Dynamical System: An Overview

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PERSPECTIVE

The idea of a dynamical system emerges from Newtonian mechanics. There, as in other natural sciences and engineering disciplines, the evolution rule of dynamical systems is an implicit relation that gives the system’s state only for a short time in the future. Finding an orbit required sophisticated mathematical techniques and could only be accomplished for a small class of dynamical systems prior to the advent of computers. The task of determining the orbits of a dynamical system has been simplified thanks to numerical methods implemented on electronic computing machines. A dynamical system is a mathematical system in which a function describes the time dependence of a point in a geometrical space. Mathematical models that describe the swinging of a clock pendulum, the flow of water in a pipe, and the number of fish in a lake each spring are some examples. A dynamical system is defined in physics as a “particle or ensemble of particles whose state changes over time and thus obeys differential equations involving time derivatives.” An analytical solution of such equations or their integration over time via computer simulation is realized to make a prediction about the system’s future behaviour. The study of dynamical systems is the focus of dynamical systems theory, which has applications in many fields including mathematics, physics, biology, chemistry, engineering, economics, history, medicine. Chaos theory, logistic map dynamics, bifurcation theory, self-assembly and self-organization processes, and the edge theory all rely on dynamical systems.

Many consider French mathematician Henri Poincaré to be the creator of dynamical systems Poincaré wrote two now-classic monographs, “New Methods ofCelestial Mechanics” (1892–1899) and “Lectures on Celestial Mechanics” (1905–1910). In them, he successfully applied the results of their research to the problem of three-body motion and studied the behaviour of solutions in detail. Many important approximation methods were developed by Aleksandr Lyapunov. His methods, which he developed in 1899, allow the stability of sets of ordinary differential equations to be defined. He developed the modern theory of dynamical system stability. Stephen Smale also made significant strides. His first contribution was the Smale horseshoe, which sparked major research in dynamical systems. He also outlined a research programme that involved a large number of people. Knowing the trajectory is often sufficient for simple dynamical systems, but most dynamical systems are too complicated to be understood in terms of individual trajectories. The problems arise because:

(a) The studied systems may only be known approximately—the system’s parameters may not be known precisely, or terms may be missing from the equations. The approximations used call the validity or relevance of numerical solutions into question. To address these issues, several notions of stability, such as Lyapunov stability or structural stability, have been introduced into the study of dynamical systems.

(b) The trajectories would be the same. The operation for comparing orbits to determine their equivalence varies depending on the notion of stability used. It is possible that the type of trajectory is more important than the specific trajectory. Some trajectories may be periodic, whereas others may wander through a variety of system states. Applications frequently necessitate enumerating or maintaining these classes.

(c) An application may require the behaviour of trajectories as a function of a parameter. The dynamical systems may have bifurcation points where the qualitative behaviour of the dynamical system changes as a parameter is varied. For example, in the transition to turbulence of a fluid, it may go from having only periodic motions to seemingly erratic behaviour. These early visionaries were the primary plane design specialists; however they were presumably totally different than the ones we know today. Since the hour of Da Vinci, splendid personalities have planned and outlined distinctive flying machines, however never carried them to fulfillment.

Smooth coordinate changes do not change the qualitative properties of dynamical systems (this is sometimes taken as a definition of qualitative): a singular point of the vector field (a point where v(x) = 0) will remain a singular point under smooth transformations; a periodic orbit is a loop in phase space, and smooth deformations of the phase space cannot change it being a loop. Even though they are fundamentally deterministic, simple nonlinear dynamical systems and even piecewise linear systems can exhibit completely unpredictable behaviour that appears random. This seemingly random behaviour has been dubbed chaos. A state space, an abstract mathematical space of points where each point

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represents a possible state of the system, is used in dynamical systems. The instantaneous values of the variables considered crucial for a complete description of the state are used to define an instantaneous state. In this account of dynamical systems and state spaces, some minor but critical assumptions are made. Specifically, that the values of the critical state space variables accurately characterize the actual state of a target system and that a physical state corresponds to a point in state space via these values.