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Second Grade Elementary School Students' Differing Performance on Combine, Change and Compare Word Problems

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Abstract

The purpose of the present study was to investigate the word problem-solving skills of 47 second-grade students by examining how they performed on combine, change and compare word problems. The results of the repeated measures ANOVA showed that students scored significantly lower on compare problems than on combine and change word problems. Based on the results of this study, we disproved the hypothesis that the students in our sample experienced more difficulties in compare problems as a result of the so-called consistency effect; in fact they performed equally well on inconsistent and consistent compare problems. The findings indicate that the core problem which the students experience might be associated with the fact that they have difficulty in general with processing relational terms like 'more than' and 'less than'. Future studies should, therefore, provide more insight into the reasons why compare problems in particular cause so many difficulties for both young and older students. This information would be helpful when it comes to developing more adequate word problem instructions that can be implemented in the curriculum of contemporary math education.

Keywords Word problem-solving; Combine word problems; Change word problems; Compare word problems; Consistency effect

Introduction

Combine, change and compare problems are frequently offered in the early grades of elementary school. It is therefore valuable to investigate the performances of elementary school children on these three word problem types. Research showed that students from first grades rarely make errors on combine and change word problems, but that difficulties often arise when these students have to solve compare problems [1]. The aim of the present study is, therefore, to provide clues as to why young, second grade students in particular experience more difficulty with compare problems than with combine and change problems.

Background

According to Realistic Math Education (RME), mathematics should be connected to realistic (verbal) contexts, stay close to children, and be relevant to society [2]. Math problems in RME (and other contemporary math approaches) are, therefore, generally presented as text rather than in a numerical format. However, students have been shown to experience more difficulties with solving these so-called word problems already in the first grades of elementary school [1,3,4]. This discrepancy between performance on verbal and numerical format problems strongly suggests that factors other than calculation ability contribute to children's word problem-solving success [1,5,6].

Many previous studies report that an important factor in how students perform on word problems is their comprehension of the text of a word problem [2,5,7-9]. The comprehension of a word problem mainly concerns the identification of (verbal and numerical) relations between the elements that are relevant for the solution, as these are used in the construction of a visual representation that reflects the structure of the word problem [1,4,10,11]. More specifically, the verbal and numerical information that is relevant for the solution of the word problem should be connected and included in a visual representation, in order to clarify the problem situation described in the word problem [5,12,13].

In the early grades of elementary school, three types of word problems are frequently offered to the students, namely, combine, change and compare problems. These three specific types of word problems play a key role in several scientific studies investigating students' word problem-solving performance [5,7,12]. Because combine, change and compare problems also play a central role in the present study, an explanation of each of these types of word problems is given below.

In the combine word problem, reflected in the first word problem example, a subset or superset must be computed given the information about two other sets.

[Word problem example 1]

Mary has 3 marbles. John has 5 marbles. How many marbles do they have altogether?

This type of problem involves understanding part-whole relationships and knowing that the whole is equal to the sum of its parts [14]. Figure 1 reflects a possible way in which the problem structure of a combine problem can be represented.

The second commonly investigated type of word problem is a change problem (see word problem example 2).

[Word problem example 2]

Mary had 3 marbles. Then John gave her 5 marbles. How many marbles does Mary have now?

Change problems are word problems in which a starting set undergoes a transfer-in or transfer-out of items, and the cardinality of a start set, transfer set, or a result set must be computed given information about two of the sets [15]. In other words, a change problem starts with a beginning set in which the object identity and the amount of the object are defined. Then a change occurs to the beginning set that results in an 'ending set' in which the new amount is defined [16] In Figure 2 an appropriate visual-schematic representation of the problem structure of a change problem is given.
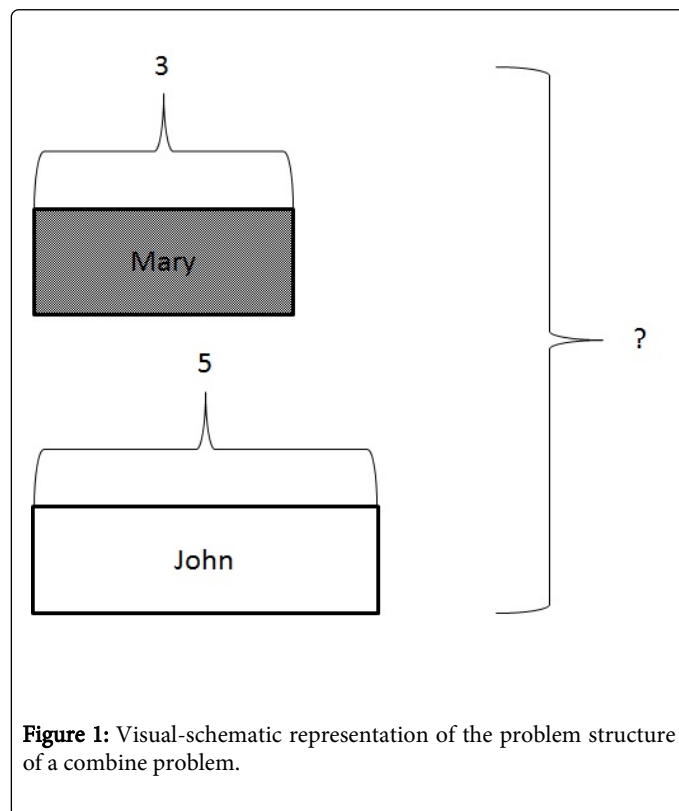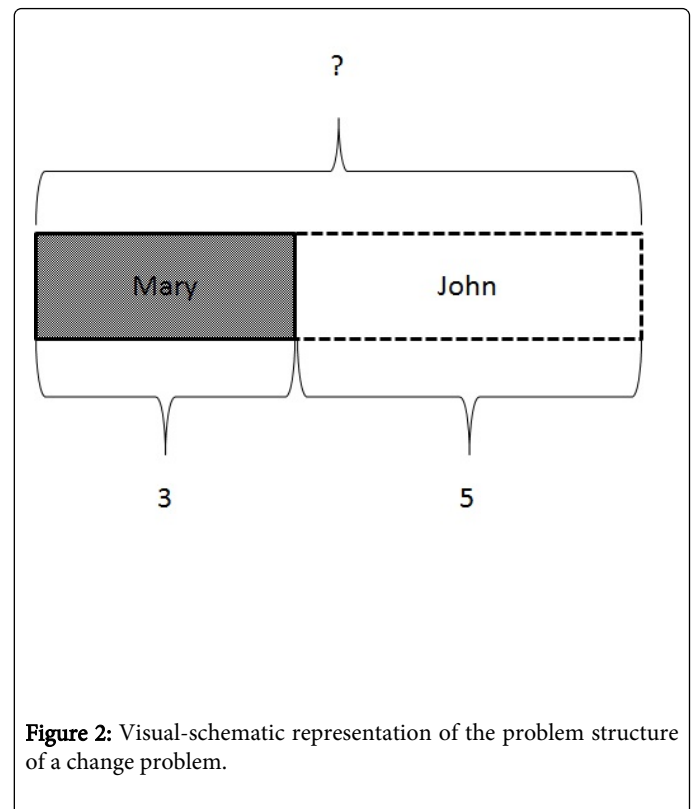

Figure 1: Visual-schematic representation of the problem structure of a combine problem.

The last word problem type that is investigated in many studies is a compare problem (see word problem example 3).

[Word problem example 3]

Mary has 5 marbles. John has 8 marbles. How many marbles does John have more than Mary?

In compare problems the cardinality of one set must be computed by comparing the information given about relative sizes of the other set sizes; one set serves as the comparison set and the other as the referent set. In this type of word problem, students often focus on relational terms like 'more than' or 'less than' to compare the two sets and identify the difference in value between the two sets [6,11,17]. Figure 3 reflects the visual-schematic representation of the problem structure of a compare problem.

Figure 2: Visual-schematic representation of the problem structure of a change problem.
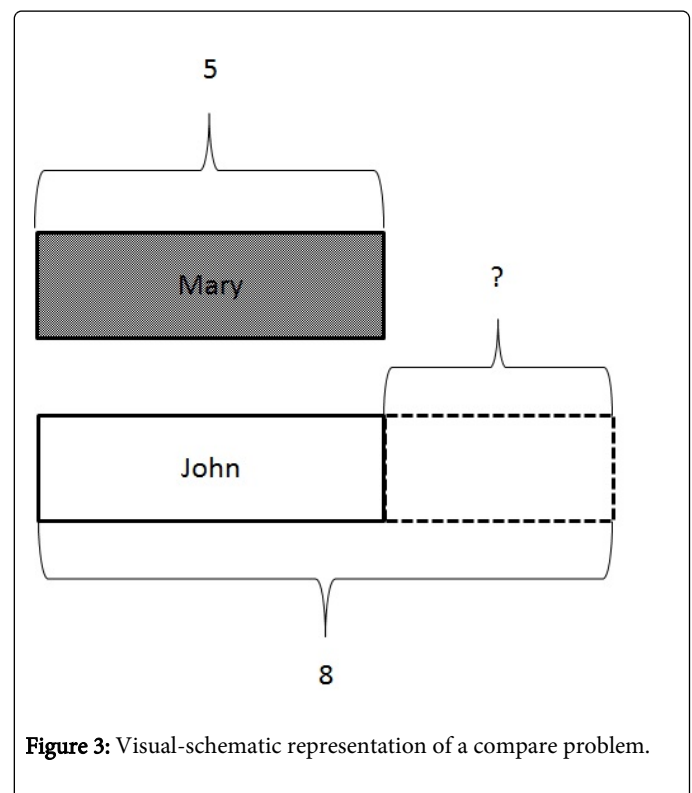
Figure 3: Visual-schematic representation of a compare problem.

Research by Cummins et al., performed in the nineteen eighties, showed that first grade students rarely make errors on combine and change word problems, but that difficulties often arise when these students have to solve compare problems. More recent studies mainly

focused on how older students, namely sixth and seventh grade students [6,11], and undergraduates [3,4] performed compare problems.

The first explanation offered for these difficulties with solving compare problems is the hypothesis that young students have not yet understood that the quantitative difference between the same sets can be expressed in parallel ways with both the terms more and fewer. Their lack of knowledge and experience with the use of language to describe relations between quantities could underlie their relatively poor performance in solving compare problems. Notably, the lack of knowledge about the symmetry of language in the case of quantitative comparisons makes it difficult for young students to perform the translation procedure correctly [17].

A second possible explanation for this difficulty with compare problems might be the extent to which the semantic relations between the given and unknown quantities of the problem are made explicit [18-21].

A third frequently investigated hypothesis that might explain the difficulties with solving compare problems involves examining whether the relational keyword of the compare problem ('more than' or 'less than') is consistent or inconsistent with the required mathematical operation [6]. In so-called inconsistent compare problems [3,4,6]. the crucial mathematical operation cannot be simply derived from the relational keyword ('more than'). The relational term in an inconsistent compare problem primes an inappropriate mathematical operation, e.g., the relational term 'more than' evokes an addition operation, while the required operation is subtraction. This accounts for the difficulty with solving inconsistent compare problems. The finding that students make more errors on inconsistent than on consistent compare problems is referred to as the 'consistency effect' [6,9,11]. Interestingly, the consistency effect has until now only been examined in students in higher elementary school grades and at university [6,11]. This raises the question whether the difficulties that young students (i.e., in lower grades of elementary school) experience with solving compare problems are confined to inconsistent compare problems, as is often the case with older students. Or, do young students experience difficulty in general with processing the verbal information contained in a compare problem?

The Present Study

Combine, change and compare problems are more frequently offered in the early grades than in the later grades of elementary school. It is therefore valuable to investigate how young elementary school children's performance on combine and change problems differs from their performance on compare problems. The only previous study on this topic was conducted in the nineteen eighties. Hence, it is relevant to evaluate whether these findings are still valid after 25 years which have seen significant adaptations in school curricula as well as changes in society.

We hypothesized that students will perform poorer on compare problems than on combine and change problems, and examined whether young students experience more difficulties with solving compare problems because of a consistency effect. Based on the findings of previous studies of older students [6,11], we hypothesized that also younger students would make more errors on inconsistent compare problems than on consistent compare problems.

Method

Participants

Forty-seven second-grade students (26 boys, 21 girls) from two classes from a mainstream elementary school in the Netherlands participated in this study. The mean chronological age of the students was 89 months (SD = 4 months; range: 79 - 96 months). Parents provided written informed consent based on printed information about the purpose of the study.

Instruments and procedure

Word problem-solving performance: Students' performances on the three different types of word problems (combine, change, and compare problems) were examined with an 18-item Word Problem-Solving test (Table 1). The WPS test was divided into two subtests containing nine word problems (three of each type of word problem). The items of each WPS subtest were presented on a different page and administered by the teacher in two classroom sessions of approximately 30 minutes. Each word problem was read out loud twice to the students to control for differences in decoding skills. After reading the word problem, students had to solve the word problem within three minutes and during this time the teacher did not speak to the student.

To examine the consistency effect in the compare word problems, both consistent and inconsistent compare problems were offered to the students. Consistency referred to whether the relational term ('more than' or 'less than') in the word problem was consistent or inconsistent with the required mathematical operation. The relational term in a consistent compare problem primed the appropriate mathematical operation (e.g., 'more than' when the required operation is addition, and 'less than' when the required operation is subtraction). The relational term in an inconsistent compare problem primed the inappropriate mathematical operation ('more than' when the required operation is subtraction, and 'less than' when the required operation is addition). The internal consistency (Cronbach's alpha) of the WPS test, measured in this study, was high (Cronbach's alpha=0.82).

Problem type	Word problem
Combine	1. Mary has 2 marbles. John has 5 marbles. How many marbles do they have altogether?
	2. Mary and John have some marbles altogether. Mary has 2 marbles. John has 4 marbles. How many marbles doe they have altogether?
	3. Mary has 4 marbles. John has some marbles. They have 7 marbles altogether. How many marbles does John have?
	4. Mary has some marbles. John has 6 marbles. They have 9 marbles altogether. How many marbles does Mary have?

	5. Mary and John have 8 marbles altogether. Mary has 7 marbles. How many marbles does John have?
	6. Mary and John have 4 marbles altogether. Mary has some marbles. John has 3 marbles. How many marbles does Mary have?
Change	
	1. Mary had 3 marbles. Then John gave her 5 marbles. How many marbles does Mary have now?
	2. Mary had 6 marbles. Then she gave 4 marbles to John. How many marbles does Mary have now?
	3. Mary had 2 marbles. Then John gave her some marbles. Now Mary has 9 marbles. Hoe many marbles did John give to her?
	4. Mary had 8 marbles. Then she gave some marbles to John. Now Mary has 3 marbles. How many marbles did she give to John?
	5. Mary had some marbles. Then John gave her 3 marbles. Now Mary has 5 marbles. How many marbles did Mary have in the beginning?
	6. Mary had some marbles. Then she gave 2 marbles to John. Now Mary has 6 marbles. How many marbles did she have in the beginning?
Compare	
	1. Mary has 5 marbles. John has 8 marbles. How many marbles does John have more than Mary?*
	2. Mary has 6 marbles. John has 2 marbles. How many marbles does John have less than Mary?
	3. Mary has 3 marbles. John has 4 marbles more than Mary. How many marbles does John have?
	4. Mary has 5 marbles. John has 3 marbles less than Mary. How many marbles does John have?
	5. Mary has 9 marbles. She has 4 marbles more than John. How many marbles does John have?*
	6. Mary has 4 marbles. She has 3 marbles less than John. How many marbles does John have?*
Note: Inconsistent compare problems are indicated with an asterisk.	

Table 1: The 18 items of the word problem-solving test.

4.3 Data analysis

To examine students' performance on the three types of word problems, a repeated measures analysis of variance (ANOVA) with type of word problem (combine, change and compare) as within subject factor was performed. Follow-up tests were performed using paired sample t-tests. Subsequently, a one sample t-test was performed to examine the existence of a consistency effect; the performance on consistent compare problems was compared with the performance on inconsistent word problems. In all analyses we tested with an alpha of 0.05. Effect sizes (partial eta-squared [$\eta p2$]) were computed to estimate the practical significance of the effects.

Results

Performance on combine, change and compare word problems

Results of the repeated measures ANOVA demonstrated a significant main effect of word problem type, $F_{(2,92)}=12.90$, $p<0.001$, $\eta p2=0.36$, indicating a large effect size [22]. Figure 4 shows the accuracy on each of the three types of word problems (combine word problems, M=4.89, SD=1.46; change problems, M=4.85, SD=1.43; compare problems, M=3.81, SD=1.53). In line with our expectations, second grade students scored significantly lower on compare word problems than on combine ($t_{(46)}=4.69$, $p<0.001$) and change ($t_{(46)}=4.90$, $p<0.001$) word problems. No differences in students' performance on combine and change problems existed ($t_{(46)}=0.27$, $p=0.79$).

A consistency effect in compare word problems

The one sample t-test on students' performance on compare problems revealed no main effect of consistency, $t_{(46)}=0.15$, $p=0.88$, indicating that a consistency effect was absent in our sample. This finding showed that students performed equally on consistent (M=1.91, SD=0.83) and inconsistent (M=1.89, SD=0.98) compare problems.

Conclusion and Discussion

The present study aimed to provide clues as to why young, second grade students in particular experience more difficulty with compare problems than with combine and change problems. As expected, second grade students made more errors on compare problems than on the other two types of word problems.
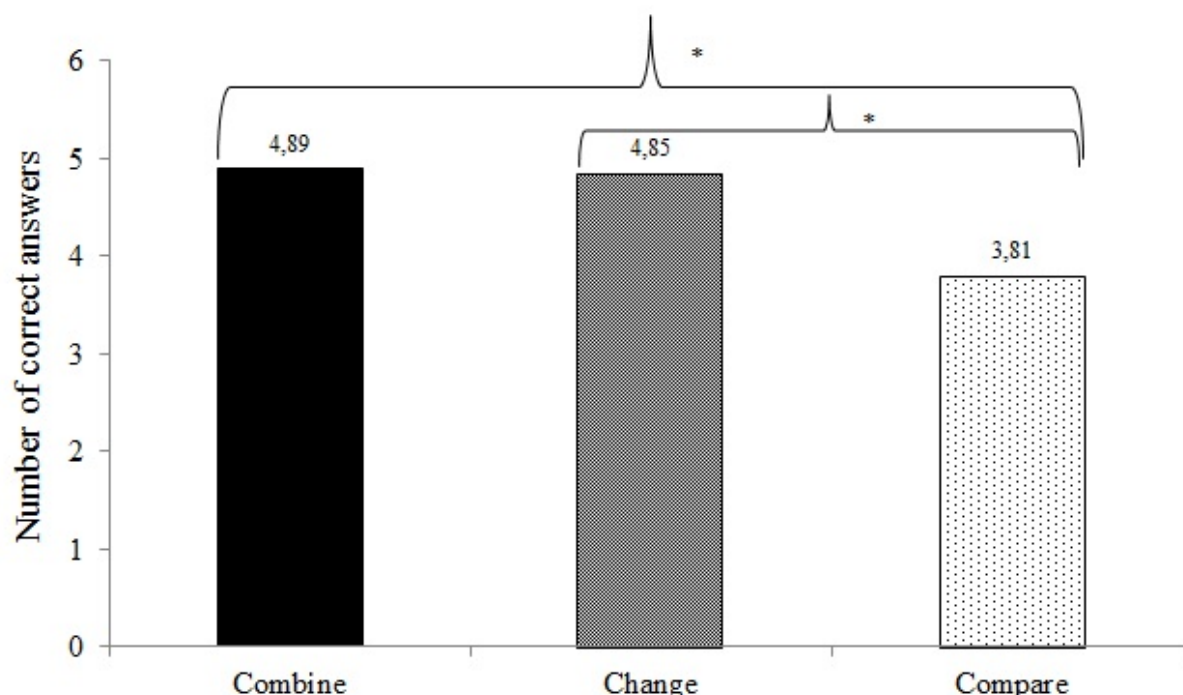
Figure 4: Results repeated measures ANOVA: students' accuracy on each word problem type, * $p<0.001$.

Importantly, a consistency effect in compare problems was not found in our study; the second grade students performed equally well on inconsistent and consistent compare problems. A difficulty in general with processing relational terms like 'more than' and 'less than', is a plausible explanation for the lack of the consistency effect in our findings. Students in the lower elementary school grades might not yet possess the conceptual knowledge required to fully understand compare problems and this might explain their difficulties with solving this particular type of word problem [23]. They apparently do not have the knowledge to comprehend and process the linguistic input of a compare problem and recall the appropriate problem structure [24]. For example, a child may understand the part-whole relationship of a combine problem, but not yet understand how the comparative verbal form (e.g., how many more Xs than Ys) maps onto the sets. As already mentioned by d'Ailley [17] students might have difficulties understanding the fact that the quantitative difference between the same sets could be expressed in parallel ways with both the terms 'more' and 'fewer'. Hence, the poorer performance on compare problems, which was found in this study, might be explained by a lack of knowledge about the symmetry of language in the case of quantitative comparisons; this makes it more difficult for young students to perform the translation procedure correctly.

Future research and implications

Future studies should, for example, examine the reasons why compare problems in particular cause so many difficulties in young students, and evaluate the possible influence of the development of higher language skills. Research has indicated that the comprehension and processing speed of complex language (i.e., students mastery of relational terms which describe linguistic relations between elements that are relevant for the solution) continue to develop beyond childhood and into adolescence [13,20].

As scientific research during the last decades has shown that the difficulties students experience when solving compare problems remain stable over time [6,11] another important topic for future research would be to focus on the development of effective word problem-solving instruction. Adequate word problem-solving instructional programs that teach students to solve these types of problems are still limited, or they have not been implemented in the educational practice of elementary schools.

Instructional approaches, like Schema-Based Instruction and the Solve It! method, that focus on explicit instruction in cognitive and metacognitive strategies to help students identify and represent the problem structure and improve their word problem-solving performance, seem promising [7,14,15,25-29]. These instructional approaches move away from keywords and superficial problem features and focus more on helping children find the underlying problem structure.

In SBI, for example, students are taught to identify and represent the problem structures of certain types of word problems (i.e., combine, change and compare problems) and are encouraged to reflect on the similarities and differences between these problem types [16,28] However, the instructional programs SBI and Solve it! are generally only used by researchers. Therefore, educational practice in regular elementary school classrooms might be improved if teachers were to implement and work with word problem instruction as well. One of the main hurdles encountered during the implementation of these instructional approaches is that they require greater effort and good classroom management skills [29]. This is an important reason why the effectiveness of SBI and Solve it! has until now been mainly

investigated in small groups of children with learning and mathematical disabilities [15,16] and not in a regular classroom setting. Therefore, before these kinds of instructional programs can be implemented in the curriculum of contemporary math education, it is essential that they are made easy to understand for both students and teachers, and that they can be implemented with a relatively small amount of effort.

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