

Tyre Contribution to Caravan Fishtailing

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ABSTRACT

The terminal end to fishtailing has ruined many a caravan vacation, caused injury and even loss of life has been reported. The only thing between the ground and the mass of caravan is a small patch on the bottom of the tyres. A serious study of fishtailing needs to consider the steering action performed by this patch. A sideways flexible tyre will exacerbate caravan sway where a laterally stiff tyre may avoid fishtailing altogether. The main contribution of this paper is to illustrate the quantitative extent to which this is evident by comparing two tyres of equal outside diameter but different wheel rim diameters. Accumulation of yaw energy that drives the oscillation in the comparison is shown to be approximately four times more in the flexible tyre compared to the stiff tyre. Methodology is developed in this paper to enable the comparison to proceed. This approach proceeds using tyre “steering” as the motivating action responsible for fishtailing. Although this paper provides comparison for only two tyres, it provides good grounding for further studies. Any forensic study for courts or civil litigation should find this paper helpful offering easy-relatable concepts needed for court digestion rather than complex vibration theory.

Keywords: Caravan; fishtailing; Snaking; Tire steering; Yaw energy; Contact patch

INTRODUCTION

One might ponder over the motivations of caravanners to trade living in a house in the suburbs with, instant hot water, microwave, dishwasher, air conditioning and a soft comfortable bed with ensuite for a mere 8 x 16 feet box on wheels, towed hundreds of miles to be squeezed into a densely populated caravan park to make small talk about the weather with the new neighbours. They swap the private air space around their homes for in-your-face communal living where every burp and cough can be heard by surrounding neighbours. Then there is the outside toilet and shower facility inconveniently placed at the distal end of the park, not to mention dogs barking at all hours of the night or unrestrained kids screaming out midnight demands.

Perhaps it is the remnant of a pioneer spirit in us or simply to escape the feeling of guilt of “must wash the curtains”, “the gutters need cleaning”, “the fence needs repairing” or any one of the dozen or more chores put off for the holidays. Nevertheless, the desire to explore the wide, wide world shows no sign of abating. The World fleet of Recreational Vehicle (RV) statistics is staggering:

- Australia, with a mere population of 25 million, shows new registrations of 11,500 caravans with 270,000 total numbers of caravans registered as at 31 Oct.1999. Australian Bureau of Statistics [1]. This number grew to 612,867 caravans by the year 2018, Caravan Industry Association of Australia [2], the increase speaking volumes on popularity.
- Europe (excluding UK) added 55,000 caravans to its fleet in 2017 showing an increase of over 8% over the previous year [3].
- The UK National Caravan Council [4] estimates over half a million touring caravans in use in the UK.
- 9 million families in the U.S. report ownership of an RV right now. The year 2018 saw an increase to the fleet of towable RV's of 119,593 units [5].

Clearly, the acquisition of RV's worldwide is not slowing. Intuitively, accidents are roughly correlated with RV numbers and, within the accident statistics, an increase of fishtailing to a terminal end is indicated thereby too.

Fatality and injury statistics are scarce as fatality statistics are typically hidden in vehicle accident statistics. Notwithstanding, the

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(US) Federal Motor Carrier Safety Administration report 70,000 injuries in 2003 rising to 75,000 injuries by 2012. The (US) Fatality Analysis Reporting System [6] indicates 212 deaths resulting from RV accidents in the years 2000 to 2007. Both statistics would indicate some intervention may be desirable. A fishtailing event goes through many of the following stages:

- A misalignment of vehicle centrelines caused by rapid steering input, wind gusts, overtaking or being overtaken and/or road surface undulations, to mention a few.
- From an articulated angle the trailers show propensity to re-align with the tow vehicle.
- This return to alignment can be violent with high forces and momenta.
- These forces and momenta are typically not acquitted at alignment.
- Residual momentum at the point of realignment will fuel sway into the opposite direction.
- If the momentum-driven sway causes sway amplitude greater than the original articulation, there is risk of runaway fishtailing.
- Not all serious fishtailing is terminal. Refer YouTube Video [7].

The lead-up to the terminal end of caravan fishtailing is a complex symphony of actions and reactions. Bevan; Smith; et al [8] succinctly summarize the influencing factors as follows:

“It was found that, to be more stable, a car/caravan combination should include a light caravan with low yaw inertia, adequate drawbar length, short hitch overhang, large car wheelbase, an optimum caravan centre of gravity, and adequate caravan tyre stiffness. A hitch damper was found to stabilize a combination approaching oscillatory instability.”

The literature offers many presentations of mathematical-mechanical formulations of models of varying complexity. These days many presentations use “black box” software such as Dads, Adams or Simpak to good avail. This paper uses published mathematical formulations as well as tedious mechanical drafting procedures in a piece-wise linear approach.

As overview, we present some details of a recent caravan rollover: “On Monday, July 29 [2019] a single vehicle rollover towing a caravan resulted in minor injuries for the 67-year-old South Australian caravaner and his passenger. The crash occurred on the Bruce Highway, Gumlu Queensland at approximately 11.55am.” [9]. The full video can be seen here:

The elapsed time between the photos in Figure 1 above was approximately 6 seconds accounting for approximately 2.5 to 3 oscillations. The frame on the right side of Figure 1 was captured just after the rig cleared the bow wave of the truck being overtaken. The oscillations started at commencement of lane change. The tyre striations on the roadway in the left-hand frame suggest the tyres in contact with the ground were at saturation. They appear to feather out at the start of the irrecoverable roll-over, leaving contact with the roadway.



Figure 1: Snapshots from dashcam video of caravan rollover.

THE CARAVAN TYRE

“An essential component for the vehicle behaviour is the tyre”, advise Lugner and Plöchl [10 pp. 219-236]. After all, the caravan tyre supports the weight of the caravan and provides direction of travel. Thus, lateral stiffness is an important performance criterion resisting aerodynamic forces as well as road-initiated actions that may threaten instability. The small area at the bottom of the tyre in contact with the ground is all there is to maintain safe travel. Even the latest Electronic Stability Control (ESC) devices are dependent on that small area of tyre.

In January 2017 the Goodyear Tire Company unveiled a dedicated tyre for trailers in the Endurance range featuring their “Durawall” technology. The key features offered for this tyre were listed as:

- “An optimized tread depth and decoupling groove, which help it remain cool while towing heavy loads;
- Rugged fabric-steel construction, which offers improved durability and carrying capacity while remaining compatible with tire pressure monitoring systems and typical tire inflation stations;
- A specialized inner liner, which minimizes air loss in trailer applications; and
- A scuff guard, which helps protect sidewalls.”

There was no mention of “good tracking”, “excellent braking performance” being the quintessential functional requirements for a trailer tyre. As it is typical for the industry, test results were not published. Thus, an informed opinion would not have been feasible from their press release.

However, but unrelated to the press release, Tim Fry, senior development engineer with Goodyear Tire & Rubber Company offered some engineering focus:

“The major difference is reflected in the polyester cords used in Special Trailer (ST) tires. These cords are bigger than they would be for a comparable Passenger (P) or Light Truck (LT) tire. Typically, the steel wire also has a larger diameter or greater tensile strength to meet the additional load requirements. Because of the heavier construction for an equal volume of air space, an ST tire is designated to carry more load than a P or LT tire” reports Hiser .

Lateral stiffness is a function of many factors chief among which are :

- Tyre size
- Tyre type (bias, radial)
- Tread pattern
- Tyre width
- Aspect ratio

- Carcass design
- Vertical load
- Inflation pressure

There are advantages and disadvantages of a wider tyre over a narrower version. A good comparison can be found in Sandberg, Formgren, et al. . We simply follow the advice of Kurz and Anderson that anyone studying car/trailer systems would be wise to pay considerable attention to the tyre forces.

THE COMPARISON TYRES

In a world of scarcity of tyre test data that is made available to the general public, test data for two tyres of identical outside diameter and different wheel size, was highly prized for this paper. Genta and Morello published test data for two such tyres: 195/65 R 15 and 225/45 R 17.

These test data are to be examined for what contribution each might make to the fishtailing phenomenon. Hewsen gives clear indication as to which tyre might provide better directional stability. The lateral tyre stiffness was implicated by treating the tread as a beam in bending. Heissing and Ersoy add: “A rim width increase of 1” can result in a sideslip stiffness increase of up to 10 %”.

Clearly if one tyre outperforms the other in the lateral stiffness criterion then that is the tyre to choose (ceteris paribus). We now introduce the tyres to be compared in (Figure 2) below:

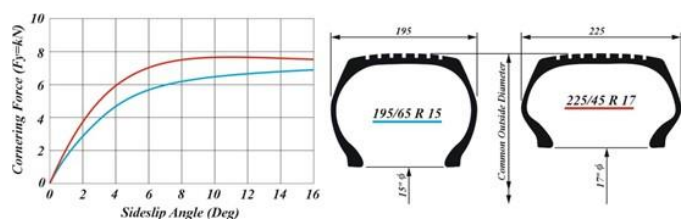


Figure 2: Comparison of cornering force vs sideslip angle for two radial tyres.

Genta and Morello [16 p. 106] noted the difference in cornering stiffness between two tyres with identical outside diameter but with different aspect ratios. They provided no quantitative guide useful to discriminate between the two. The data from their graphs was read and fitted to Pacejka’s “Magic Formula” in (Figure 2). Blundell and Harty remind us that the “Magic Formula” is nota predictive tool but rather it is to empirically represent test data for a physical tyre. They add that this formula (Pacejka’s) had been specifically developed to relate:

- The lateral force F_y as a function of slip angle α
- The aligning moment M_z as a function of slip angle α
- The longitudinal force F_x as a function of longitudinal slip α

A scale sketch of the tyre cross-sections was added to Figure 2 while shows their relative contact patches. These tyre curves form the basis of comparison in the caravan context. It is assumed that the inflation pressures are the same so that the area of the two contact patches in are nominally identical. In this regard, we hasten to point out that tyres are not some pumped up birthday balloon that exhibits contact area fully proportional to inflation pressure. Rather, they are a mechanical structure that responds to

internal pressure. Thus, the idea of identical pressures/areas is a simplification.

In the late 70’s Paul Simon popularized the song “Slip Slidin’ Away” as if the terms slip and sliding had the same meaning. In the context of tyre behaviour, however, they have very different meanings. The behaviour of the contact patch is complex and is often idealized in the literature into two surfaces, even though the line of demarcation is fuzzy. Sliding refers to a surface in a state of shear with the ground, shown dark grey in while slip in sideslip context refers to the surface in rolling contact with the ground. The rolling surface gives direction while sliding provides the most friction and damping.

As speed and sideslip increase, the dark shaded area increases and the light-shaded area decreases in size until the whole area is involved in sliding, a state often referred to as saturation. Black tyre striations often seen in dashcam movie clips are indicative of a tyre at saturation. E.g. YouTube Video . There is little directional stability in a tyre at saturation. The choice of study velocity for the caravan model was made on the basis of avoiding full saturation but maximizing demonstrable effect.

A further semantic clarification is addressed by Pauwelussen , referring to the two contact patch surfaces as sliding and adhesion. As the latter terminology is in semantic conflict with adhesive friction (as distinct from Coulomb friction) the descriptors of Zhou et al. as sliding and rolling are preferred.

SAMPLE CARAVAN

A real caravan typically pitches and rolls in addition to fishtailing. In addition, there is an offloading of one tyre at the expense of loading the opposite tyre. To simplify the study, both wheels of the sample caravan at each end of the axle work with the same sideslip angle and there is no body roll to transfer load from one wheel to the other. In this ideal state, the comparison tyres are set in a nominal caravan configuration as described below .

To load a caravan such that the weight is concentrated at the intersection of the axle and drawbar would be a disastrous situation. Rule of thumb in a practical setting is a hitchpoint load of approximately 10% of gross vehicle mass. Genta and Genta [26 pp 553] note: “If the centre of mass is exactly on the axle, the trailer has no effect on the steady state behaviour of the tractor”. For this study, simplification is the prize (Table 1).

Table 1: Example caravan attributes

Item	Symbol	Value	Units
Mass of caravan	m	1,600	kg
Drawbar length	a	4	m
Mass moment of Inertia at CofG1	J0	3867	kgm ²
Mass moment of Inertia at Hitchpoint2	JH	29,467	kgm ²
Velocity at Arrow	VH	15	m/s
Trailer axle sideslip stiffness (195/65 R15)	C195	1,88,800	N/rad
Trailer axle sideslip stiffness (225/45 R17)	C225	2,41,600	N/rad

¹Engineersedge.com [J0 = m(L²+B²)/12] ²Parallel axis theorem [JH = J0+m*a²]

A STATE OF INSTABILITY

Car-Caravan Combinations (CCC's) exhibit dynamic critical speeds by which stability boundaries for the combinations are defined. The most dominant factor in trailer stability was found to be trailer yaw inertia. Other factors found by them were trailer mass distribution and drawbar length both of which are beyond the scope of this paper which uses a straight-line path for the hitchpoint travel. So too, excluded from discussion here is the phenomenon of divergent instability where the caravan can jack-knife without oscillating.

Karnopp reminds us that Eigenvalue analysis to be the "primary tool" to evaluate stability. He cautiously adds that in such endeavours: "...there is no substitute for physical experimentation". The small-step piece-wise approach of this paper is seen as a close approximation to such experimentation.

The danger with instability in a real setting is that trailers and caravans can merrily appear to be stable until at a certain speed, without warning, a stable/unstable threshold is exceeded. Such a speed is termed the critical velocity VCR occurring when damping ratio is zero.

Another way of determining the boundary between stable and unstable is by hooking up a physical caravan to test with a prime mover initiating sway by a sudden steering input. The procedure is set out in International Standard ISO 9815: 2010 where for safety reasons the final zero damping condition is determined by straight-line extrapolation of test data.

An amplitude gain/loss on each successive sway cycle as visualized Figure 5 is indicative of stability/instability, as appropriate.

Each CCC has inherent damping produced by tyre friction and windage on the outgoing side. Some CCC's are equipped with various devices that mitigate out-of-control yaw oscillation. An example of inherent damping is demonstrated by . Their

CCC test involved impulse steering inputs by the driver to initiate fishtailing. This is shown in which was redrawn from their graphed values and superposed to highlight response lag. They found a high inverse relationship of damping and speed .

A complete study of the fishtailing phenomenon that includes the behaviour of the tow vehicle as well is found in Hac et al. . Their study included a modifiable test trailer used to calibrate and verify their theories. Such a study is beyond the current brief.

DAMPING

Some tyres dissipate more energy than others. The more energy that is dissipated as the caravan cycles left and right, the less likely for out-of-control fishtailing to occur. A good measure for comparison is the so-called damping ratio. This non-dimensional ratio is an indicator of the energy dissipation. A unity ratio marks the point where there is no rebound in the mass-spring oscillator analogy. A zero damping ratio indicates the onset of instability. ISO 9815 provides for a 'Reference-damping speed' that results in a damping level of 5% ($D=0.05$). Heissing and Ersoy offer an empirical equation that can be used to calculate the damping ratio while a similar result is obtained from .

Where C is trailer axle sideslip stiffness, C_{195} or C_{225} as appropriate for the tyre. Equation [1] is simplified by elimination of

a term not needed as CofG position is defined to be on the trailer axis. It is plotted for hitchpoint velocity (VH) as the independent variable and other details as described in Table 1 and posted to Figure 6 where a higher damping ratio means quicker recovery from fishtailing .

The Sway-Time graph inset of Figure 6 (INTERPRETATION) is merely indicative in a general sense and not meant to reflect the two comparison tyres, the values for which are to be read vertically for any given hitchpoint velocity. The importance of showing that the wider tyre offers more damping lies in the need to show in the conclusions drawn here that damping does not undermine by a potential reversal of damping efficacy. Other than showing the general damping improvement over the narrower tyre, the actual damping is not relevant to the conclusions.

The 5% damping level for ISO 9815 requirements was also calculated to but found to be well beyond a normal travelling speed of a caravan. Thus the caravan described in Figure 4 is a stable configuration for normal road speeds. To emphasize, the highlight of Figure 6 is in the demonstration that the wider tyre simply offers more damping than the narrow tyre.

THE SWAY CYCLE

A caravan or trailer whose centreline is at an angle with the prime mover centreline experiences two forces, viz centrifugal action and wind forces both reflecting the speed of the prime mover. These forces indirectly cause the caravan to sway left and right by the steering effect of sideslip. Central to this paper is the idea that external forces affect the tyres to "steer" the caravan into oscillation.

The nature of the sway cycle can be considered in two phases. The ingoing phase runs from extremity articulation to the point of zero articulation, being the total ingoing angular displacement, or put simply, yaw. At extremity yaw, the yaw velocity is momentarily zero and then it reverses, accumulating angular momentum towards zero yaw angle where the tow vehicle centreline axis is collinear with the caravan or trailer axis. This then, is the yaw energy accumulation phase. Yaw energy peaks as it crosses the zero-yaw line. The zero-yaw line marks the theoretical beginning of the discharge cycle where the accumulated yaw energy is applied to the damping forces of the outgoing phase .

The accumulation phase conserves energy via rotation of the caravan's mass moment of inertia about the Z-Axis (vertical) at the hitchpoint. In addition, more energy is acquired as a result of centrifugal action similar to a vehicle involved in cornering.

The swaying caravan applies lateral forces on the hitch point that can cause the tow vehicle to steer in directions not intended by the driver. There is value here in assuming the tow vehicle is of great mass with a long wheelbase, thus the tow vehicle is deemed unaffected by the oscillating motions of the caravan. This simplifying assumption empowers focus on trailer/caravan response in isolation.

Damping to the oscillating trailer is supplied by windage during the discharge phase but not during the accumulation phase where it fuels the oscillation cycles. In both phases tyre friction provides energy loss as yaw damping.

The external forces are resisted by the reaction of the contact patch as a basic assumption of this paper. In the event of exceeding tyre saturation as shown in a video by the caravan becomes airborne

and tyres no longer play a relevant role.

In a cause-effect sequence of events, windage and centrifugal action cause tyres to distort and direct the caravan along its sideslip path. The path is continuous but if discretized, each step to the new position presents a new radius and different windage and new centrifugal action. A new sideslip angle emerges for the next step in the cycle to reflect the new forces. Naturally smaller step sizes emulate the continuous process better.

So, tyre sideslip angle gives the trailer direction from the previous yaw position. During the ingoing phase the angular momentum builds reflecting the conservation laws. During the outgoing phase the sideslip angle serves to increase the yaw angle towards the terminal end if there is insufficient damping in the windage and tyre lateral friction forces.

It would be tidy under an assumption that the yaw momentum would commence discharging only after the zero-yaw displacement line. Momentum discharge occurs some time prior to the zero-yaw line. As the discharge phase (whenever it begins in earnest) is not to be studied (or needed) to compare the tyres, this early discharge phenomenon remains for future study.

TRACTRIX INFLUENCE

A tractrix curve (Latin trahere meaning to pull or drag) is the curve made by an object attached to and pulled by a tractor unit (or person) moving in a straight path. Maybe it was Leibnitz in the 17th Century who caught his daughter dragging her schoolbooks through the mud or maybe it was one of the other mathematicians like Huygens, Leibnitz or Bernoulli who became obsessed with the tractrix curve? Figure 8 is an artist's impression of such a child dragging schoolbooks through the mud making a tractrix path .

The mathematical relationship for a tractrix curve is as follows, after Orloff is regarded as a transcendental equation and so finding a closed form solution is unlikely. Accordingly, Eqn [2] is solved numerically. Nomenclature is best understood.

The tractrix curve is asymptotic to the X-axis, necessitating some practical cut-off point. The influence of the tractrix curve on towed objects has been known for hundreds of years. Not so well known is how the tractrix interacts with tyre sideslip. To illustrate this interaction, the reader is asked to consider how a rigid solid tyre with almost no

sideslip might compare with a laterally 'soft' inflatable tyre. The flexible tyre will quickly depart from the tractrix while the solid rubber tyre would remain almost true to the tractrix path, both experiencing the same forces of windage and centrifugal action. The tractrix curve can thus be described as the path a trailer will follow at zero sideslip.

TYRE RUN-UP and RELAXATION ZONE

The interchange from discharge to accumulation phase does not occur instantly as might be wrongly inferred from Figure 10. Rather it is a process dealing with the run-up and relaxation length properties of the tyre. The winding down from the distorted shape and the ramping up to the distortion in the other direction occurs in the zone described as the 'Tyre Relaxation Zone'.

At the Tangent Point to the tractrix and beyond, we note the following:

- Tyres under lateral force do not instantly deflect (or undeflect) into the distorted shape shown in Figure 11 but achieve the shape progressively during a distance called relaxation length. Maurice and Pacejka .
- For a normal passenger tyre the relaxation length ranges between 0.2 m and 0.8 m depending on weight carried and amount of sideslip. Maurice, et al.
- Heissing and Ersoy weigh in with their estimate of relaxation length: "Typical passenger car tire run-in lengths are between 0.2 and 0.7 m".
- Pacejka gives a rule of thumb that "at nominal vertical load the relaxation length is of the order of magnitude of the wheel radius" corresponding to between
- 0.14 m and 0.45 m the higher values for higher velocities and heavier loads.
- Pacejka noted a sharp decrease in relaxation length at large slip angles adding to the variability scope of relaxation length.
- The wheelbase point P is no longer tangent to the sideslip-affected path after the point P(x,y).
- The angle (α) peaks prior to P(x,y), the tangent point to the tractrix.
- After the tangent point P(x,y) the angle (α) decreases creating rotation of the caravan in the direction as shown by the right-hand rule.
- The change in rotation gives rise to an angular velocity ($\dot{\alpha}$) and in turn angular momentum.
- The exact shape of the interchange path is not known but is discretized in Detail Z of Figure 11 where the three points A, B & C are horizontally equi-spaced according to hitchpoint steps for modelling.

There is no tractrix effect on the discharge phase. Thus, the yaw energy produced by the tractrix action etc during the accumulation phase produces the discharge phase. It is worthy of note that the discharge phase may commence sooner than arrival at the zero-yaw line shown on account of build-up of yaw momentum.

WINDAGE

To determine the effects of windage on the resisting tyres, a bluff body sized 2.0*2.0*5 m long, was analysed in a virtual wind tunnel as part of a Computational Fluid Dynamics (CFD) analysis. A rendition of the 3D model used is found. The prime mover was included in the CFD analysis to catch the eddies and vortices that might affect the air flow over the caravan. CFD analyses were performed for different articulation angle, viz. 10, 20, 30 degrees. Each position was subjected to air velocities of 10, 15, 20 & 25 m/s.

The model of the bluff body comprised 41 surfaces, mainly flat but also curved and spherical at the corners. Typical output for each surface is found in .

Our interest is in moments about the Z-Axis by the forces F_x & F_y .

CFD models were created for 10, 15, 20 & 25 m/s for each of 10, 20 & 30 degree articulation angles. These were processed in a

spreadsheet to derive a windage force resolved to the contact patch at road level. The results of these calculations are posted to.

REFERENCE ANGLE FOR SIDESLIP

Understanding of this section is essential to understanding the process of following the top-to-tail journeys of the trailer as it steps away from the tractrix. These observations lead to the following self-evident truth:

Axiom: "A disturbed tractrix object being pulled from the same asymptote,

will form its own tractrix trajectory in relation to that asymptote."

The example here is where the trailing point on a tractrix is disturbed by moving the trailing point upwards maintaining the leading point on the asymptote. (i.e. keep the finger on the chain end and push the watch up.) This is illustrated in using the classic tractrix example of Perrault's fob watch from around the year AD1670. Osterman and Wanner.

It should be apparent that the path of the fob watch would follow a new tractrix, starting from the new position but reflecting a relocated Y-Axis. Similarly, the position of a trailer is "disturbed" by a sideslip action of the tyres, only in this case the new location is below the tractrix. This is illustrated in where the first sideslip moves from Point P to Point Q.

If the reader were to entertain the notion of a sideslip switch which could be flicked to an OFF position, then the path of the trailer would continue after Point Q and follow the new relocated tractrix. The tangent angle of the trailer centreline would then become the reference angle from which any subsequent slip angle should be measured.

PIECEWISE LINEAR COMPARISON

By way of overview to this process of deviating from the tractrix, we offer an early scaled drawing of the paths of the two tyres under sideslip in . The manual drafting was performed at a lower hitch point velocity to derive a velocity for presentation of maximum sideslip without incurring complete tyre saturation. The tedium of the drafting exercise encouraged a simpler mathematical approach. See also Brell and Thambiratnam .

The work involved to produce was tedious and relatively inflexible. A similar process is developed mathematically permitting lower step distances and offering more flexibility and is expounded below.

As a first step in the mathematical presentations was the preparation of a diagram. is a representation of the caravan CofG journey from point P to point Pnew. This journey is accomplished by hitchpoint travel from Q to Qnew under the forces of centrifugal and windage action at Point P causing sideslip or steer. These forces "steer" the wheels from a trailer articulated position away from the tractrix accumulating a greater angular momentum in the caravan's moment of inertia. Most importantly, the sideslip "steering" provides the successive positions from which to calculate the results of the ensuing steps of hitchpoint travel in an iterative fashion where the "new" positions become the "old" positions in successive iterations.

The next step was to develop a mathematical relationship between hitchpoint travel and the locus of point P. The point Pnew is unknown but sits on the intersection of a ray from point P and an arc centred on Qnew and of radius (a). We derive a pair of

simultaneous equations for the dimension (x), where (s) is hitchpoint step size:

Knowing that $(\alpha = \alpha \alpha \alpha \alpha)$ and combining Eqn (3) with (4) we get:

There appears no closed form solution to Eqn [5] and will thus be solved numerically for each hitchpoint step.

Travel on the tractrix is slower than hitchpoint velocity. Instantaneous velocity and instantaneous radius of curvature at point P is given by the following .

There is need to calculate centripetal and windage forces acting at the tyre contact patch. Windage was determined earlier with the following curve fit polynomial:

Since each tyre is defined to carry half of the side force

We note the windage forces to be relatively small, supporting Kurz and

Anderso: "Aerodynamic forces are less significant than tire forces in modelling vehicles, but they do have noticeable effects and are thus often included."

The relevant values of Table 3 and Table 4 are graphed in (Figure 20). An arbitrary census line is drawn over these graphs to show how far the hitchpoint has had to travel to cross a given articulation angle point.

At the census line in (Figure 20) the caravan swayed from 9.350 to 7.000 = 2.350. The according distances travelled are 0.56, 0.72 & 1.27 m for 195, 225 wide tyres and respectively, 1.27 m for the no-slip curve. At 15 m/s hitchpoint velocity, the time taken is calculated at 37, 48 & 85 milliseconds. The average angular velocity follows at 63.5,

49.0 & 27.6 degrees/second ($\alpha = 1.10, 0.86 \& 0.46$ radians/second respective to the prior presentation order). Angular momenta (L) are determined by $L = JH \alpha$.

\ The increased momenta shown in show how much more momentum is carried forward to the discharge cycle of caravan oscillation. The improvement for the wider tyre may mean the difference between recovery and terminal. The zero-slip improvement highlights that there is still potential for improvement by the tyre-makers.

Another viewport of tyre performance comparison is found in a parametric look, where the parameter is hitch travel.

Thus, when the hitch has travelled a distance from maximum articulation (9.350), the wider tyre caravan configuration has travelled say, 3.0 degrees. The narrower tyre configuration has already covered 3.5 degrees. This improvement for the wider tyre can be seen in the graph for the full length of hitch travel studied.

Finally, we determine the rotational kinetic energy in yawing the caravan an amount equal to the angular displacement as stepped in using Work/Energy equivalence, according to using angular velocities .

67.3 & 15.6 kJ for the narrow & wider tyres respectively. In summary, at the practical end of hitch travel assessment (1.5 m), the sample caravan configured with the 195/65 R15 tyre builds 67.3 kJ of kinetic yaw energy to fuel the subsequent discharge phase

of oscillation. By contrast, the 225/45 R17 configured caravan builds only 15.6 kJ of kinetic yaw energy to be acquitted and has better damping characteristics compared to the narrow tyre, as well.

CONCLUSION

To test the contribution of different tyres to the towing stability of a trailer/caravan a sample caravan model was devised and was badly loaded (i.e. trailer mass on drawbar and axle centrelines).

To address concerns that poor damping performance of the better side-slip performing tyre may detract from gains shown, damping was calculated to literature sourced equations. The wider tyre showed better damping performance as compared to the narrower tyre.

Using a modest externally forced articulation angle and a modest hitch speed, the sample caravan model tracking behaviour was calculated for windage and centrifugal action. Angular momentum at an arbitrary census point was shown to be lower for the wider low-profile tyre. This would result in lesser available “fuel” for the remainder of the oscillation cycle.

At low angles of articulation, windage and centrifugal action were found to be low. Accordingly, the calculation of rotational kinetic energy was ignored beyond a practical hitch travel of 1.5 m. This presented only a minimal effect to the quantitative conclusion and did not affect the qualitative conclusion.

The rotational mechanical work done in rotating the caravan from approximately 10 degrees of articulation to the commencement of the next phase of oscillation was found to be about four times higher for the narrower tyre.

Given that a lesser amount of frictional work done (damping) is subtracted from the kinetic energy equivalent, an easy conclusion follows that the wider low-profile tyre offered significant increase in performance in the directional stability context.

The net rotational kinetic energy built up at the zero-articulation point is discharged against windage and tyre friction on the opposite side of the oscillation cycle.

If then, the articulation angle thereby created on the opposite side is greater than the original and ingoing articulation, runaway oscillation may prevail.

Whilst the wider low-profile tyre may not prevent fishtailing in a scenario where speed, loading and body roll are strongly implicated, it provides a good defence against the fishtailing phenomenon generally.

The contribution of this paper lies in the comparison of driving energy of the tyres into the fishtailing phenomenon. The paper provides a methodology to perform this comparison mathematically by linear step-wise methods using tyre steering as motivating action for the caravan oscillation.

Future work could include the effects of the increased yaw energy in the discharge part of the fishtailing cycle. Critical articulation angle could then be defined as the point where ingoing and outgoing are the same. An outgoing articulation angle greater than the previous ingoing angle would indicate oscillation instability.

In the piece-wise analysis of this paper, yaw energy accumulation (fuelling fishtailing) is greater at the higher articulation angles where high slip angles are associated with greater angular velocity. The Eigenvalue pathway uses only the linear portion of the lateral slip curves to explain oscillating behaviour. Future work might explain the implications of using only a part of a rapidly changing cornering coefficient.

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