Rough Sets Based Product Mix Analysis

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Abstract

In this paper, the TOC approach to the product mix optimization is modeled using rough set theory. Rough set theory is a new mathematical tool for imperfect data analysis and supports approximations in decision-making. The product mix adjustments reduce the volume of some products to maximize the revenue by producing the products with high profitability. TOC heuristics provides a solution that is, implicit, and rough sets provide an explicit solution by rule extraction from the information system. The exact concepts described by *lower* and *upper approximations* are determined by an indiscernibility relation (equivalence) on the domain, which in turn may be induced, by a given set of attributes ascribed to the objects of the domain. The extensional description and intentional description is studied here for feature and rule extraction.

Keywords: Rough sets; rule extraction; indiscernibility; K-means clustering.

1. Introduction

The product mix problem considered in this paper is useful for small and medium enterprises (SME's) for aggregate planning and production control for decision making by managers where proprietary software may be less affordable due economical reasons. The constraints on the resources availability enforces decisions, to determine feasibility estimates on demand, bottle neck recognition, outsourcing and the product mix adjustments that reduces the volume of some products, to maximize the revenue by producing the product with high profitability.

The paper analyzes and demonstrates, using a case study, the application of rough sets for a product mix problem [13] [14], involving the performance criteria and the decision of selecting the volume and mix of products that maximize the profit within market and technical constraints. Rough sets a recent emergence introduced by Z. Pawlak [7] [8] in the early eighties as a major mathematical approach to manage uncertainty, due to imprecision, noisy or incomplete information. The main focus is in the area of knowledge reduction, and ambiguity caused by limited discernibility of objects in the domain of discourse. Rough sets intent to approximate a rough (imprecise) concept in the domain of discourse by a pair of exact concepts, called the *lower and upper approximations.* These exact concepts are determined by an indiscernibility relation on the domain, which in turn may be induced, by a given set of attributes ascribed to the objects of the domain. The extensional description finds the reduct by removing the dispensable attributes without reducing the content of the information system. Intentional description extracts rules based on the set of rules that describe the scope of the category.

1.1 Supply/ Demand Chain for DIP molding case

Dip molding is exactly what the name suggests; a former is preheated and then dipped into the PVC plastisol and allowed to dwell. The former is withdrawn with a gelled coating of the plastisol around it and placed into an oven where it is cured. After cooling the molding is stripped from the former by hand, tool and/or compressed air. It is probably the most simple of all the molding processes but certainly it is a very flexible and cost effective one that has applications in many if not all industries.

Dip molding systems relies on the quality and timely supply of raw materials, design of molds as required in the industries, that can satisfy three different segments, namely consumer, automobile and industrial needs. The demands in the above segments is quit large, and is determined by the orders placed by the industry as in automobile or industrial application, having a specific design and specifications, and a variety of products to be satisfied. The demands in the consumer segment are mainly controlled by the distributors geographically situated in the country and abroad and require standard products. The supply chain for the dip mold industry largely depends on the several participating companies including the supply of raw material, tooling parts and its design which is a specialized industry, manufacturers and the special process equipments, and the distribution system.

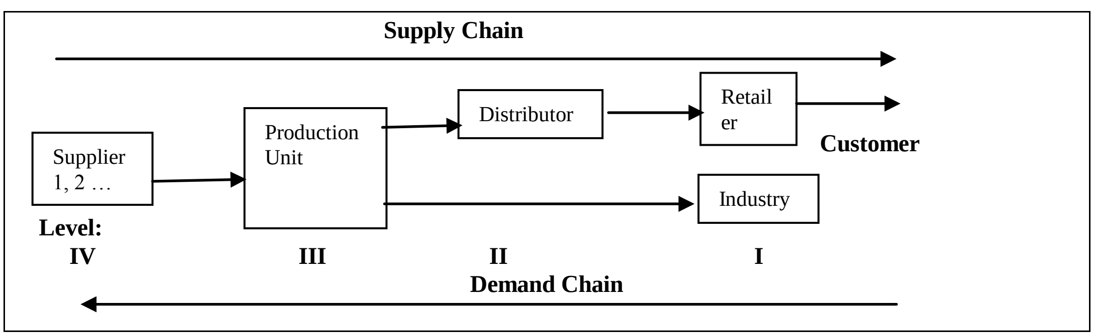


Figure 1: Supply/Demand Chain Model

The supply/demand chain, see Fig 1 activities [34] include suppliers, manufacturers and distributors, or an industry (customer) such as an automobile or an industrial application. Outsourcing of tooling to a tooling industry is important and can save on time and cost for the manufacturer. The main objective of the case study is the quantitative analysis the Information System of product mix, to design of knowledge base to make the supply chain robust, reliable and responsive to the market demands.

The paper is organized as follows: Review of literature is discussed in section 2, the basics of rough sets theory in section 3. Product Mix problem for DIP Molding and Information System is given in section 4 and 5 respectively. The Proposed methodology using rough sets, and the results and conclusions are discussed in section 6 and 7 respectively.

2. Review of the Literature

Theory of constraints (TOC), conceived by Goldratt [1] [2] [3] [4] has drawn attention from all corners of the manufacturing and academicians. TOC clarifies that constraints play an important role in the organization's goal and limits the system's throughput. The systems can gain from its optimization only when all the constraints are exploited and utilized to the fullest of its capacity. The product mix problem have been dealt with by many researchers using the linear programming approach with TOC as the starting point as we can see in Balakrishnan and Cheng [14] identifies and exploits the critically constraint resource to maximize the output using the theory of constraints (TOC) for a product mix problem. Decision making in TOC is implicit or infeasible when multiple resource constraints exist. The paper proposes a genetic algorithm to make the TOC problem explicit, and is also studied in Luebbe and Finch [15].

The TOC product mix problem is implicit when multiple constraints exist. Mathematically formulated as Linear Programming (LP) model, provide an explicit solution for small to medium size problems within reasonable computation time. The TOC problem is approached by using a TOC heuristic with some search techniques. TOC heuristics as given by Luebbe and Finch [15], Lee and Plenert [18] can be applied to solve product mix optimization problems with the focused five steps through breaking the systems constraint limitations and maximizing systems throughput. According to Luebbe and Finch [15], Lea and Plenert [16], Plenert [17] the TOC heuristic was inefficient and produced non-optimal solution when certain new product alternatives are available. Also Posnack [19] and Maday [20] that partial product should be allowed to be manufactured in the next planning horizon, and that the product quantity must be slacked to real number and not integer. Lea and Fredendall [21] first determined a feasible solution rather than an optimal solution and latter with a neighborhood search increased the product quantity so as to eliminate the idle time for each constraint until all constraints could be fully solved.

Hsu and Chung [22] using the TOC heuristic obtained feasible solution by identifying a new constraint if the previous constraint did not give a feasible solution, and iteratively tried to decrease the quantity of the lowest priority product type, hence reducing the overload on the constraint, and utilizing fully the constraints by adjusting the products' quantity to satisfaction. All the approaches mentioned above had some shortcomings: the computation of work load was large and could not be easily applied to large scale product mix optimization.

It is obvious that with multiple constraints, TOC heuristic might produce an optimal solution or an infeasible solution. The immune algorithm (IA) devised by Wang et al. [23], for a product mix optimization search solution, is useful in small /large scale situation. The immune mechanism, improves the search ability and adaptability, and increases the global convergence of immune evolution.

The genetic algorithm in Onwubolu and Matungi [25] and Godfrey et al. [26] gives an explicit solution for large real world problems the genetic algorithm which has a large parallel processing capacity to deal with combinatorial problem.

The decision making is uncertain under the constraints of technical and market demand, which are fuzzy, is approached by a smooth logistic membership function. The vagueness and the level of satisfaction for theory of constraints (TOC) for a product mix problem are studied in Bhattacharya et al. [13] to include the flexibility in manufacturing, to meet the customer's demands.

The fuzzy linear approach is used for optimization of product mix by Pandian Vasant [27] and Pandian et al. [28] has considered the interaction between the analyst, the decision maker and implementer in production planning management to find an optimal solution good enough for the decision maker in a fuzzy environment.

The study in Susanto et al. [29] has remodeled the LP model using fuzzy sets to optimize the product-mix problem applied to the real world data of a food processing industry. The degree of satisfaction is in terms of the total profit obtained and least wastages.

The study by Vasant et al. [32], a tripartite interactive Fuzzy Linear Programming (FLP) is used to solve a real-world industrial production planning. Decision making in profit optimization is solved interactively between the analyst, decision maker and implementer in a fuzzy environment using the S-curve function. Mula et al. [31] have studied the production planning including ambiguous situations in the production processes such as breakdown of machines, skills of employee, lack of efficient information and intuition of a decision maker. Gaur and Ravindran [30] considers excessive inventories as worse cost to squeeze profit of company and mange to reduce it. Hasuike [33] considers the product mix problems under randomness and fuzziness.

TOC heuristic produces unrealizable solution when a manufacturing plant has multiple resource constraints. Tabu search method by Onwubolu et al. [24] is applied to TOC product mix heuristic to determine the optimal or near optimal product mix and results for small to medium size problems are comparable to the optimal methods. Large size problems have no feasible solution by optimal methods were solved in reasonable times and with quality solutions. This approach is appropriate for production planners for product mix problem in manufacturing industry.

The work in this paper relates to the application of rough sets, for extracting rule from the knowledge base, in which the superfluous attributes are removed keeping intact the contents of the information system. This novel approach analyzes and demonstrates an explicit method, for representing an information system for the product mix in a simple and easy to implement modular rules.

3. Rough Sets Theory- Basics

This theory initially, developed for a finite universe of discourse partitions the knowledge base using the equivalence relation defined on the universe of discourse. The data in rough sets is organized in a table called the decision table, in which the rows correspond to objects and the columns correspond to attributes. Equivalence class is used for partitioning of the data set based on the decision attributes and the conditional attributes.

Rough set defines three regions based on the equivalent classes induced by the attribute values: *lower approximations*, *upper approximations* and *boundary*. Lower approximation contains all the objects, which are classified surely based on data set. Upper approximation contains all the objects, which can be classified as probable, and the boundary class is the difference between the upper approximation and the lower approximation.

Let U be a non-empty finite set called the universe of discourse and A a non-empty finite set of attributes. Then an information system can be represented as a pair $I_B = <U, A>$. A decision system is any information system of the form $A = (U, A \cup \{d\})$ where $d \notin A$ is a decision attribute. The *attribute-value* table represents an information system, in which the rows are labeled by objects of the universe and the columns by attributes. The equivalence relation I_B, for every subset of attribute $B \subseteq A$ can be associated on U, $I_B = \{(x, y) \in U : \forall a \in B, a(x) = a(y)\}$. Then $I_B = \bigcap_{a \in B} I_a$. If $X \subseteq U$, sets $\{x \in U : [x]_B \subseteq X\}$ and $\{x \in U : [x]_B \cap X = \phi\}$, where $[x]_B$ denotes the equivalence class of object $x \in U$ relative to I_B, are called $B-lower$ and $B-upper\ approximations$ of X in S and denoted by $\underline{B}X, \overline{B}X$ respectively. $X \subseteq U$ is $B-exact$ or $B-definable$ in S if $\underline{B}X = \overline{B}X$. It can be observed that $\underline{B}X$ is greatest $B-definable$ set contained in X and $\overline{B}X$ is a smallest $B-definable$ set containing X. Further the notion of knowledge reduction aims to obtain irreducible but essential parts of knowledge encoded by given *information system*, which constitutes the reducts of the system. Thus the essence of the information is intact, with superfluous attributes being removed are characterized by discernibility matrices and discernibility functions.

This method represents a category providing intentional description based on the set of rules that describe the scope of the category. The choice of such rules is not unique. The rule-extraction method we use is based on Ziarko and Shan [9].

Let S be an information system with *n* objects. The discernibility matrix *M* is asymmetric $n \times n$ matrix with entries c_{ij} as given below:

$$c_{ij} = \{a \in A \mid a(x_i) \neq a(x_j)\} \ for i, j = 1,2,...,n$$

Each entry in the matrix consists of the set attributes upon which objects x_i and x_j differ. A discernibility function f_S for an information system S is a Boolean function of m Boolean variables $a_1^*,...,a_m^*$ corresponding to attributes $a_1,...,a_m$, is defined as follows:

$$f_s(a_1^*,...,a_m^*) = \wedge \{ \vee c_{ij}^* \mid 1 \leq j \leq i \leq n, c_{ij} \neq \phi\}$$

Where $c_{ij}^* = \{a^* \mid a \in c_{ij}\}$ and the set of prime implicants of f_S determines the set of all reducts of S.

4. Product Mix Problem for DIP Molding

The current focus is on level III (See Fig. 1), in which the product mix problem and the system specification and product information is simulated in LPP. The system attributes such as demand, resource capacity, batch sizes etc and the performance index such as yield, profit and customer satisfaction of the system are analyzed.

4.1 Process Structure

The typical although simplified description of a dip molding flow manufacturing process is shown in Figure 2. The flow process structure consists of 8 (eight) resources denoted alphabetically: A (Mold Former), B (Heating), C (Dipping), D (Leaching), E (Texturing), F (Cooling) and H (Ejection). In addition, tooling and certain materials also have their effect as well as processing equipment, release agents and post molding operations that may have effects upon the dipping cycle. The ejection process may require an automated robot for cooling and systematic removal of the final mold and transfer to packaging bins.

The most obvious limitation of dip molding is the use of a male only type former which gives a hollow molding. As can be appreciated, the action of immersion, dwell and withdrawal will lead to a tapered wall thickness which will be more prominent over longer lengths. However, this can be reduced to a minimum, and in some applications it is most desirable. Internally however, faithful reproduction of the formers dimensions and details can be reproduced as long as the dip and/or tooling have been carried out to meet the process requirements.

Having reviewed the disadvantages it is time to take a look at the positive side of dip molding. The time and cost required to prototype, tool and go into production is very much where the process shines. The simplicity of tooling and its ease of manufacture will show a significant reduction in both these areas. Where a component starts life as a low volume requirement and so did not warrant a high investment in tooling it is a simple matter of adding formers to the tool mass as the requirement grows. The larger tool mass will enable the part to be run on larger capacity automatic plant giving a twofold improvement as output will be increased by the extra tooling and faster cycle times available from the automatic plant. This of course forms a base for automating the plant design.

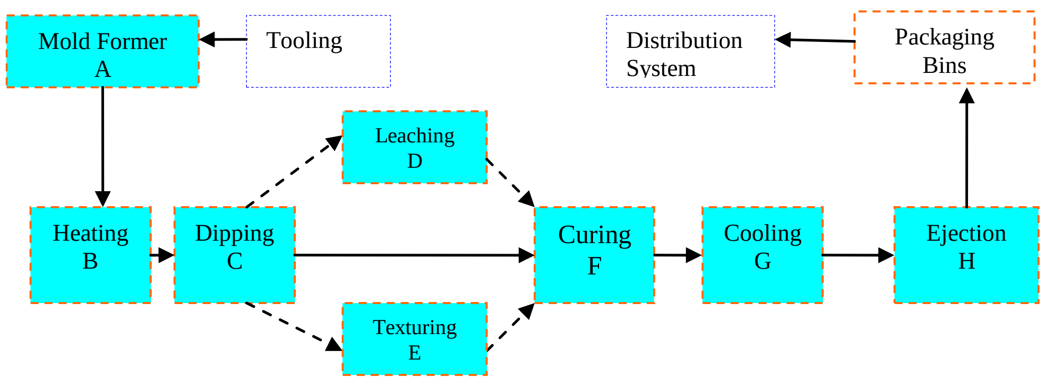


Figure 2: Process in DIP molding

Dip molding produces very attractive high gloss finished items that display no seams or flash. Internally they will reproduce tool detail to the extent where moldings can be turned inside out to display a detailed surface. This can be taken a step further as this detail can be used as reinforcement, stiffening or a physical stop.

4.2 Theory of Constraints

The Theory of Constraints (TOC), a modern approach [5] [6], focuses on the system constraints and proposes a set of principles and concepts to manage the constraints. The concept of TOC is based on three simple global operational measures: *Throughput*, *Inventory* and *Operating Expense*. The TOC approach consists of the five steps for identifying and managing the system constraints: i) Identify systems constraint/s ii) Decide how to exploit system constraint/s, iii) Subordinate everything else to the above decision, iv) Elevate system constraints, and v) Go back to step i) and do not allow inertia to be a constraint. Thus, identifying and exploiting the constraint resource to adjust the capacity of that resource to increase the production and profit of the company. However, in the process the resource constraint may shift to another location in the process. A continuous improvement in the process can increase the productivity and the systems performance.

The TOC heuristic based product mix decision-making is implicit considering a multiple constraint resources. A correctly formulated LPP approach gives better results compared to TOC heuristic based product mix approach.

4.3 Decision Making in Product-Mix using Heuristics

Decision–making in product-mix problem, considering the TOC heuristics is the starting point to identify the constraint resources. The process times (see Fig. 2) for each process for each of the nine products is given in Table 1 is designed as per the company's data. The main objective is to optimize the throughput of the system. The throughput of the system is given as the selling price/unit minus the raw material cost/unit.

Table 1: Process time required for each process

PROCESS TIME(MIN)/ PRODUCTS	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
P1	15	15	5	5	-	20	10	20
P2	15	18	5	5	-	25	10	18
P3	12	15	5	5	-	20	12	18
P4	10	10	3	5	-	18	15	20
P5	20	12	3	-	-	15	12	18
P6	18	10	4	-	10	18	10	18
P7	20	10	4	-	10	20	12	20
P8	15	12	4	-	10	18	15	18
P9	15	12	4	-	10	20	12	18

The resource capacity denotes the capacity of the resource, which is 9360 minutes, calculated for 26 days available in a particular month, and working hours in a shift for 360 minutes per day considering allowances for breaks and down-time due to power failures. The real data is collected from the manufacturer from the process times for each resource. The product data is given in Table 3. The flow process caters to more than 250 products of the company.

Table 3: Product data

PRODUCTS	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Market Potential	2500	1500	1200	1000	3000	4500	2500	960	660
Selling Price/unit	65	18	15	35	18	16	28	22	16
Raw Material cost/unit	10.5	10.8	7.6	15.6	11.8	6.8	14.3	9.5	8.7
Number of units per cycle	20	25	25	8	40	60	20	20	60
Number of Batches required	125	60	48	125	75	75	125	48	11

The load calculation for each resource is shown in Table 4. The constraint resources **A, F** and **H** are determined. The capacity utilization for the most constraint resource is **F**, which exceeds the available capacity determines the priority based on the heuristic; throughput per constraint minute. The highly ranked products are selected to produce the maximum volume meeting the demand. While the lower ranked products are considered to produce only a part of the volume, and not satisfying the demand. In this the constraint resource **F** is exploited as the capacity utilization is 141.7%, thus ranking the product for the most constraint resource **F** as: P1, P4, P8, P7, P6, P5, P3, P9, and P2. The intelligence in TOC using this heuristics determines the volume of product mix considering the fact that at least 50% of the products be produced to satisfy all customers. The detail revised calculation is given in Table 5. According to this ranking of products, the volume of each product to be produced uses the Knapsack method given in column 2 and system reaches an optimal value with a maximum throughput with the constraints moving to an another resource. The throughput calculated is **₹ 2, 28,521** satisfying at least 50% of the customers demand. It is observed that the constraint resource **H** is still critical, which can be further exploited and therefore a further reduction in the yield and the throughput as well. Hence the TOC heuristic process is considered to be implicit, when multiple constraints exist.

Table 4: Load calculation and constraint resources

PRODUCTS	**CAPACITY OF RESOURCES**							
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
P1	1875	1875	625	625	-	2500	1250	2500
P2	900	1080	300	300	-	1500	1250	1080
P3	576	720	240	240	-	960	576	864
P4	1250	1250	375	625	-	2250	1875	2500
P5	1500	900	225	-	-	1125	900	1350
P6	1350	750	300	-	750	1350	750	1350
P7	2500	1250	500	-	1250	2500	1500	2500
P8	720	576	192	-	480	864	720	864
P9	165	132	44	-	110	220	132	198
Total Time	**10836**	8533	2801	1790	2590	**13269**	8953	**13206**
Avail. Cap.	9360	9360	9360	9360	9360	9360	9360	9360
% Capacity Utilization	**115.7**	91.1	29.9	19.1	27.6	**141.7**	95.6	**141.0**

Table 5: Load calculation (revised) and constraint resources

PRODUCTS	PRODUCT MIX	**CAPACITY OF RESOURCES**							
	Volume	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
P1	2500	1875	1875	625	625	-	2500	1250	2500
P2	750	450	540	150	150	-	750	300	540
P3	600	288	360	120	120	-	480	288	432
P4	1000	1250	1250	375	625	-	2250	1875	2500
P5	1520	760	456	152	-	-	570	456	684
P6	2280	684	380	152	-	380	570	380	684
P7	1240	1240	620	248	-	620	1240	744	1240
P8	866	649.5	519.6	173.2	-	433	779.4	649.5	779.4
P9	660	165	132	44	-	110	220	132	198
Total Time		7361.5	6132.6	2039.2	1520	1543	**9359.4**	6074.5	**9557.4**
Total	**11416**								

5. Information System for a Product Mix

Further we exploit the product mix problem, to determine the effect of the parameters such as demand, resource capacity and batch sizes/ cycle time on the systems performance such as, the yield or the throughput. Using the LP model, the above parameters are varied as per the data obtained from the company for TEN different scenarios as given in Table 6, assuming same or similar products are requested by the customer. To satisfy the customers' continually varying requirements and at the same time the availability of the resources at the shop floor pose the problem of selecting the product mix. The main objective function is to determine an optimal solution of the throughput under the constraints of technical constraints given in Table 1 and the resource constraints, subject to the market constraints, that is, the demand. Thus to satisfy this demand, the optimal yield is determined giving the level of satisfaction of the customers.

The Table 6 gives TEN different scenarios which are used to obtain the optimal throughput using Linear Programming to obtain solution to 30 different periods. The experimental data is given in table 7.

Table 6: Data for varying Demand and Batch Sizes

PRODUCTS/ PARAMETERS	PROD1	PROD2	PROD3	PROD4	PROD5	PROD6	PROD7	PROD8	PROD9	TOTAL
1 Demand	2500	1500	1200	1000	3000	4500	2500	960	660	17820
Batch Sizes	20	25	25	8	40	60	20	20	60	338
2 Demand	2500	1750	1250	1000	2800	4200	2500	960	660	17260
Batch Sizes	20	25	25	10	40	60	25	20	60	285
3 Demand	2700	1500	1500	1280	2600	4560	2500	960	660	18260
Batch Sizes	20	25	25	8	40	60	25	40	30	273
4 Demand	2400	1200	1250	960	2880	4500	2500	960	640	17290
Batch Sizes	20	25	25	8	40	60	25	30	40	273
5 Demand	2400	1600	1200	800	2750	4225	2500	950	650	17075
Batch Sizes	25	40	25	8	25	65	20	25	25	258
6 Demand	2000	1000	800	1000	2500	4000	2000	900	600	14800
Batch Sizes	25	25	20	10	20	50	25	30	20	225
7 Demand	1800	1100	1240	975	2650	3800	2250	950	700	15465
Batch Sizes	25	25	20	15	25	40	25	25	20	220
8 Demand	1500	850	750	720	2000	3750	2000	900	500	12970
Batch Sizes	25	25	25	8	40	50	25	30	25	253
9 Demand	1250	750	600	760	1750	3600	1600	750	600	11660
Batch Sizes	25	25	25	8	35	40	25	30	25	238
10 Demand	1000	1000	750	1000	2025	3550	1200	950	750	12225
Batch Sizes	20	40	25	8	45	50	30	25	25	268

This experimental data form the basis for gathering knowledge about the systems attributes, system specifications, and product information and performance indices. The data in Table 7 represents an information system for the product mix problem, which is used for extracting features and the rule set using the rough set theory. The alphabet **U** represents objects in the data set and parameters **Demand, Resource Capacity** and **Batch Sizes** are the *conditional attributes* and **Yield** is the *decision attribute*.

Table 7: Experimental data used for data model

U	DEMAND	RESOURCE CAPACITY	BATCH SIZE	YIELD	U	DEMAND	RESOURCE	BATCH SIZE	YIELD
1	17820	9360	338	10280	16	12970	7440	253	10189
2	17620	9280	285	14195	17	17620	8680	285	11318
3	18260	9200	273	10333	18	17820	8600	338	7746.7
4	17290	9120	273	11867	19	15465	8520	220	11583
5	17075	9000	258	11883	20	12970	8440	253	12071
6	15465	9480	220	12826	21	17290	8360	273	10100
7	11660	9560	238	11660	22	14800	8280	225	11122
8	17290	9600	273	12534	23	12225	7520	268	9595.6
9	14800	9680	225	12811	24	12970	7480	253	10887
10	17620	9720	285	13197	25	12225	8080	268	10920
11	18260	9800	273	12139	26	17820	8000	338	6595
12	17075	9840	258	13187	27	12225	7280	268	9113.9
13	18260	9920	273	12306	28	17075	7880	258	9452.2
14	15465	8920	220	12072	29	18260	7840	273	6825
15	11660	8840	238	11660	30	11660	7800	238	11660

6. Proposed Methodology using Rough sets

The conditional and decision attributes given in Table 7 are fuzzified using the triangular function [35] [36]. The membership values of each crisp attribute is classified ad High (H), Medium (M) and Low (L) given below is used and the decision table is represented in Table 8 which contains 26 objects after merging of the common objects from the 52 objects obtained. The decision table represents an information system for the product mix required for further analysis.

(Low): D= [11000 13600 15100]; R= [7000 7580 8200]; B= [200 230 260]; Y= [6500 8040 10300].
(Med): D= [13450 15800 17700]; R= [7810 8540 9020]; B = [230 270 300]; Y= [8960 10800 12740].
(High): D= [15900 17600 19450]; R= [8800 9400 10000]; B= [290 320 350]; Y= [11100 12870 14400].

6.1 Set Approximations

In this section, we partition the data for individual attributes having indiscernibility relation on the domain.

a) Partition due to demand P = {D}
$P = \{\{1,2,35,10,16,19,24,25\}_H , \{4,6,11,15,18,20\}_M ,\{7,8,9,12,13,14,17,21,22,23,26\}_L\}$
b) Partition due to resource capacity P = {R}
$P = \{\{1,2,3,4,5,6,7,8,9,10\}_H ,\{11,12,15,16,17,18,19,20,23\}_M ,\{13,14,21,22,24,25,26\}_L \}$
c) Partition due to batch size P = {B}
$P = \{\{1,10,16\}_H , \{2,3,4,7,14,15,17,18,19,21,23,25\}_M , \{5,6,8,9,11,12,13,20,22,24,26\}_L \}$
d) The target set X (decision attribute) is given by,

X = {{2,4,6,7,9,10,17,20,26}$_H$, {1,3,5,8,11,12,14,15,19,22,23}$_M$, {13,16,18,21,24,25}$_L$}
Thus the lower and upper approximations are as follows:
$\underline{P}X$ (D) $=\phi$; $\underline{P}X$ (R) $=\phi$; $\underline{P}X$ (B) = ϕ and
$\overline{P}X$ = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26}

Table 8: Decision table

U	DEMAND (P)	RESOURCE CAPACITY (R)	BATCH SIZE (B)	YIELD (Y)		U	DEMAND (P)	RESOURCE CAPACITY (R)	BATCH SIZE (B)	YIELD (Y)
1	H	H	H	M		14	L	L	M	M
2	H	H	M	H		15	M	M	M	M
3	H	H	M	M		16	H	M	H	L
4	M	H	M	H		17	L	M	M	H
5	H	H	L	M		18	M	M	M	L
6	M	H	L	H		19	H	M	M	M
7	L	H	M	H		20	M	M	L	H
8	L	H	L	M		21	L	L	M	L
9	L	H	L	H		22	L	L	L	M
10	H	H	H	H		23	L	M	M	M
11	M	M	L	M		24	H	L	L	L
12	L	M	L	M		25	H	L	M	L
13	L	L	L	L		26	L	L	L	H

The accuracy of the approximations is, $\alpha_P(X)=0$ i.e. ratio of the lower and upper approximation gives the measure of closeness of the rough set, is approximating the target set. Thus X is said to be rough (vague) with respect to P.

e) The partition based on the attributes Demand and Resource Capacity, $P=\{D,R\}$ for the data set U is,
$U/P=U/IND(D)\otimes U/IND(R)$
={{1,2,3,5,10,16 19, 24, 25}$_H$,{4, 6, 11, 15, 18, 20}$_M$,{7, 8, 9, 12,13,14,17,21, 22,23,26}$_L$}$_D$
$\otimes$ {{1,2,3,4,5, 6, 7,8 9, 10}$_H$, {11,12, 15,16, 17, 18, 19, 20, 23}$_M$, {13, 14, 21, 22, 24, 25,26}$_L$}$_R$

= {{1, 2, 3, 5, 10}, (16, 19}, {24, 25}}, {4, 6}, {11, 15, 18, 20}, {ϕ},{7, 8, 9}, {12, 17, 23}, {13, 14, 21, 22, 26}}

The partition due to decision attribute for membership values of X (High, Medium, Low) is,

X (Y=H) = {2, 4, 6, 7, 9, 10, 17, 20, 26}
X (Y=M) = {1, 3, 5, 8, 11, 12, 14, 15, 19, 22, 23}
X (Y=L) = {13, 16, 18, 21, 24, 25}

The P-lower approximations that classify as member X are certain:
$\underline{P}X$ = {{24, 25}, {4, 6} {ϕ}} = {4, 6, 24, 25}.

The P-upper approximations that classify as a member X are possibility:
$\overline{P}X$ ={{1,2,3,5,10}, {16,19}, {24,25}, {4,6}, {11,15, 18, 20}, {7,8,9}, {12,17,23}, {13, 14, 21, 22, 23}}.

={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26}

The P-boundary region is the difference of the upper approximations and the lower approximations that cannot decisively classify as a member of X on the basis of knowledge P:

BN = {1, 2, 3, 5, 7, 8, 9,10,11,12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26}

The accuracy of the approximations is,

$$\alpha_P(X) = 4/26 = 0.15384$$

f) Partition due to P = {D, B} = ϕ and P = {R, B} = ϕ

g) Partition due all the attributes P = (D, R, B} = 1; and P = {D, R, B, Y} = 1.

The above extensional approach does not provide the necessary reduction in the knowledge base and the method of reduct using the QuickReduct algorithms [10] [12] show the dependency on all the three conditional attributes in the information system Thus the information system is irreducible as all the three attributes are necessary in the representation of the information system and are indispensable.

6.2 Rule extraction from the information table

Another dimension is the intentional description of the category, is based on a set of rules that describe the scope of the category. The method of rule extraction procedure is based on Ziarko & Shan [9]. We wish to determine a minimal set of consistent rules (logical implications) that characterize the data set given in Table 8. Thus for a set of condition attributes $P = \{P_1, P_2, .., P_n\}$ and decision attribute Q, $Q \notin P$, these rule should have the form $P_i^a P_j^b ... P_k^c \rightarrow Q^d$, which same as $(P_i = a) \wedge (P_j = b) \wedge ... \wedge (P_k = c) \rightarrow (Q = d)$ where $\{a, b, c, ..\}$ are the legitimate values from the domains of their respective attributes. Typically, the association rules and the number of items in U which match the condition/antecedent is called the support for the rule.

The method of extracting rules is to form a matrix corresponding to each individual value d of the decision attribute Q. This means, that the decision matrix for value d of decision attribute Q lists all attribute-value pair that differ between the objects having Q = d and $Q \neq d$.

Thus the decision table in Table 8 is arranged using the K-means cluster algorithm with K=3, is again represented in Table 9. For each object in U the elements with Y= H are paired with elements Y $\neq$ H and the list all the differences. The positive objects (Y = H) form rows, while the negative objects (Y $\neq$ H) form columns.

The terms in the matrix in Table 10 are represented as conjunction of the conditional attribute name and its superscript is the value. All terms are represented as the disjunction represents an expression.

The list of object 2 is as given below:

O^2= (B^M) (ϕ) (B^M) $(D^H \vee B^M)$ $(D^H \vee R^H \vee B^M)$ $(D^H \vee R^H \vee B^M)$ $(D^H \vee R^H)$ $(D^H \vee R^H)$ (R^H) $(D^H \vee R^H \vee B^M)$ $(D^H \vee R^H)$ $(D^H \vee R^H \vee B^M)$ $(R^H \vee B^M)$ $(D^H \vee R^H)$ $(D^H \vee R^H)$ $(R^H \vee B^M)$ (R^H).

The above expression can be reduced using the laws of Boolean algebra.

$= (R^H)\ (B^M)\ (D^H \vee B^M)\ (D^H \vee R^H)\ (R^H \vee B^M)\ (D^H \vee R^H \vee B^M)$
$= (R^H)\ (B^M)\ (D^H \vee R^H)$.
$= (R^H \wedge B^M) \rightarrow Y^H$.
The implication for object 2 can be written as rule in the form,
2. (Resource Capacity = H) $\wedge$ (Batch Size = M) $\rightarrow$ (Yield = H)

Table 9: Decision table

U	DEMAND (P)	RESOURCE CAPACITY (R)	BATCH SIZE (B)	YIELD (Y)		U	DEMAND (P)	RESOURCE CAPACITY (R)	BATCH SIZE (B)	YIELD (Y)
2	H	H	M	H		11	M	M	L	M
4	M	H	M	H		12	L	M	L	M
6	M	H	L	H		14	L	L	M	M
7	L	H	M	H		15	M	M	M	M
9	L	H	L	H		19	H	M	M	M
10	H	H	H	H		22	L	L	L	M
17	L	M	M	H		23	L	M	M	M
20	M	M	L	H		13	L	L	L	L
26	L	L	L	H		16	H	M	H	L
1	H	H	H	M		18	M	M	M	L
3	H	H	M	M		21	L	L	M	L
5	H	H	L	M		24	H	L	L	L
8	L	H	L	M		25	H	L	M	L

The remaining 8 objects are reduced and the rules are as given below:
4. (Demand = M) $\wedge$ (Resource Capacity = H) $\rightarrow$ (Yield =H)
or (Resource Capacity=H) $\wedge$ (Batch Size = M) $\rightarrow$ (Yield = H)
6. (Demand = M) $\wedge$ (Resource Capacity = H) $\rightarrow$ (Yield =H)
7. (Demand = L) $\wedge$ (Resource Capacity = H) $\wedge$ (Batch Size = M) $\rightarrow$ (Yield =H).
9. (Demand = L) $\wedge$ (Resource Capacity = H) $\rightarrow$ (Yield =H).
10. (Resource Capacity = H) $\wedge$ (Batch Size = H) $\rightarrow$ (Yield =H).
17. (Demand = L) $\wedge$ (Resource Capacity = M) $\wedge$ (Batch Size = M) $\rightarrow$ (Yield =H).
20. (Demand = M) $\wedge$ (Batch Size = L) $\rightarrow$ (Yield =H).
26. (Demand = L) $\wedge$ (Resource Capacity = L) $\wedge$ (Batch Size = L) $\rightarrow$ (Yield =H).

The above process is repeated for (Y = M) and (Y = L) to obtain the logical implications. The rule base for the complete system is represented in Table 11. The rule base in Table 11 is implemented using the Mamdani Method and the performance parameter, the Yield is determined. The Fuzzy Tool box in MATLAB® is selected for the implementation for its versatility and graphic representation.

Table 10: Matrix representation for List of Objects (Y=H)

U	[1]	[3]	[5]	[8]	[11]	[12]	[14]	[15]	[19]	[22]	[23]	[13]	[16]	[18]	[21]	[24]	[25]
[2]	B^M	ϕ	B^M	D^H, B^M	D^H, R^H, B^M	D^H, R^H, B^M	D^H, R^H	D^H, R^H	R^H	D^H, R^H, B^M	D^H, R^H	D^H, R^H, B^M	R^H, B^M	D^H, R^H	D^H, R^H	R^H, B^M	R^H
[4]	D^M, B^M	D^M	D^M, B^M	D^M, B^M	R^H, B^M	D^M, R^H, B^M	D^M, R^H	D^M, R^H	D^M, R^H	D^M, R^H, B^M	D^M, R^H	D^M, R^H, B^M	D^M, R^H, B^M	D^M, R^H	D^M, R^H	D^M, R^H, B^M	D^M, R^H
[6]	D^M, B^L	D^M, B^L	D^M	D^M	R^H	D^M, R^H	D^M, R^H, B^L	R^H, B^L	D^M, R^H, B^L	D^M, R^H	D^M, R^H, B^L	D^M, R^H	D^M, R^H, B^L	R^H, B^L	D^M, R^H, B^L	D^M, R^H	D^M, R^H, B^L
[7]	D^L, B^M	D^L	D^L, B^M	B^M	D^L, R^H, B^M	R^H, B^M	R^H	D^L, R^H	D^L, R^H	R^H, B^M	R^H	R^H, B^M	D^L, R^H, B^M	D^L, R^H	R^H	D^L, R^H, B^M	D^L, R^H
[9]	D^L, B^L	D^L, B^L	D^L	ϕ	D^L, R^H	R^H	R^H, B^L	D^L, R^H, B^L	D^L, R^H, B^L	D^L	R^H, B^L	R^H	D^L, R^H, B^L	D^L, R^H, B^L	R^H, B^L	D^L, R^H	D^L, R^H, B^L
[10]	ϕ	B^H	B^H	D^H, B^H	D^H, R^H, B^H	D^H, R^H, B^H	D^H, R^H, B^H	D^H, R^H, B^H	R^H, B^H	D^H, R^H, B^H	D^H, R^H, B^H	D^H, R^H, B^H	R^H	D^H, R^H, B^H	D^H, R^H, B^H	R^H, B^H	R^H, B^H
[17]	D^L, R^M, B^M	D^L, R^M	D^L, R^M, B	R^M, B^M	D^L, B^M	B^M	R^M	D^L	D^L	R^M, B^M	ϕ	R^M, B^M	D^L, B^M	D^L	R^M	D^L, R^M, B^M	D^L, R^M
[20]	D^M, R^M, B^L	D^M, R^M, B^L	D^M, R^M	D^M, R^M	ϕ	D^M	D^M, R^M, B^L	B^M	D^M, B^L	D^M, R^M	D^M, B^L	D^M, R^M	D^M, B^L	B^L	D^M, R^M, B^L	D^M, R^M	D^M, R^M, B^L
[26]	D^L, R^L, B^L	D^L, R^L, B^L	D^L, R^L	R^L	D^L, R^L	R^L	B^L	D^L, R^L, B^L	D^L, R^L, B^L	ϕ	R^L, B^L	ϕ	D^L, R^L, B^L	D^L, R^L, B^L	B^L	D^L	D^L, B^L

7. Results and Conclusions

Table 12 gives the performance of the system using the reduced rule base.

The neuro-fuzzy approach using the Mamdani Method is used for implementing the rule set obtained in Table 11 gives in the performance of the system. The average yield obtained by the proposed method is 11182.4 compared to the model used in table 6 which is found to be 11070. It can be noted that there are no rules to guide, as to how to fuzzify the parameters and the area of overlap that may be appropriate.

However the implementer and decision maker can interact with each other to obtain a satisfying solution. The reduced rule base contain only those features i.e. the attributes that are indispensable are retained. The decision attributes fairly agree with that of the information system (see Table 7). The proposed method can be useful in large information systems by extraction of features and to reduce the rule base [11].

Table 11: Reduced rule base

U	DEMAND (P)	RESOURCE CAPACITY (R)	BATCH SIZE (B)	YIELD (Y)		U	DEMAND (P)	RESOURCE CAPACITY (R)	BATCH SIZE (B)	YIELD (Y)
2	-	H	M	H		11	-	M	L	M
4	M	H	-	H		12	L	M	L	M
4	M	-	M	H		14	L	L	M	M
6	M	H	-	H		15	M	M	M	M
7	L	H	M	H		19	H	M	M	M
9	L	H	-	H		22	L	L	L	M
10	-	H	H	H		23	L	M	-	M
17	L	M	M	H		13	-	L	L	L
20	M	-	L	H		16	H	M	-	L
26	L	L	L	H		16	-	M	H	L
1	-	H	H	M		18	M	M	M	L
3	H	H	H	M		21	-	L	M	L
5	H	H	L	M		24	H	L	-	L
8	L	H	L	M		25	H	L	-	L

Table 12: Performance of the system with reduced rule base

U	DEMAND	RESOURCE CAPACITY	BATCH SIZE	YIELD		U	DEMAND	RESOURCE	BATCH SIZE	YIELD
1	17820	9360	338	11709		16	12970	7440	253	10028
2	17620	9280	285	12776		17	17620	8680	285	9591
3	18260	9200	273	12783		18	17820	8600	338	8293
4	17290	9120	273	12778		19	15465	8520	220	11722
5	17075	9000	258	12059		20	12970	8440	253	11644
6	15465	9480	220	12788		21	17290	8360	273	9923
7	11660	9560	238	11699		22	14800	8280	225	11651
8	17290	9600	273	12783		23	12225	7520	268	9338
9	14800	9680	225	12163		24	12970	7480	253	10028
10	17620	9720	285	12775		25	12225	8080	268	10969
11	18260	9800	273	12786		26	17820	8000	338	8350
12	17075	9840	258	12404		27	12225	7280	268	9563
13	18260	9920	273	12756		28	17075	7880	258	10056
14	15465	8920	220	12256		29	18260	7840	273	8525
15	11660	8840	238	11488		30	11660	7800	238	9788

Fig 3 shows the surface view of the rule base implemented in Mamdani method. The performance of Demand, Resource Capacity and Batch Sizes with respect to the output Yield is shown in Fig 4. The intentional approach demonstrates the application of rough sets in reducing certain superfluous parameters without affecting the contents of the information system.

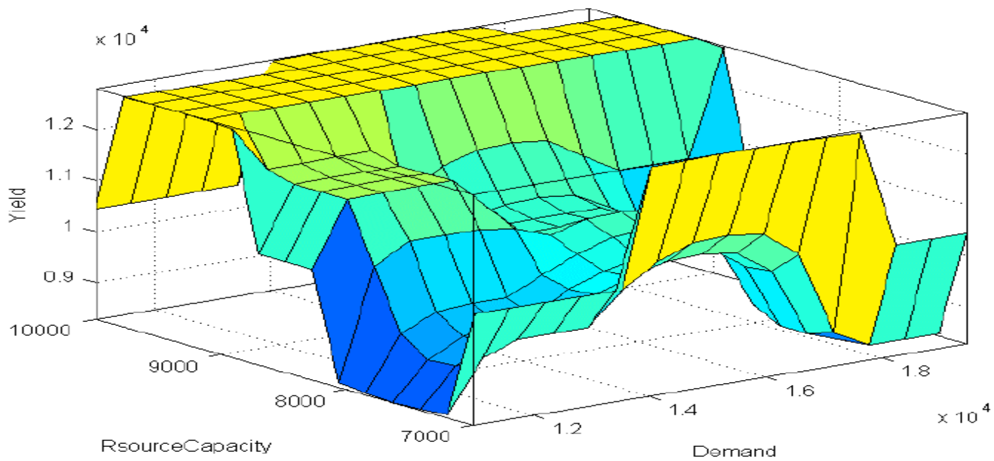
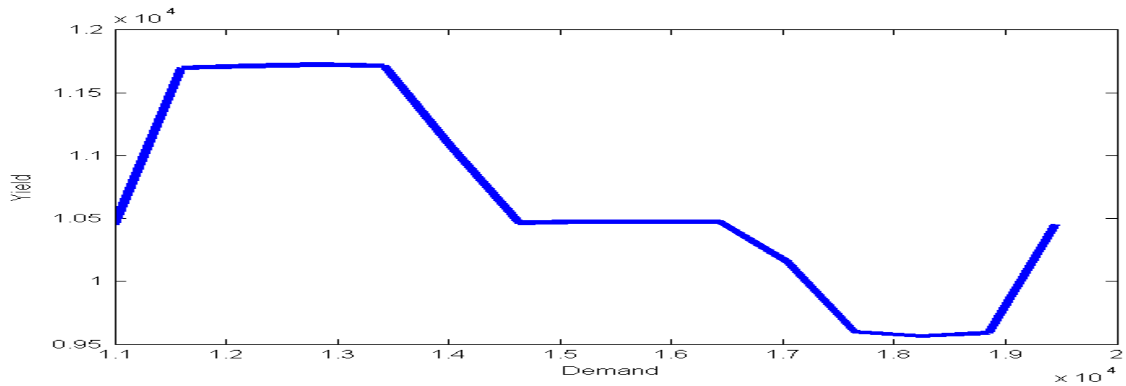
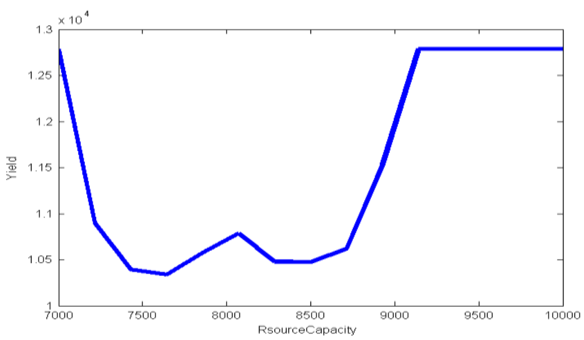
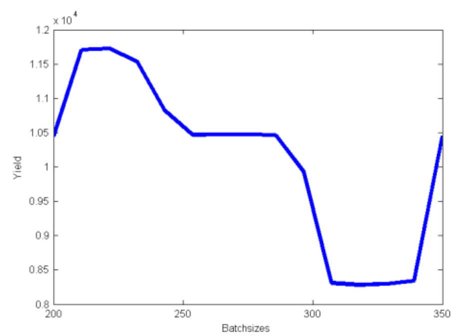


Figure 3: Surface View for the rules



(a)





(b) (c)

Figure 4: Plot of Yield vs. Demand, Resource Capacity and Batch Sizes.

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