta} + \frac{1}{2}|\mathbf{E}-\tilde{\mathbf{E}}|^2 + \mu\left\{-(\mathbf{E}-\tilde{\mathbf{E}})\cdot\mathbf{J} + \frac{1}{2}|\mathbf{E}-\tilde{\mathbf{E}}|^2\right\} \tag{2.13}$$

Where μ

is a real number which will be determined later? In terms of $(2.1)_2$ and the boundary condition (1.4), we get,

$$\begin{aligned} &-\frac{d}{dt}\int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})\cdot\mathbf{J}dx \\ &= -\int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})_t\cdot\mathbf{J}dx - \int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})\cdot\mathbf{J}_t dx \\ &= \int_\Omega \left|(\mathbf{E}-\tilde{\mathbf{E}})_t\right|^2 dx + \int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})\cdot\left\{\nabla\cdot\left(\frac{\mathbf{J}\otimes\mathbf{J}}{n}\right) + \nabla(n-\tilde{n}) - (n\mathbf{E}-\tilde{n}\tilde{\mathbf{E}}) + \mathbf{J}\right\}dx. \end{aligned}$$

We calculate the right side of (2.14) item by item. Firstly, using Young's inequality, we have,

$$\begin{aligned} \int_\Omega \left|(\mathbf{E}-\tilde{\mathbf{E}})_t\right|^2 dx &= -\int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})_t\cdot\mathbf{J}dx \\ &\le \frac{1}{2}\int_\Omega \left|(\mathbf{E}-\tilde{\mathbf{E}})_t\right|^2 dx + \frac{1}{2}\int_\Omega |\mathbf{J}|^2 dx, \end{aligned} \tag{2.15}$$

Which gives,

$$\int_\Omega \left|(\mathbf{E}-\tilde{\mathbf{E}})_t\right|^2 dx \le \int_\Omega |\mathbf{J}|^2 dx \tag{2.16}$$

We also have,

$$\begin{aligned} \int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})\cdot\left\{\nabla\cdot\left(\frac{\mathbf{J}\otimes\mathbf{J}}{n}\right)\right\}dx &= C_4\int_\Omega \frac{1}{n}(\nabla\cdot(\mathbf{E}-\tilde{\mathbf{E}})\cdot|\mathbf{J}|^2 dx \\ &\le C_4\int_\Omega \left|\frac{n-\tilde{n}}{n}\right||\mathbf{J}|^2 dx, \end{aligned} \tag{2.17}$$

$$\begin{aligned} \int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})\cdot(\nabla(n-\tilde{n}))dx &= -\int_\Omega \nabla\cdot(\mathbf{E}-\tilde{\mathbf{E}})\cdot(n-\tilde{n})dx \\ &= -\int_\Omega (n-\tilde{n})^2 dx. \end{aligned} \tag{2.18}$$

Notice

$$\begin{aligned} -\int_\Omega (n-\tilde{n})\tilde{\mathbf{E}}\cdot(\mathbf{E}-\tilde{\mathbf{E}})dx &= -\int_\Omega (\nabla\cdot(\mathbf{E}-\tilde{\mathbf{E}}))\tilde{\mathbf{E}}\cdot(\mathbf{E}-\tilde{\mathbf{E}})dx \\ &= \int_\Omega \frac{\nabla\cdot\tilde{\mathbf{E}}}{2}|\mathbf{E}-\tilde{\mathbf{E}}|^2 dx, \end{aligned} \tag{2.19}$$

We obtain

$$\begin{aligned} &-\int_\Omega (\mathbf{E}-\tilde{\mathbf{E}})\cdot(n\mathbf{E}-\tilde{n}\tilde{\mathbf{E}})dx \\ &= -\int_\Omega \tilde{n}|\mathbf{E}-\tilde{\mathbf{E}}|^2 dx - \int_\Omega (n-\tilde{n})\tilde{\mathbf{E}}\cdot(\mathbf{E}-\tilde{\mathbf{E}})dx - \int_\Omega (n-\tilde{n})|\mathbf{E}-\tilde{\mathbf{E}}|^2 dx \\ &= \int_\Omega \left(\frac{\nabla\cdot\tilde{\mathbf{E}}}{2} - \tilde{n}\right)|\mathbf{E}-\tilde{\mathbf{E}}|^2 dx \le -\left(\frac{N_*}{2} + \frac{D_*}{2}\right)\int_\Omega |\mathbf{E}-\tilde{\mathbf{E}}|^2 dx, \end{aligned} \tag{2.20}$$

Where we have used $(1.5)_2$ and the fact that

$$\frac{\tilde{n}}{2} + \frac{D(x)}{2} \ge \frac{N_*}{2} + \frac{D_*}{2}.$$

From the above analysis (2.14)-(2.20) and (2.11) (2.12), we deduce

$$\frac{d}{dt}\int_\Omega W dx \le -\int_\Omega Y dx, \tag{2.21}$$

Which

$$Y = (1 - \mu n - \mu C_4|n-\tilde{n}|)\frac{|\mathbf{J}|^2}{n} + \mu(n-\tilde{n})^2 + \mu\left(\frac{N_*}{2} + \frac{D_*}{2}\right)|\mathbf{E}-\tilde{\mathbf{E}}|^2. \tag{2.22}$$

As the proof in [2], we can choose μ

small enough such that W and Y are positive definite quadratic forms. So there exist positive constants K_W and K_Y such that,

$$K_w\int_\Omega \left((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n} + |\mathbf{E}-\tilde{\mathbf{E}}|^2\right)dx \le \int_\Omega W dx, \tag{2.23}$$

$$K_y\int_\Omega \left((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n} + |\mathbf{E}-\tilde{\mathbf{E}}|^2\right)dx \le \int_\Omega Y dx, \tag{2.24}$$

The estimate (2.21) turns into

$$\frac{d}{dt}\int_\Omega W dx \le -K_y\int_\Omega \left((n-\tilde{n})^2 + \frac{|\mathbf{J}|^2}{n} + |\mathbf{E}-\tilde{\mathbf{E}}|^2\right)dx, \tag{2.25}$$

From Gronwall inequality we get Theorem 2.2, with $\beta = K_y/K_w$

By Theorem 2.2 we can easily deduce Theorem 1.1.

REFERENCES

1. Ali G. Global existence of smooth solutions of the N-dimensional Euler-Poisson model, SIAM J Math Anal. 2006;35(2):389-422.
2. Ali G, Jungel A. Global smooth solutions to the multi-dimensional hydrodynamic model for two-carrier plasmas. J Differential Equations. 2003;190(2):663-685.
3. Chen G, Wang D. Formation of singularities in compressible Euler-Poisson fluids with heat diffusion and damping relaxation, J Differential Equations. 1998;144(1):44-65.
4. Degond P, Markowich P. On a one-dimensional steady-state hydrodynamic model for semiconductors, Appl Math Lett. 1990;3(3):25-29.
5. Gamaba I. Stationary transonic solutions of a one-dimensional hydrodynamic model for semiconductors. Comm Partial Differential Equations. 1992;17(3):553-577.
6. Guo Y, Strauss W. Stability of semiconductor states with insulating and contact boundary conditions. Arch Ration Mech Anal. 2006;179(1):1-30.
7. Huang F, Li T, Yu H. Weak solutions to isothermal hydrodynamic model for semiconductor devices. J Differential Equations. 2009;247(11):3070- 3099.
8. Huang F, Mei M, Wang Y, Yu H. Asymptotic convergence to stationary waves for unipolar hydrodynamic model of semiconductors. SIAM J Math Anal. 2011;43(1):411-429.
9. Huang F, Pan R, Yu H. Large time behavior of Euler-Poisson system for semiconductor. Sci China Series A. 2008;51(5):965-972.
10. Huang F, Mei M, Wang Y, Yu H. Asymptotic convergence to planar stationary waves for multi-dimensional unipolar hydrodynamic model of semiconductors. J Differential Equations. 2011;251(4):1305-1331.
11. Hsiao L, Markowich PA, Wang S. The asymptotic behavior of global smooth solutions of the multidimensional isentropic hydrodynamic model for semiconductors. J Differential Equations. 2003;192(1):111-133.
12. Hsiao L, Wang S. The asymptotic behavior of global smooth solutions to the hydrodynamic model with spherical symmetry. Nonlinear Anal. 2003;52(3):827-850.
13. Hsiao L, Yang T. Asymptotics of initial boundary value problems for hydrodynamic and drift diffusion models for semiconductors, J. Differential Equations. 2001;170(2):472-493.
14. Jungel A, Peng Y. Zero-relaxation-time limits in the hydrodynamic

equations for plasmas revisited. Z Angew Math Phys. 2000;51(3):385-396.

15. Li T. Convergence of the Lax–Friedrichs scheme for isothermal gas dynamics with semiconductor devices. Z Angew Math Phys. 2005;57(1):12-32.
16. Li H, Markowich P. A review of hydrodynamic models for semiconductor asymptotic behavior. Bol Soc Brasil Math. 2001;32(3):1-22.
17. Li H, Markowich P, Mei M. Asymptotic behavior of solutions of the hydrodynamic model of semiconductors. Proc Roy Soc Edinburgh Sect A. 2002;132(2):359-378.
18. Li H, Markowich P, Mei M. Asymptotic behavior of subsonic shock solutions of the isentropic Euler-Poisson equations. Quart Appl Math. 2002;60(4):773-796.
19. Li Y. Global existence and asymptotic behavior for a multidimensional nonisentropic hydrodynamic semiconductor model with the heat source. J Differential Equations. 2006;225:134-167.
20. Li Y. Diffusion relaxation limit of a nonisentropic hydrodynamic model for semiconductors. Math Methods App Sci. 2007;30:2247-2261.
21. Luo T, Natalini R, Xin Z. Large-time behavior of the solutions to a hydrodynamic model for semiconductors. SIAM J Appl Math. 2009;59(3):810-830.
22. Li J, Yu H. Large time behavior of solutions to a bipolar hydrodynamic model with big data and vacuum. Nonlinear Anal. 2017;34:446-458.
23. Marcati P, Natalini R. Weak solutions to a hydrodynamic model for semiconductors: the Cauchy problem. Proc Roy Soc Edinburgh. 1995;125:115-131.
24. Marcati P, Natalini R. Weak solutions to a hydrodynamic model for semiconductors and relaxation to the drift-diffusion equation. J Arch Rational Mech Anal. 1995;129(2):129-145.
25. Yu H. Large time behavior of entropy solutions to a unipolar hydrodynamic model of semiconductors, Commun Math Sci. 2016;14:69-82.
26. Zhang B. Convergence of the Godunov scheme for a simplified one- dimensional hydrodynamic model for semiconductor devices. Commun Math Phys. 1993;157(1):1-22.