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Journal of
Aeronautics & Aerospace Engineering

Research Article

Effects of Cubic Wing Loading Parameter on Airplane Wing Sizing and Parasitic Drag

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ABSTRACT

This study is about the applicability of cubic wing loading (CWL) in conceptual aircraft design and flight performance. Cubic wing loading is a dimensionless parameter and is related to the square cube law. Cubic wing loading, or volumetric wing loading, offers a size-independent density factor for comparative study of aircraft made of similar building materials. CWL, density factor, is naturally depends on aircraft building materials. Also, CWL is concerned with the ratio of fuselage to wing size in aircraft because wings and fuselage have different stiffness and density values. Considering the aircraft mass as a whole, the density values of large wing aircraft are lower than those of small wing aircraft. Cubic wing loading is more applicable for initial wing sizing than wing loading. The aspect ratio is a dimensionless wing shape parameter and CWL is a dimensionless density factor. Thus, the AR-CWL graph can be used for the comparative study of aircraft. Also, CWL is a measure of relative wetted area i.e., wetted area over wing area. Low CWL means the aircraft need less power-to-weight ratio (better fuel economy) because high relative wetted area means high parasitic drag.

Keywords: Cubic wing loading; Wing cube loading; Density factor; Conceptual design; Parasitic drag

INTRODUCTION

The cubic wing loading (CWL) factor is a size independent parameter used for grouping and comparing radio controlled (RC) planes according to flight characteristics. CWL is found by dividing the mass of the aircraft with the power of 3/2 of the wing area. Aircraft with close CWL have similar flight characteristics regardless of their mass and size features provided that they are made of similar materials. CWL is obtained by converting the 2-dimensional wing area to the 3-dimensional mathematically equivalent volume. This volume is not relevant with actual 3D aircraft volume. Because CWL is a measure of density, it is also related to strength and stiffness of aircraft. Cubic wing loading (CWL) is obtained from dimensional analysis or scaling study. In birds and bats, wing size is correlated with mass and the scaling study between mass and size is based on the idea that the unit length scale is proportional to the power of one-third of the mass [1]. Liu [2] applied the square to cube law scaling methodology to birds, propeller/turboprop aircraft, and jet transports and the empirical estimates made regarding wingspan, wing area, flight speed, and power consumption. Reynolds [3] is the person who used term “wing cube loading” and brought it into literature. He also revealed the physical meaning of CWL in RC airplane design. The fly-ability factor, density factor, cube loading, wing volume loading, and volumetric loading are other terms or nomenclatures for CWL. Because the wing loading varies according to the size (mass), it is useless in comparative study at different scales; on the other hand, CWL is a size independent of and can be used in comparative study at different scales. Grouping of aircraft according to increased CWL value: Gliders around 4, Trainers around 6, Sport Aerobatic around 9, Racers around 12, Scale around 10-15 [3]. CWL is the most considered value in model aircraft designs [4-9]. In studies where drag estimates are made for an airship, the volumetric drag coefficient is proportional to the wetted area, i.e., 2/3 power of the airship volume [10].

Küchemann [11] claimed that the parameter-volume/(wing surface area)$^{3/2}$ which is a kind of square-cube law parameter is an important factor for evaluating aircraft flight performance. Furthermore, Küchemann [11] argued that such a parameter could be used to enhance the lift to drag (L/D) ratio. Kundu et al. [12] argued that larger aircraft provide better power-to-weight ratio and better fuel economy than aircraft designed by the square-cube law. Aircraft designed considering square-cube law provide better structural efficiency and reduce the maximum take-off weight and the empty mass ratio. Despite all this, CWL is still not defined as an aerodynamic design parameter. This study aims to reveal

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Received: October 06, 2020, **Accepted:** October 20, 2020, **Published:** October 27, 2020

Citation: Durmus S (2020) Effects of Cubic Wing Loading Parameter on Airplane Wing Sizing and Parasitic Drag. J Aeronaut Aerospace Eng. 9:226. doi: 10.35248/2168-9792.20.9.226

the physical meaning of CWL and for clarity it is shown on the examined data of 81 aircraft. Later, it was aimed to demonstrate the applicability of cubic wing loading (CWL) to wing sizing in conceptual aircraft design and algebraically show the effect of cubic wing loading on relative wetted area.

MATERIALS AND METHODS

The square-cube law says that the increase in volume (i.e., mass) is faster than the increase in surface area. The square-cube law is illustrated in Figure 1 for ease of understanding.

Wing aspect ratio (AR) as shape factor and cubic wing loading (CWL) as density factor are dimensionless parameters. Hence, AR-CWL charts can be used in comparative study analysis of aircraft. The best glide ratio is proportional to the square root of AR, so it can be concluded that high AR is a criterion for good gliding [13]. Wing loading (WL) is a direct measure of mass over wing size. Cubic wing loading (CWL) is derived from dimensional analysis by taking mass and size properties into consideration together. Dimensional analysis is based on the idea that unit length (l) is proportional to the power of one-third of the mass (m).

$$l \sim m^{1/3} \ldots \quad (1)$$

Thus, the wing area (S) is a power of 2; the mass (m) is a power of 3 of unit length scale.

$$S \sim l^2,\ m \sim l^3 \ldots \quad (2)$$

The wing loading is the ratio mass to wing area given in Equation 3.

Wing loading has a significant effect on the aircraft's stall speed, take off distance, turning radius, and cruising speed.

$$\text{Wing Loading} = m/s \text{ or } = mg/s \text{ or } = w/s \ldots \quad (3)$$

Dimension analysis for wing loading (WL) is given in Equation 4. The numerator is in cubic form and denominator is in square form, so WL is a mass or size-based parameter. The SI unit of WL is the same as pressure unit kg/m^2 or N/m^2(Pa).

$$\text{Wing Loading (WL)} = l^3/l^2 = l \ldots \quad (4)$$

Unlike WL; CWL offers a dimensionless parameter and refers to the density factor which is independent on aircraft mass or size but building material and rigidity. The mass value to be taken for aircraft is the mass at maximum takeoff weight (MTOW).

$$\text{Cubic Wing Loading (CWL)} = MTOW/S^{3/2} \ldots \quad (5)$$

Dimension analysis for density factor is given in Equation 6. The numerator with denominator is in cubic form, so CWL is constant coefficient independent of size.

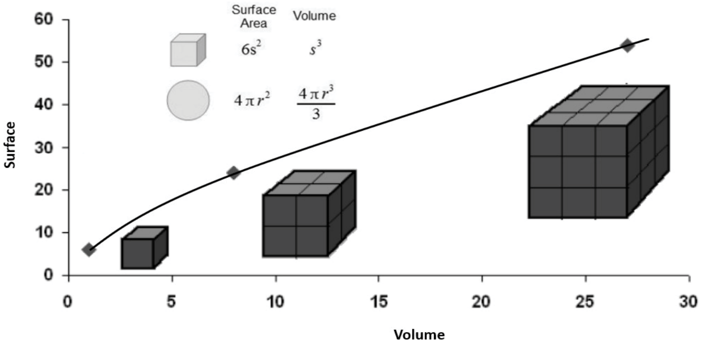

Figure 1: Demonstration of square-cube (2/3) law.

$$\text{Cubic Wing Loading (CWL)} = \frac{l^{2\left(\frac{3}{2}\right)} - l^3}{l^{2\left(\frac{3}{2}\right)}\ l^3} = cons\tan t \cdots \quad (6)$$

Unit analysis for CWL, density factor, is given in Equation 7. The cubic wing loading unit is mass per unit volume and it is the same as volumetric mass density. Both CWL and volumetric mass density are a degree of relative weight. The SI unit of CWL is the same kg/m^3 as unit of volumetric mass density.

$$\text{Cubic Wing Loading (CWL)} = \frac{ki\,\text{logram}}{meter^{2\left(\frac{3}{2}\right)}} = \frac{kg}{m^3} \cdots \quad (7)$$

Wing area estimation by density factor (CWL) is done as in Equation (8) for take-off flight phase. It is possible to predict wing size of an aircraft with the average CWL value of aircraft produced from similar materials. MTOW is maximum takeoff mass of aircraft. It is also possible to use WCL for initial mass estimate, if the wing area is known.

$$\left(\frac{MTOW}{WCL_{mean}}\right)^{2/3} = Swing_{est} \cdots \quad (8)$$

Küchemann [11] defined a volume parameter as shown in Equation 9 and claimed that the parasitic drag was related to this parameter. Similarly, CWL is proportional to ratio of wetted area to wing area and parasitic drag. The wetted area of the aircraft can briefly be defined as the exposed surface area that interacts with the air.

$$Volume\ Parameter\ (\tau) = Volume/S^{3/2} \ldots \quad (9)$$

When the author compared the CWL values of the aircraft, CWL was found to be highly proportional to relative wetted area (Swet/Swing). Relative wetted area estimation can be done by density factor or CWL as shown in Equation 10.

CWL˜Relative Wetted Area=Cw. (Swet/Swing)... (10)

Equation 11 is obtained by rewriting the Equation 10 parasite drag in terms of skin friction drag (C_{fe}). High density (CWL) aircraft are related to high relative wetted area (Swet/Swing) and parasitic drag (C_{D0}).

$$C_{D0} = C_{fe}\left(\frac{Swet}{Swing}\right) = C_{fe} * WLC \cdots \quad (11)$$

RESULTS AND DISCUSSION

Aircraft building material affecting CWL. Most of RC plane (low density, low CWL) is made from depron foam, balsa wood, plywood and carbon fiber rod. Most general aviation aircraft and commercial airliners' aircraft (relatively low density, low CWL) are made of composites and most of the Fighters (relatively high density, high CWL) are made predominantly of metallic alloys. The second factor affecting CWL is the fuselage size to wing size. Given unit MTOW value, the aircraft has larger wings compared to the fuselage size, it has lower CWL or lower density. For instance, the CWL value of flying wings (B-49) and blended wing bodies (Boeing X-48) is lower than conventional airplanes consist of tube and wings. Interestingly, CWL is a parameter more commonly used by RC aircraft designers. Table 1 includes the CWL value of some models with their types. Most RC aircraft hobbyists have experienced that cubic wing loading is more practical than wing loading (WL) parameter. So why is this so ? Aircraft with low WCL are lighter (low density), can fly slower, sensitive to lift, highly maneuverable and can carry more payloads, but they are vulnerable to strong winds and turbulence. Aircraft with high WCL are heavier (low density), stable, fly at high speeds, tolerant to turbulence and strong winds, but are less maneuverable and have faster landing speeds. The author collected model aircraft data. The results show that WCL values were between 0-3 for indoor models (low wind speed), 3-5 for backyard flyers (low speed, small range), 5-7 for park flyers, powered gliders and slow trainers; 7-10 for trainers, sports, aerobatics, 10+ for scales, racers and warbirds (high speed). As can be seen from the Table 2, WCL value is more significative than WL on flight characteristics.

What results do we get if this scale used in model aircraft is applied to different types of real aircraft? Below, the AR-CWL diagram (Figure 2) and AR-WL diagram (Figure 3) are shown for 81 examined aircraft in different groups. When we split the aircraft wings as delta wings and regular wings; the B-47E aircraft has the highest CWL (S_{wet}/S_{wing}=7.8 by Raymer [14]) values in regular wings.

Yet another, high AR and low CWL is a sign for unpowered or low

Table 1: WL and CWL of value of some RC model aircraft.

Model Type	Model Name	Mass (gr)	Wing Area (dm^2)	WL (gr/dm^2)	CWL
Indoor Flyers (Low Wind Speed)	T-IFO	173	27.8	6.2	1.18
	Ikarus Su-27XXL	454	30.5	14.9	2.69
	Electrifly Extra 330SC	193	17.2	11.2	2.7
Backyard Flyers	Tech One Malibu F3P	295	18.1	16.3	3.84
	Sky Surfer 2m	1350	46.5	29	4.3
	Sig Kadet Senior EG	3175	76.1	41.7	4.78
Park Flyers Powered Sailplanes Slow Trainers	NES Gabby E	1025	33	31.1	5.42
	E-Flite L-4 Grasshopper	278	13.5	20.5	5.51
	Mountain Models Etana	822	27	30.5	5.82
	Trainer Pilot RC 90"	4900	79.8	61.4	6.9
Trainers Aerobatics Sports	RC Extra 330L	624	19.3	32.3	7.3
	Phoenix Classic .61	2800	50.5	55.4	7.8
	IMEX Spacewalker EP-400	436	14.5	30.2	7.95
	Mid-West Citabria	4218	62.5	67.4	8.52
	Phoenix Model Domino	2600	44.3	58.7	8.8
	Cox EP 380	839	20.2	41.5	9.22
	Sportman Aviation Sonic 500	1871	33	56.6	9.86
Scales Racers Warbirds	Art Tech Cessna 182	1350	23	58.7	12.2
	A-1 Skyraider	1100	19.8	55.6	12.5
	P-51D Mustang	4540	46.5	97.6	14.3
	L39 Albatros	3900	33	118.2	20.6

Table 2: The CWL and AR value of the aircraft examined. Mean value of the bombers was not given, as the group had unusual designs (Avro Vulcan B.2, B-47E).

Aircraft Type	CWL Margin	Mean CWL	Aspect Ratio
BWB	8	8	4.1
Sailplanes	Oct-15	12	21-51
Flying Wing	Dec-16	14	5.8-7.2
Large Delta Fighters	13-20	16	1.8-31
GAA	17-23	20	06-Oct
Supersonics Transportation	18-27	22.5	1.6-1.7
An-225 and A380	23	23	7.5-8.6
Utility jets	32-36	34	07-Oct
Wide Body and Cargoes	30-41	37	7.5-12
Narrow Body	43-61	51	7.8-9.6
BJ and Regional Airliner	47-60	53	5.5-12
Stealth Fighters	50-56	53	1.8-2.5
Bombers	13-68	~	~
Multirole Fighters	60-160	90	2.4-4.0

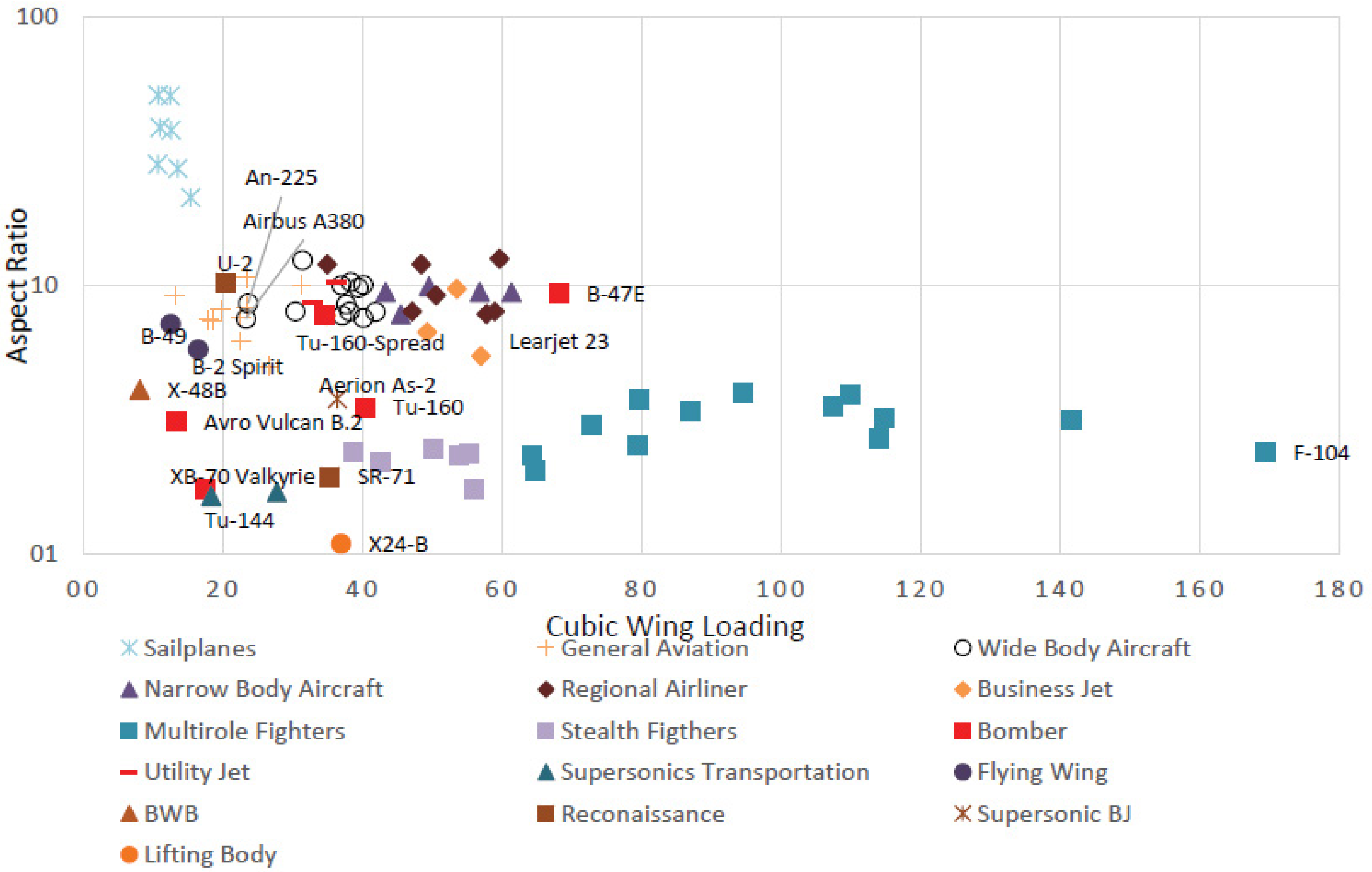

Figure 2: Aspect ratio and cubic wing loading chart of 81 aircraft. A base 10 log. Scale for AR was used for the readability of the scatter plot. The top left corner of the diagram is for good fliers (requires low thrust) and the bottom right corner is for poor fliers (requires high thrust).

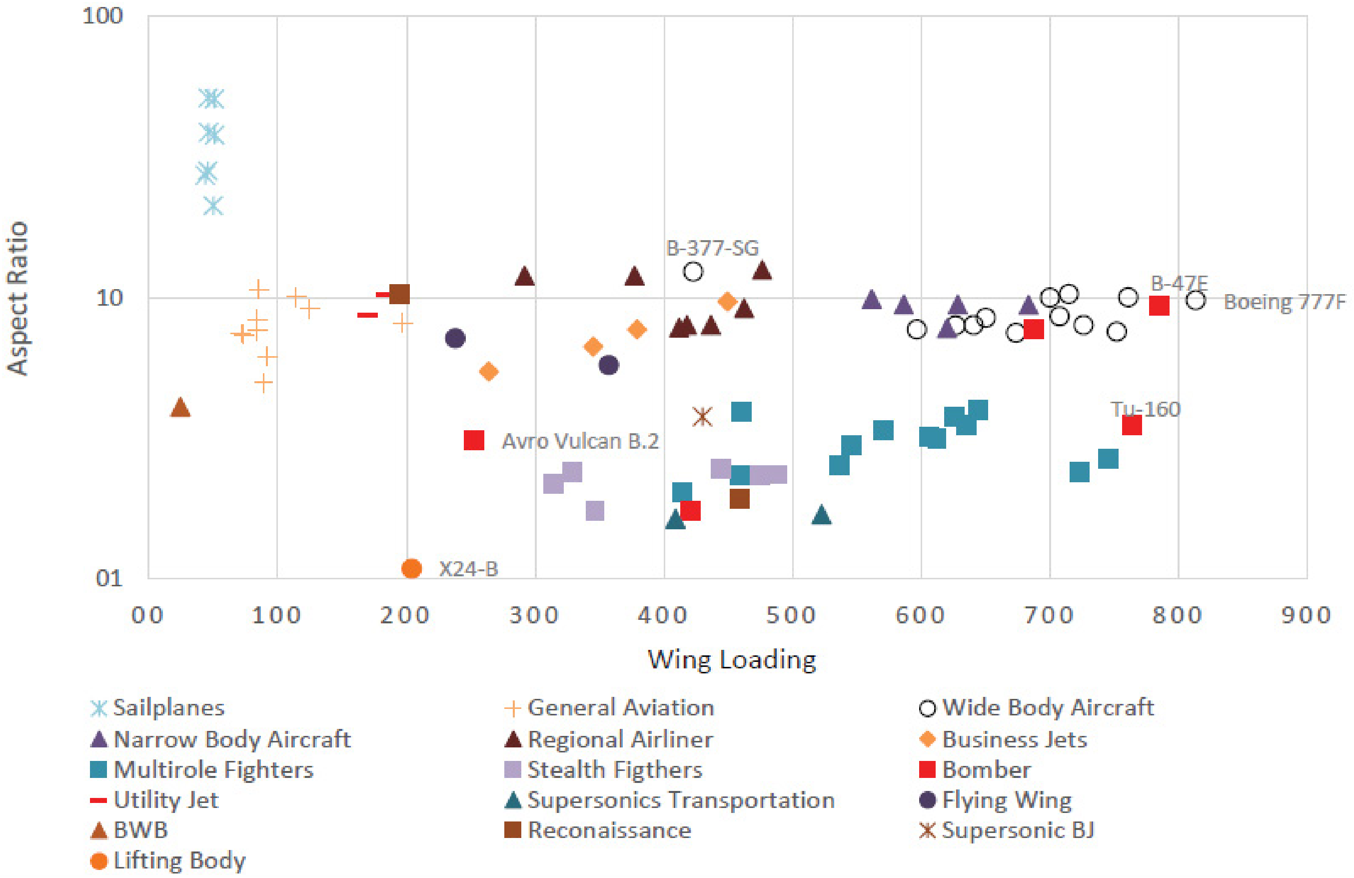

Figure 3: Aspect ratio and wing loading chart of 81 aircraft. 10 based log. Scale for AR scale was used for the readability of the scatter plot.

Table 3: Good and poor design according to the square cube law.

Density Factors	Poor Design According to the Square Cube Law	CWL Value	Good Design According to the Square Cube Law	Mean CWL
Low Density	Airbus A380	23.2	Airbus Beluga	37
	An-225	23.5	Ilyushin Il-76	37.5
	DC-3	13	CL-215	19.6
High Density	Cirrus SR22	31.1	Cessna 172	17.7
	F-104	169.3	F-16	114.7

thrust required flight; unlike low AR and high CWL is a sign for high thrust required flight. Therefore, if an airplane is designed at lower WCL values contrary to the square cube law (An-225, Airbus A380); the power to weight ratio and fuel economy may be the reason.

The ranking of increasing density factor (CWL) in civilian aircraft: Sailplanes, general aviation aircraft (GAA), supersonic transportation (Concorde), utility jets, wide body commercial aircraft, narrow body commercial aircraft, business jets and regional airliners. The ranking of increasing density factor in Fighter and experimental aircraft: Blended wing bodies (BWB), flying wings, large delta Fighter aircraft (Avro Vulcan, XB-70), stealth Fighters and multirole Fighters. CWL margin and average CWL values for different aircraft types studied are given in Table 3 with wing aspect ratio. After the initial weight estimate, wing sizing of the aircraft can be done according to mean CWL values specified in Equation 8. Although the evaluation has been made in a separate category, the narrow body aircraft and jet airliner have close CWL values and CWL range.

So, what is practicability of CWL apart from wing sizing and relative wetted area estimation? The answer to this question is undoubtedly to from an opinion of the average density values of aircraft and based on this, to have an idea about bad design according to the square cube law. Aircraft with low CWL (low density aircraft) may be poor designs i.e., wing have designed in larger sizes than it should be. Aircraft with high CWL (high density aircraft) have designed smaller wings than they should have. Design of Airbus A380, An-225 and DC-3 defies the square-cube law; but in this case it is situated better power-to-weight ratio and better fuel economy is targeted. Yet another, the design of the Cirrus SR22 and F-104 is questionable in terms of power-to-weight ratio. The accident rates of high CWL aircraft may be a possible subject.

The answer to the question of whether it is the effect that determines the value of CWL in aircraft with different fuselage size to wing size made of different materials is not certain. Therefore, the WCL should not be seen as a constant value independent of aircraft building materials. CWL depends on aircraft materials, so the CWL grouping of aircraft can be done only for similar aircraft materials. For instance, the CWL value of model RC sailplane (foam, balsa wood) is around 4; however, that of a true scale composite sailplane is around 12.

Therefore, a glider made entirely of metal may have a CWL value of about 20 or more. Kundu [12] claimed that the design of aircraft with 2/3 rule provided better structural efficiency to reduce the maximum take-off weight and the empty mass ratio although better power-to-weight ratio, better fuel economy opposed the square-cube law [14-16].

CONCLUSION

This is the explanation for why Airbus A380 and An-225 produced in large wing sizes are against the square-cube law. Physical meaning of CWL is the density factor in aircraft design and is mathematically approved by dimension analysis. Yet another, relationship between the CWL and relative wetted area or relationship between CWL and parasitic drag could be improved for future studies. Finally, a comparative study could be conducted between UAVs and true scale aircraft in the future.

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