Hybrid Bayesian Network Modeling of Slips and Falls for Forensic Analysis in Civil Litigation

Richard E. Hughes*
Departments of Orthopaedic Surgery, Biomedical Engineering, and Industrial & Operations Engineering, University of Michigan, Ann Arbor, MI, USA.

ABSTRACT
Experts in biomechanics and human factors are often asked to opine about the cause of a slip and fall event that resulted in an injury. The question posed to the expert is fundamentally different than the question answered by the traditional engineering analysis methods. Instead of trying to predict the probability of slip given available coefficient of friction (aCOF) and required coefficient of friction (rCOF), the expert knows a slip occurred and is trying to make inferences about the causes of aCOR and rCOF. It is apparent that what links the traditional engineering approach and needs of the litigation expert is Bayes’ theorem. A hypothetical case study was used to illustrate how a hybrid Bayesian network could be developed to compute a probabilistic statement about two competing theories of injury causation, one put forward by the plaintiff and one by the defense. The resulting probability aligns well with the requirement that the expert deliver an opinion based on the civil litigation standard of “more probable than not.”

Key words: Slip and fall; Bayesian network

BACKGROUND
Biomechanics has been extensively used to analyze and predict slip and fall events [1]. Physical parameters used include the available coefficient of friction (aCOF), which is a measure of the slipperiness of the shoe-floor interface. It is a standard quantity in tribology and can be measured with a variety of instruments. The required coefficient of friction (rCOF) is the ratio of the shear force to normal force acting on the floor by the person while walking or running. In classic deterministic biomechanical analyses, a slip occurs when the rCOF exceeds then aCOF. This concept has been extended to predict the probability of a slip by numerous authors [2, 3]. In the stochastic versions of the slip model, the goal is to predict the probability of a slip given aCOF and rCOF.

Experts in biomechanics and human factors are often asked to opine about the cause of a slip and fall event that resulted in an injury. The question posed to the expert is fundamentally different than the question answered by the traditional engineering analysis methods. Instead of trying to predict the probability of slip given aCOF and rCOF, the expert knows a slip occurred and is trying to make inferences about the causes of aCOR and rCOF. It is apparent that what links the traditional engineering approach and needs of the litigation expert is Bayes’ theorem. Bayesian network models have been developed for use in the evaluation and interpretation of evidence in criminal law [4]. Thus, Bayesian network modeling methods have been applied in a legal context and may be able to help address the problem of inferring causation in a slip and fall in personal injury litigation case.

The purpose of this project was to demonstrate how to develop a hybrid Bayesian network model for computing the probabilities of two competing theories (plaintiff and defense) of the cause of a slip and fall injury. The probabilistic framework aligns well with the expectation that the expert opine based on the civil standard of “more probable than not.”

METHODS
Case Study
This work will be motivated by the following case study: Plaintiff brings a civil suit against a business claiming that it failed to adequately maintain a walkway on its premises, resulting in him slipping, falling, and fracturing his humerus. The plaintiff claims he was walking at a normal speed but slipped because the walkway was covered with a slippery fluid resembling cleaning fluid. The defendant claims that the walkway had been recently washed with water and a “no running” sign was set in front of the cleaned area. The defendant claims that the walkway was covered with a slippery fluid resembling cleaning fluid. The defendant claims that the walkway had been recently washed with water and a “no running” sign was set in front of the cleaned area. The defendant claims that the walkway had been recently washed with water and a “no running” sign was set in front of the cleaned area. The defendant claims that the walkway had been recently washed with water and a “no running” sign was set in front of the cleaned area.
Site investigation performed by the expert months later indicated that the company did have the cleaning fluid on hand and a “no running sign.” However, the expert could not determine from security video or other evidence whether the sign was present at the time of the injury. The expert was able to measure the available coefficient of friction (aCOF) of the walkway made of ridged quarry flooring under the two conditions (wet with water only and with the cleaning fluid). Both the plaintiff and defendant stipulate there was an injury resulting from a fall, and the fall was caused by a slip. The contested issue the expert is asked to opine about is the cause of the slip.

For the purposes of this work, assume the expert has been asked by the retaining attorney to opine about the cause of the slip and fall. There are two competing hypotheses, one supporting the plaintiff (Hₙ) and the other argued by the defense (Hᵣ):

Hₙ: Plaintiff was walking on an excessively slippery surface, and the low aCOF was due to the cleaning fluid contaminant on the walking surface.

Hᵣ: Plaintiff was running on a walkway that had been recently washed with water only.

Development of hybrid Bayesian network model

A Bayesian network (BN) is a probabilistic graphical modeling method that was initially developed in artificial intelligence for probabilistic reasoning [5] and has been applied to a wide range of fields. A hybrid Bayesian network is a subclass of models that can incorporate both discrete and continuous probability distributions. A Bayesian network consists of nodes, directed edges, and node probability tables [6]. The network is represented as a directed acyclic graph, so there cannot be any directed cycles. Each node represents a random variable, and the directed edges encode the conditional independence relationships between random variables. Each node has an associated table that encodes the conditional probabilities for that node. That is, it specifies the probability of the random variable associated with the node taking on a specific value given the states of the random variables associated with nodes having edges incident on the node in question. Once the directed acyclic graph and associated node probability tables have been defined, calculations can be performed. The most straightforward calculations are in the direction of the edges because they involve just the law of total probability applied at each node. However, evidence can be entered at nodes in the graph including nodes that are descendants of other nodes, i.e. have directed edges pointing into them from other nodes. In this case the effect of this evidence can be propagated through the network using special algorithms. All of these calculation algorithms are explained in detail in [6, 7] and implemented in software.

The heart of this model is the slip_and_fall node at the bottom of Figure 1 (note that node names will be placed in italics). It represents a Boolean random variable that takes on a value of true if an only if a slip occurs (the hypothetical assumes both parties agree the fall was caused by a slip so the slip event is equivalent to a fall event). Mechanically, this is when aCOF < rCOF. The model implements a stochastic slip model developed by Chang [8]:

\[
P[\text{slip}] = \int_{-\infty}^{\infty} f_{rCOF}(x) \int_{-\infty}^{x} f_{aCOF}(y) \, dy \, dx
\]

where \( f_{aCOF}(y) \) and \( f_{rCOF}(x) \) are the probability density functions of aCOF and rCOF, respectively. The hybrid Bayesian network implementation of this probabilistic slip model is based on the hybrid Bayesian network model implementation of Chang’s model [8] described in Hughes [9]. The inputs to the slip_and_fall node are the rCOF and aCOF nodes, and the probability table for that node is in Table 1. The rCOF node has a node probability table that is contingent on the running_as_cause node above it (Table 2). The aCOF node, which is also a parent of the slip node, has a node probability table conditional on the running_as_cause node (Table 3). The logic of the competing hypothesis of the plaintiff and defense is represented in the topmost node, running_as_cause. This is a Boolean variable in which true represents Hₙ and false represents Hᵣ.

Table 1: Node probability table for slip_and_fall node.

<table>
<thead>
<tr>
<th>Predecessor nodes: aCOF and rCOF</th>
<th>slip_and_fall takes on value of true if aCOF &lt; rCOF</th>
<th>slip_and_fall takes on value of false if aCOF ≥ rCOF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>False</td>
<td>True</td>
</tr>
<tr>
<td>Lognormal</td>
<td>LN(-1.46,0.00387)</td>
<td>LN(0.689,0.138)</td>
</tr>
<tr>
<td>Weibull</td>
<td>shape = 20.0</td>
<td>shape = 2.82</td>
</tr>
<tr>
<td></td>
<td>scale = 0.239</td>
<td>scale = 0.604</td>
</tr>
</tbody>
</table>

Table 2: Node probability table for rCOF node.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Predecessor node: running_as_cause</th>
<th>False</th>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lognormal</td>
<td>LN(-0.720,0.270)</td>
<td>LN(0.391,0.00136)</td>
<td></td>
</tr>
<tr>
<td>Weibull</td>
<td>shape = 1.87</td>
<td>shape = 34.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>scale = 0.627</td>
<td>scale = 0.688</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Node probability table for aCOF node.

Table 4: Probability of running_as_cause taking on value of true.

<table>
<thead>
<tr>
<th>rCOF</th>
<th>aCOF Distribution</th>
<th>Lognormal</th>
<th>Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.729</td>
<td>0.762</td>
<td></td>
</tr>
<tr>
<td>Lognormal</td>
<td>0.593</td>
<td>0.635</td>
<td></td>
</tr>
</tbody>
</table>

Data

In reality, aCOF would be measured on-site. Because this model is based on a hypothetical case, data on aCOF for wet and contaminated ridged quarry surfaces were taken from Chang et al. [10]. Mean and standard deviation data for rCOF during walking was taken from Chang et al. [11]. Median and interquartile range data on rCOF during running was abstracted from Vidal et al. [12]. Mean and standard deviation values were estimated from Vidal et al.’s data using methods described by Wan et al. [13]. Parameters for the lognormal and Weibull distributions (Table 1) were computed from the means and standard deviations using custom MATLAB.
Figure 1: Hybrid Bayesian network model of slip and fall in the presence of two competing theories of causation. The node `running_as_cause` represents the defense hypothesis ($H_D$) when it takes on a value of true; a value of false corresponds to the plaintiff hypothesis ($H_P$).

Figure 2: Distributions of $aCOF$ and $rCOF$ nodes when there is a flat prior for `running_as_cause` node.
functions (MATLAB v. 2020b 9.9.0.1467703, The Mathworks, Natick, MA). Both aCOF and rCOF were modeled using lognormal and Weibull distributions.

Simulations
The model was solved using a naïve prior in which the running_as_cause Boolean random variable was set to true (to model the hypothesis H₉ without any knowledge of whether a slip and fall occurred) assuming rCOF and aCOF were both lognormally distributed. The purpose of this simulation was to provide intuition about how a “forward” hybrid Bayesian network model would perform. The result was 0.214. The simulation was repeated with running_as_cause set to false to simulate the case where we know that the cause was walking (hypothesis H₉ with no knowledge of slip and fall). The probability of slip and fall was 0.079. Then a “flat” or “uninformative” prior was used in the simulation (the probability of running_as_cause being true was set to 0.5). The probability of slip and fall was 0.146. A closer examination of the probability distributions (Figure 2) shows how aCOF was a convex combination of two lognormally distributed random variables (one for the wet condition and one for the contaminated condition). Similarly, rCOF was a convex combination of the distributions for running and walking. The weights in the convex combination were 0.5 and 0.5 because of the flat prior.

Finally, the simulation used to inform the development of the expert opinion was performed. The node slip_and_fall was set to true to represent the knowledge that a slip and fall occurred. Bayes’ theorem was then used by the software to compute the probabilities “backwards” in the network, going against the direction of the directed edges. The results were that the probability of running_as_cause taking on the value of true ranged from 0.593 to 0.762 (Table 2) depending on the assumed distributions for aCOF and rCOF.

CONCLUSIONS
The purpose of this project was to develop a model of a slip event that can be used to opine about the cause of the slip in a way that is consistent with being asked to deliver an opinion about the likelihood of two competing theories (hypotheses) of causation in a civil case. Because the First Circuit found in Burke v. Town of Walpole [14], that “reasonable scientific certainty” means “more likely than not,” a probabilistic model is ideally suited for developing an opinion that should be expressed in terms of a “reasonable scientific certainty.” The model developed here computes the probability of one of two competing (plaintiff v. defendant) theories being correct, so it is ideally suited for developing an opinion that can be stated to a “reasonable scientific certainty” in a civil case. These simulations showed that the predicted probability of H₉, being true ranged from 0.593 to 0.762. This range of values was completely greater than 0.5, making the opinion stated to within a “reasonable scientific certainty” robust.

The modeling approach proposed here addresses both aleatoric and epistemic uncertainty. Aleatoric uncertainty represents natural randomness in a process and is represented by a probability distribution; epistemic uncertainty refers to limited data and knowledge of the process and is often modeled by using alternative probability density functions. The stochastic slip model of Chang [8] explicitly addresses aleatoric uncertainty because it uses probability distributions to describe aCOF and rCOF. Epistemic uncertainty is addressed by modeling aCOF using lognormal and Weibull distributions and rCOF using the lognormal and Weibull distributions. Lognormal and Weibull distributions were selected because they have been used in the literature to COF data [10, 11], and the normal distribution allows for unrealistic negative values. Finally, it could be argued that the assessment of epistemic uncertainty in a stochastic slip model is the most relevant to assessing the “known or potential rate of error” mentioned in Daubert v. Dow Merrell Dow Pharmaceuticals, Inc.

The novelty of this modeling approach can be appreciated by comparing it to the body of work applying Bayesian network modeling in law. Bayesian networks have been compared to Wigmore charts for the analysis of criminal cases [15]. Additional work has explained how Bayesian networks can be used to analyze and construct legal arguments in criminal contexts [16]. Many papers have been published on the use of Bayesian networks in modeling criminal cases, with special emphasis on evidence [17]. Decarboxyribonucleic acid (DNA) evidence has been a special area of application of Bayesian networks to criminal cases [18]. Papers specific to other types of evidence, such as gunshot residues [19], handwriting [20], documents [21], and glass fragments [22] have also been developed.

By comparison, there are much fewer papers related to civil litigation and even fewer related to personal injury. Analysis of DNA to determine paternity using Bayesian networks [23] may support cases involving family law. Insurance litigation that involves arson could use the Bayesian network models developed for fire investigations [24]. Bayesian network models for employment discrimination [25] and public health enforcement [26] have also been reported. Hughes [9] is the closest to the current work, as it proposed a method for constructing hybrid Bayesian network models for the analysis of slip and fall events. However, its formulation focused on the analysis of a claim of negligence rather than competing plaintiff and defense hypothesis regarding injury causation.

SOURCES OF FUNDING
Salary support for the author during the preparation of this manuscript was provided by the Department of Orthopaedic Surgery of the University of Michigan.

CONFLICT OF INTEREST
None to report.

REFERENCES


