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PI Controller Design for a Coupled Tank System Using LMI Approach: An Experimental Study

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Abstract

This paper presents a Linear Matrix Inequality (LMI) tuned PI controller for real-time control of a coupled-tank liquid level system. The proposed approach is based on the transformation of the PI controller design problem to a state feedback controller design problem, which is further solved using convex optimization approach. The model of the coupled tank system has been developed based on system identification technique that employs least square error method (LS) for parameter estimation. The proposed controller algorithm has been applied on the identified model. The performance of the proposed control algorithm has been compared with that of a Ziegler-Nichols tuned PI controller. From both the simulation as well as the experimental results, it is observed that the performance of the proposed PI control is more efficient than the widely used Ziegler-Nichols approach.

Keywords: System identification; Least square estimation; Linear matrix inequality (LMI); Coupled tank system

Introduction

Controlling liquid level and flow in tanks of a coupled tank system is considered as an important benchmark control platform due to their wide spread applications in the process control industries [1]. The control objective of the coupled tank system is to maintain the liquid level at the desired level. The coupled tank system dynamics has interaction characteristics as it is a MIMO system [2]. This dynamics is nonlinear due to valve characteristics and also exhibits non-minimum phase behaviour [3]. The nonlinearity and the non-minimum phase behaviour makes the associated control problem very challenging. A number of PI controllers have been extensively used in process control industries. These PI controllers exploit several tuning methods for obtaining appropriate control parameters. Among the several methods reported, Ziegler-Nichols based tuning method is widely used because of its simple structure [4-6]. However, this Ziegler-Nichols based tuning method fails to provide appropriate system response. To overcome this difficulty, various control techniques have been proposed in literature, these includes an auto adjustable PI controller using MRAC technique [7], a standard two DOF PID with decoupling [8], PI controller tuning by CRA technique (characteristics ratio assignment) [9], an inverting decoupling technique [10]. The existing techniques are inadequate to provide robust responses, in the presence of disturbance in the plant dynamics. To eliminate the drawbacks of above-reported controllers, this paper presents a design methodology for tuning of a PI controller by using LMI (an optimization approach) in order to provide appropriate response considering disturbances. In recent times, LMI has emerged as a powerful design tool for solving many convex problems [11,12]. The basic idea behind LMI the problem is that to translate a given control problem to a standard semidefinite problem (SDP) [11-15]. In general, the systems are modelled by the common approach i.e., state space or a system matrix form [16]. In most of the applications, the system matrix is used with a low order transfer function that leads to the loss of some accuracy in the modelling. Hence, modelling of nonlinear systems such as NARMAX [17], and ANFIS model [18] have been suggested for better accuracy. It is, in general, difficult to treat various nonlinearities under a unified framework. Also in some practical situations, due to limited knowledge about certain nonlinear physical phenomena, it is difficult to describe the nonlinearities precisely. Due to this complexity, the nonlinear controllers are very rarely used in industries. In this paper, modelling is accomplished by linear system

identification technique [19,20]. Here the performances are compared with conventional PI Controller in terms of time domain specifications and also different performance index criteria such as Integral Square Error (ISE) and Integral Average Error (IAE).

The rest of the paper is organized as follows. Section 2 provides development of a simplified mathematical model of coupled tank system (CTS). Section 3 presents the control algorithm. Section 4 presents both the simulation and experimental results. Finally, conclusions are made in section 5.

Coupled tank process modelling

Figure 1 gives a sketch of the experimental set-up of the coupled tank system used in the present work. It is a challenging benchmark control device that is commonly used in many process control industries. The control objective of the coupled tank system is to maintain the level of the tanks at the desired level, during inflow and outflow of water. It consists four translucent tanks, and each tank is fitted with an outlet pipe to transmit the over flow water to the reservoir. In this process, the fifth tank is used for water storage purposes i.e., as a reservoir. A level sensor is also attached at the base of each tank to measure the water level of the corresponding tank. The output of the level sensor is converted to 0-5 volt DC by the help of a signal conditioning circuit. There are two pumps installed in the reservoir to drive the water from bottom to top of the tank. A scale is attached in front of all individual tanks for the purpose of monitoring the water level. It works under two basic modes of operations i.e., local mode and remote mode. In local mode, two tanks are controlled by two separate potentiometers that are applied to two tanks to drive water to respective tanks (Figures 4-6). In the present work, the system is used in the remote mode of

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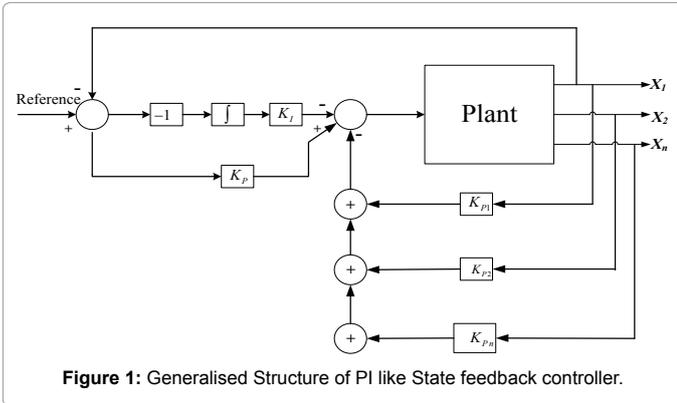
Figure 1: Generalised Structure of PI like State feedback controller.
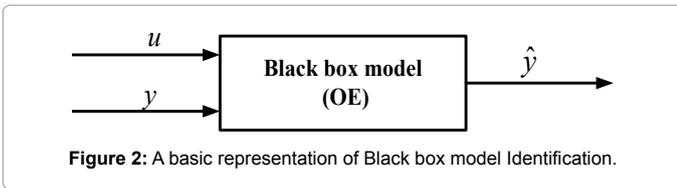

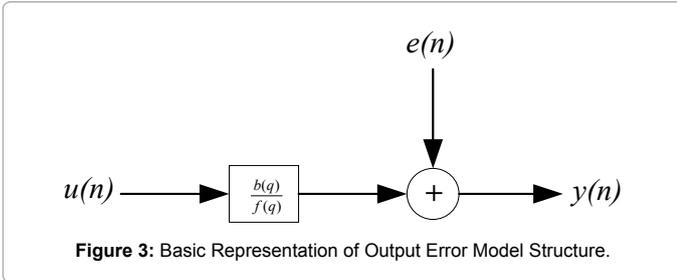**Figure 2:** A basic representation of Black box model Identification.

Figure 3: Basic Representation of Output Error Model Structure.

operation for carrying out experiments. The simplest nonlinear model of the coupled tank system can be obtained by considering the mass balance equation, which is relating the water level h_1, h_2 and the applied voltage 'u' to the pump.

$$\frac{dh_1}{dt} = -\frac{a}{A}\sqrt{2gh_1(t)} + \eta u(t) \tag{1}$$

$$\frac{dh_2}{dt} = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)} \tag{2}$$

where

h_1=water level in tank 1

h_2=water level in tank 2

a_1=outlet area of tank 1

a_2=outlet area of tank 2

A=cross-sectional area of tanks

g=gravitational constant

η=constant relating to the control voltage

For real time implementation a linear model is considered for the controller design. Hence, the above nonlinear model can be converted into a linear model by using Taylor series expansion using two working points.

$$h_{10} = \frac{1}{2g}\left(\frac{\eta u_o A}{a_1}\right), \; h_{20} = \left(\frac{a_1}{a_2}\right)^2 h_{10} \tag{3}$$

$$\Delta\dot{h}_2(t) = \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}.\Delta h_1(t) - \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}.\Delta h_2(t) \tag{4}$$

$$\Delta\dot{h}_2(t) = \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}.\Delta h_1(t) - \left(\frac{a1}{A}\right)^2 \frac{g}{\eta u_0}.\Delta h_2(t) \tag{5}$$

In order to obtain, transfer function model, we take Laplace transformation of above equations (4) and (5) which yields the followings.

$$s\Delta H_1(s) = -\left(\frac{a}{A}\right)^2 \frac{g}{\eta u_0}.\Delta H_1(s) + \eta.\Delta U(s) \tag{6}$$
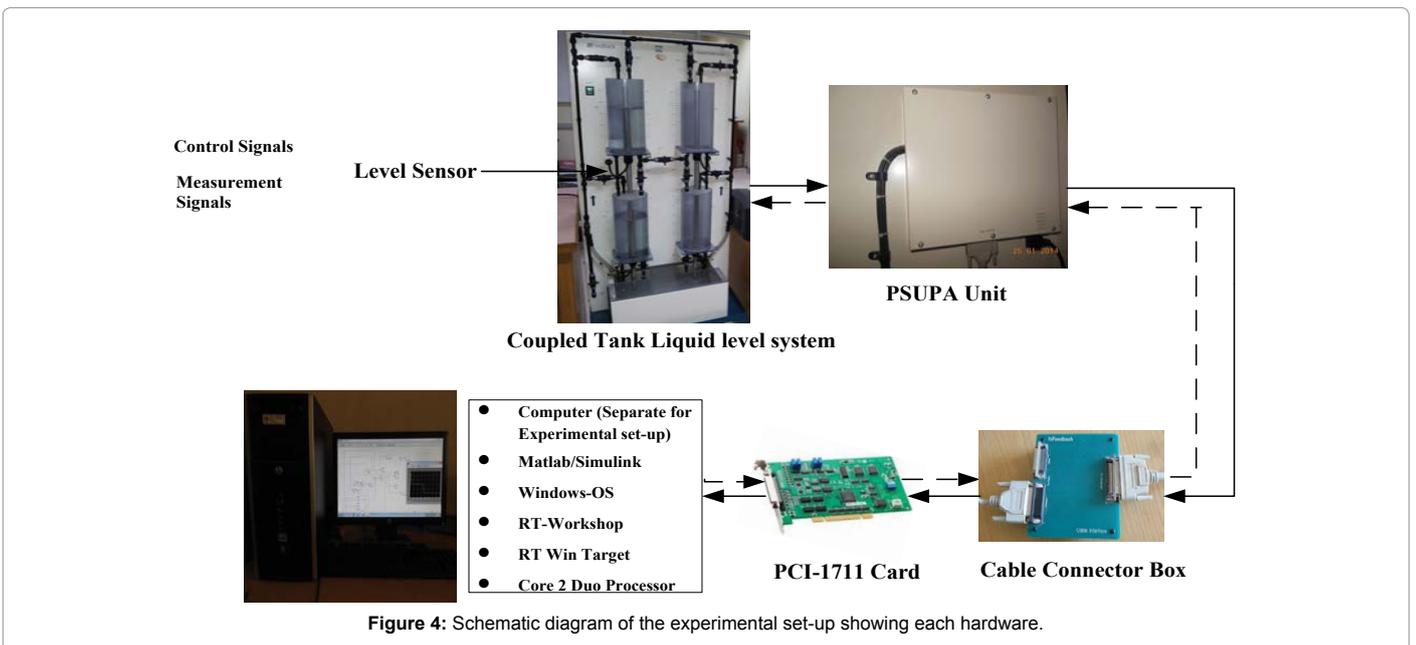

Figure 4: Schematic diagram of the experimental set-up showing each hardware.

$$s\Delta H_2(s) = \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta H_1(s) - \left(\frac{a_2}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta H_2(s) \qquad (7)$$

$$\frac{\Delta H_1(s)}{\Delta U(s)} = \frac{\eta}{s + \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}} \qquad (8)$$

$$\frac{\Delta H_2(s)}{\Delta H_1(s)} = \frac{\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}}{s + \left(\frac{a_2}{A}\right)^2 \frac{g}{\eta u_0}} \qquad (9)$$

During linearization by Taylor series expansion, higher order terms are omitted (1) and (2). Also some parameter of the coupled tank system are not known perfectly. Hence there is an obvious need of obtaining an accurate dynamics model of the system. To overcome the above mentioned difficulties, in this paper system identification is accomplished using a black box model approach. We consider pump control voltage as an input and water level as the output for the system identification. An output error model is considered and parameters of the model are determined by using least square estimation algorithm [19]. The obtained model is validated by different validation techniques such as mean square error and residual analysis (Figure 2). In the present work, a second order output error (OE) model has been considered [20] for model identification since it is found to give a better fit to experimental data as compared to ARMAX and ARX model. The output error model structure (Figure 3) describes the system dynamics separately and also in this model structure, no parameters are used for modelling the disturbance characteristics. In the simplest form, the OE model is represented as below.

$$y(t) = \frac{b}{f} u(t) \qquad (10)$$

The parameter b and f are determined by a least square estimation algorithm, as it minimizes the error between the model and plant output in the sense of minimum square error. The parameter vecter can be obtained using the following equation.

$$\hat{\theta}_{Ls} = \left(\varphi^T \varphi\right)^{-1} \varphi^T y \qquad (11)$$

$$b_1(q) = -0.004711\hat{q}^{-1} + 0.004877\hat{q}^{-2}$$
$$f_1(q) = 1 - 1.991\hat{q}^{-1} + 0.9911\hat{q}^{-2}$$
$$b_2(q) = 0.06481\hat{q}^{-1} - 0.062\hat{q}^{-2} \qquad (12)$$
$$f_2(q) = 1 - 1.901\hat{q}^{-1} + 0.9017\hat{q}^{-2}$$

Control Algorithm

The PI controller design problem is converted to a state feedback control design problem and further solved using convex constraint optimization approach. Convex optimization can be efficiently solved by interior point LMI solver (MINCX) [11-13]. It is a state space approach control design technique, here PI controller design is considered as a static feedback controller and the static feedback gain 'K' vector contains all the parameters of PI Controller.

Consider an LTI system given by

$$\dot{x} = A(t)x + B(t)u$$
$$y = Cx \qquad (13)$$

In the feedback control strategy (u=Kx), the static feedback gain K

contains only proportional gain for inner and outer feedback loops. To transform the system into a PI like a framework, an extra state variable must be included which is usually chosen as the integral of necessary output, for which zero steady state error is desired.

Let x_1 is the selected output, for which zero steady state error is desired. If the augmented state is chosen as $\tau = -\int(r - x_1)$, where r is the desired trajectory for z_1 output, then the PI like state feedback control problem can be described as depicted in Figure 5.

Let us consider an objective function for equation (13) such as

$$J = \int_0^\infty x(t)^T Q x(t) + u(t)^T R u(t) dt \qquad (14)$$

where Q and R are symmetric positive semi-definite matrices.

The control law is given by

$$u = -Kx = -R^{-1}B^T P \qquad (15)$$

In the above P is a positive definite solution of the Algebraic Riccati Equation (ARE).

$$A^T P + PA - PBR^{-1}B^T P = -Q \qquad (16)$$

The minimum quadratic cost is given by

$$J_{min} = x_0^T P x_0 \qquad (17)$$

The above LQR problem can be recast as an optimization problem over $\hat{P}$ and Y, which is stated as follows.

$$\min_{\hat{P}Y} \ x^T(0)\hat{P}^{-1}x(0)$$

Subject to

$$\begin{bmatrix} AP + A^T P + BY + Y^T B^T & \hat{P} & Y^T \\ \hat{P} & -Q^{-1} & * \\ Y & * & -R^{-1} \end{bmatrix} \leq 0, \hat{P} > 0 \qquad (18)$$

The cost objective (17) can be rewritten as

$$x_0^T P x_0 = x_0^T \hat{P}^{-1} x_0 \leq \gamma \qquad (19)$$

By using Schur, compliment the above equation (19) can be written as

$$\begin{bmatrix} \gamma & x_0^T \\ x_0^T & \hat{P} \end{bmatrix} \geq 0 \qquad (20)$$

$$K = -Y^*(P)^{-1}, K = [K_p, K_I] \qquad (21)$$

Results and Discussion

The controllers, LQR-LMI based PI and Ziegler-Nichols based PI controller were implemented on the real -time coupled tank set up. In the present work a step input is given to the system and the response of both controllers is recorded and compared with each other. All the identification results as well as the model validation results are also presented here. The parameters of the CTS (Coupled tank system) as given in Table 1 are used for simulation studies (Tables 1-3).

The coupled tank system is excited by white noise for performing system that covers a broad range of frequencies for whole dynamics in

Symbol	Quantity	Value
A	Cross-sectional area of the tanks	0.01389
a_1	Tank 1 outlet area	0.1245
a_2	Tank 2 outlet area	0.1245
g	Gravitational Constant	9.8
η	Constant relating the control voltage with the water flow from the pump	0.1194

Table 1: Parameters of the coupled tank system.

Performance Index	Tuning based on ZN Method	Tuning Based on LMI Method
ISE	43.55 (Tank 1) 65.20 (Tank 2)	27.81 (Tank 1) 17.81 (Tank 2)
IAE	65.599 (Tank 1) 59.33 (Tank 2)	62.82 (Tank 1) 43.22 (Tank 2)

Table 2: Response analysis by different performance index.

Performance (Sec)	Tuning based on ZN Method	Tuning Based on LMI Method
Rise Time	5.123 (Tank 1) 15.5 (Tank 2)	2.9 (Tank 1) 1.78 (Tank 2)
Peak Time	20.87 (Tank1) 27.88 (Tank 2)	21.5 (Tank 1) 26.5 (Tank 2)
Settling Time	27.77 (Tank 1) 113.8 (Tank 2)	10.5 (Tank 1) 20.93 (Tank 2)

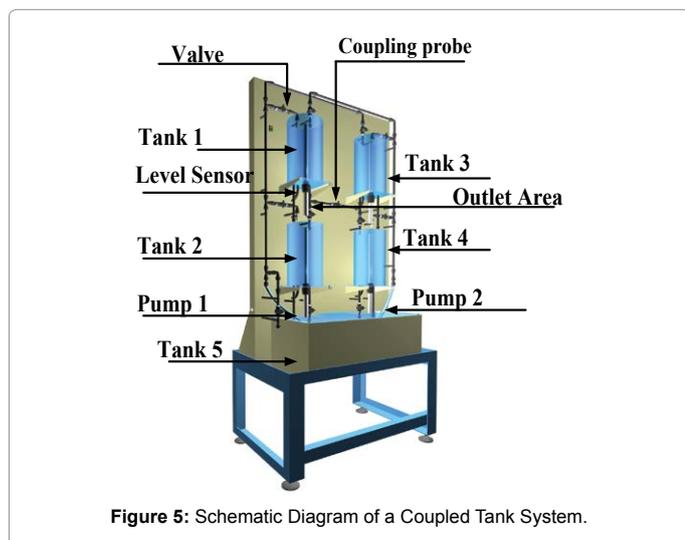**Table 3:** Response analysis by Time domain specification.

Figure 5: Schematic Diagram of a Coupled Tank System.

the identification of parameters. The excited input signal is as shown in Figure 8. The outputs of the both tanks are as shown in Figures 9 and 10. The experiment is performed for 250 secs with a sampling time of 0.1 sec i.e., a record of 2500 experimental samples is considered. After getting the identified model, the model was validated by using Mean Square Error (MSE) approach that is shown in Figure 11 and nonparametric approach technique i.e., auto-correlation method. In Figure 12, the mean square error (MSE) validation technique (Figure 11) ensures that the output error model i.e., (OE-221) gives superior fitting characteristics as compared to the corresponding ARMAX (2 2 2 1]), [Auto-Regressive Exogenous Moving Average] and ARX (2 2 1), [Auto Regressive Exogenous] model. Where Auto-Regressive means current output has a relationship to the previous values of the output, Exogenous stands mean that the system relies not only on the current input values but also the history of output and moving average is stands for noise [19]. Usually, the output error model structure defines the

dynamics of a system separately and also in this model structure no parameters are used for modelling the disturbance characteristics. On the other hand, in both the ARX and ARMAX model structure the dominating disturbance is a part of the process input. In generally both the ARX and ARMAX model structure will be considered whenever a higher order model is desirable. In above-reported model (2 2 1), (2 2) represents the order of the model parameter and (1) represents the input-output delay.

From Auto correlation analysis (Figure 12) it is observed that all lags (which is the time difference (in samples) between the signals at which the correlation is lies inside 90% of the confidence interval (where the 90% confidence level means, the region around zero represents the range of residual values that have a 90% probability of being statistical insignificant). Hence, from both the obtained responses it is envisaged that the selected model is the suitable one for the controller design (Figures 7-15).

Figures 8 and 9 illustrate simulations of these two tank models (Tank-1 and Tank-2) compared to real time data obtained from the model identification. It is clearly observed that from the simulation results shown Figures 13 and 14 as well as from the experimental response results (Figure 15) that the performances of the proposed control algorithm deliver best performances as compared to the Ziegler-Nicholas based conventional PI controller. Since by the presented LMI tuned PI controller algorithm the system can reach the set point in a short time with no overshoot. Also that also the system is settled in the steady state with a small settling time. On this contrary, the Ziegler-Nicholas based conventional PI controller system can reach the set point with a significant rise time and more steady state error as compared to LMI tuned PI Controller and along with the system is settled at steady state by taking large settling time with more overshoot as compared to the proposed controller algorithm. The gain parameters obtained from the presented algorithm is given as Eq. (22).

$$K = \{ \; [0.2761 \quad 0.4542] , \atop [0.8265 \quad 0.6640] \; \} \tag{22}$$

Conclusions

In this paper, an approach i.e., Linear Matrix Inequalities is addressed for tuning of a PI controller for controlling liquid levels in a coupled tank system. From the obtained results it is observed that when the conventional PI controller is applied in real- time it takes more time to settle the desired level. On the other hand, the proposed control algorithm requires less control effort to achieve the desired level with no overshoot. Thus, the obtainable control algorithm provides better liquid level control performance.
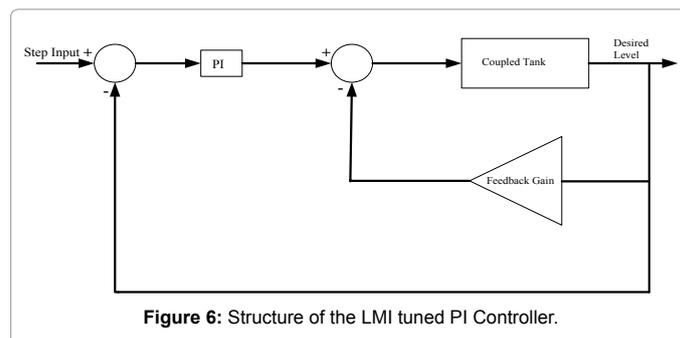

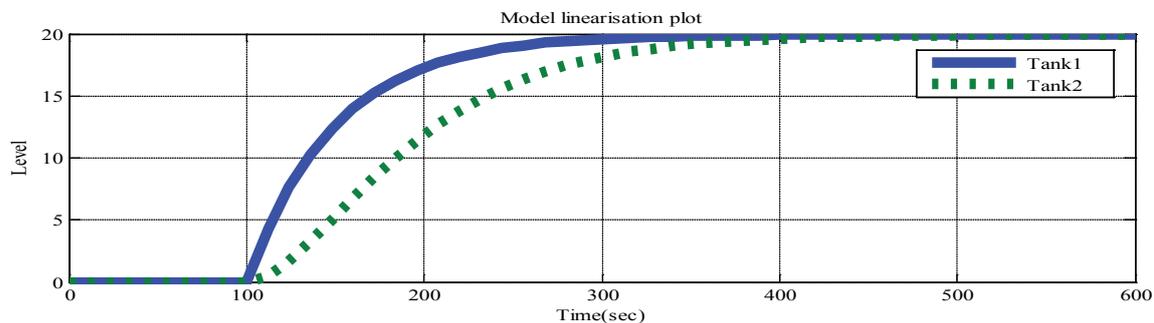**Figure 6:** Structure of the LMI tuned PI Controller.
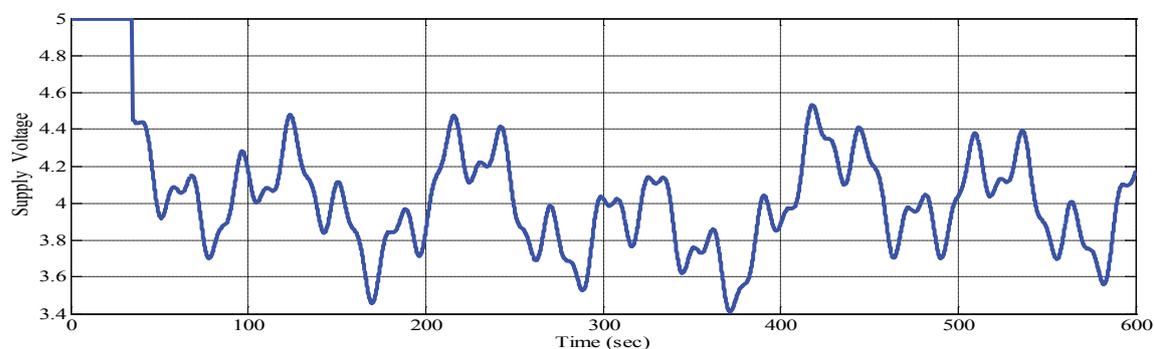
Figure 7: Linearization Response plot of both tanks.

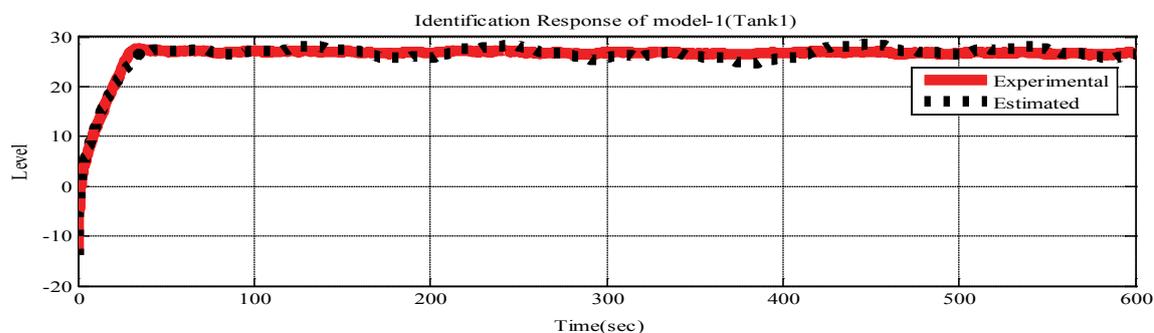
Figure 8: Experimental Input Data.

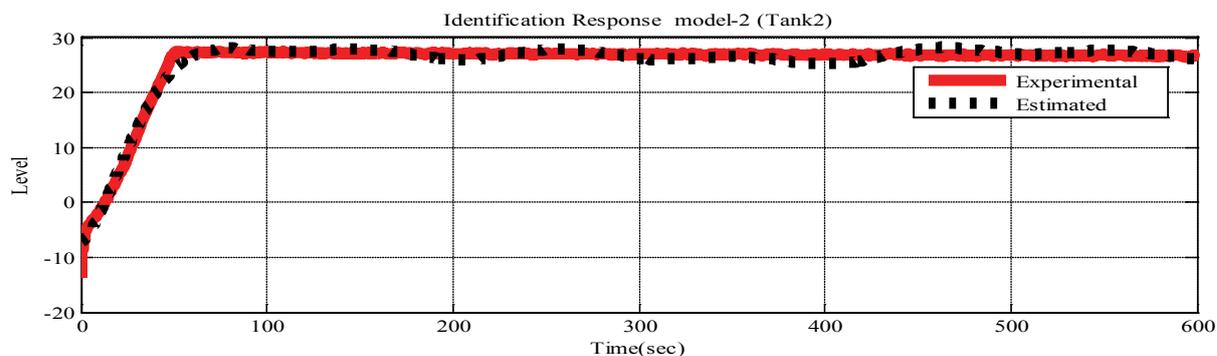**Figure 9:** System Identification response plot of Tank 1.

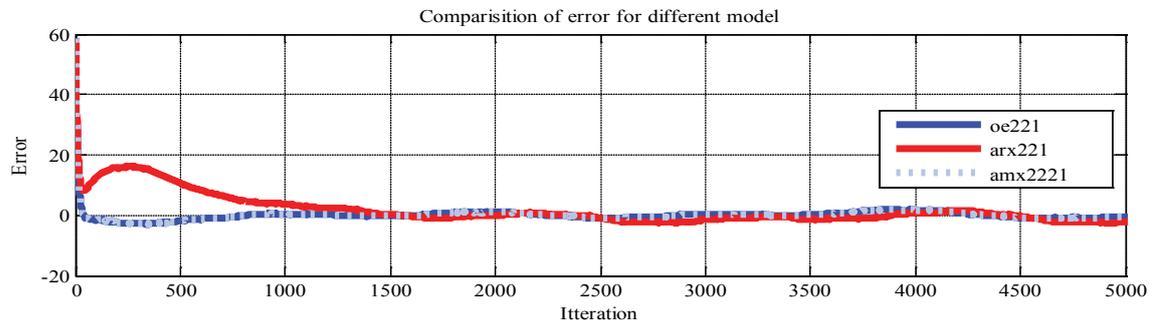**Figure 10:** System Identification response plot of Tank 2.

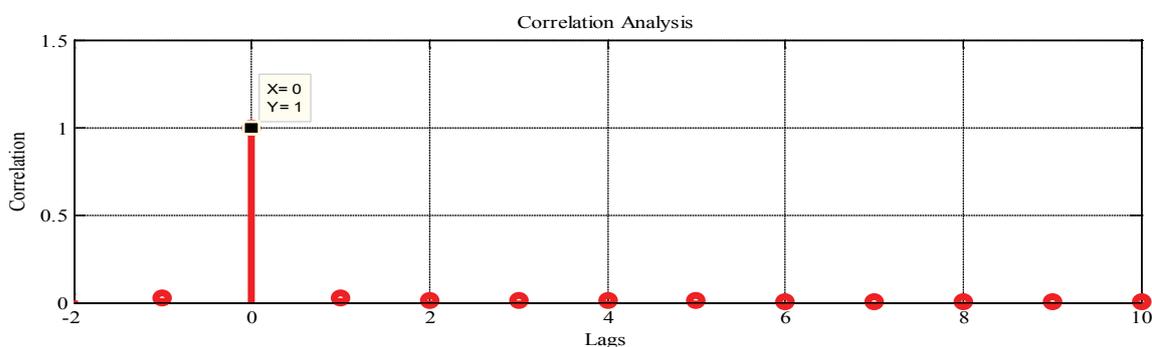**Figure 11:** Model validation response by Mean Square Analysis (MSE).

Figure 12: Model validation response by correlation analysis.
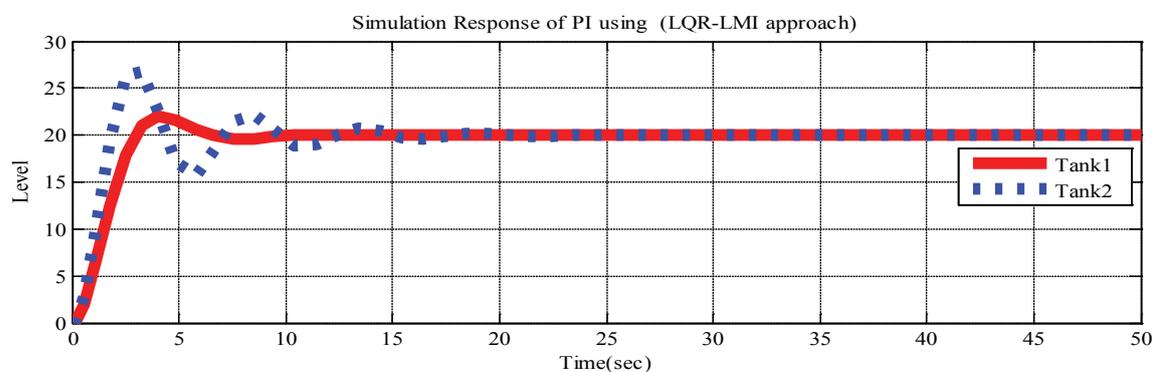

Figure 13: Simulation response of PI using (LQR-LMI) approach of both Tanks.
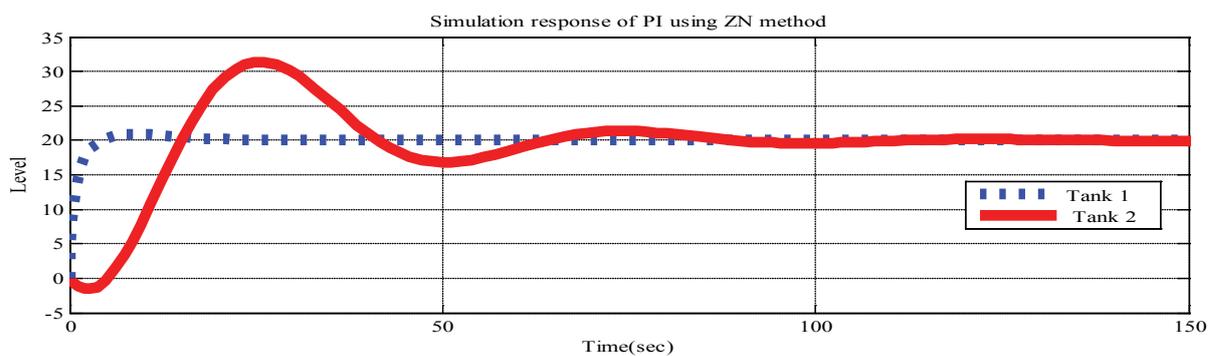

Figure 14: Simulation Response of PI using (Ziegler and Nicholas) approach of both Tanks.
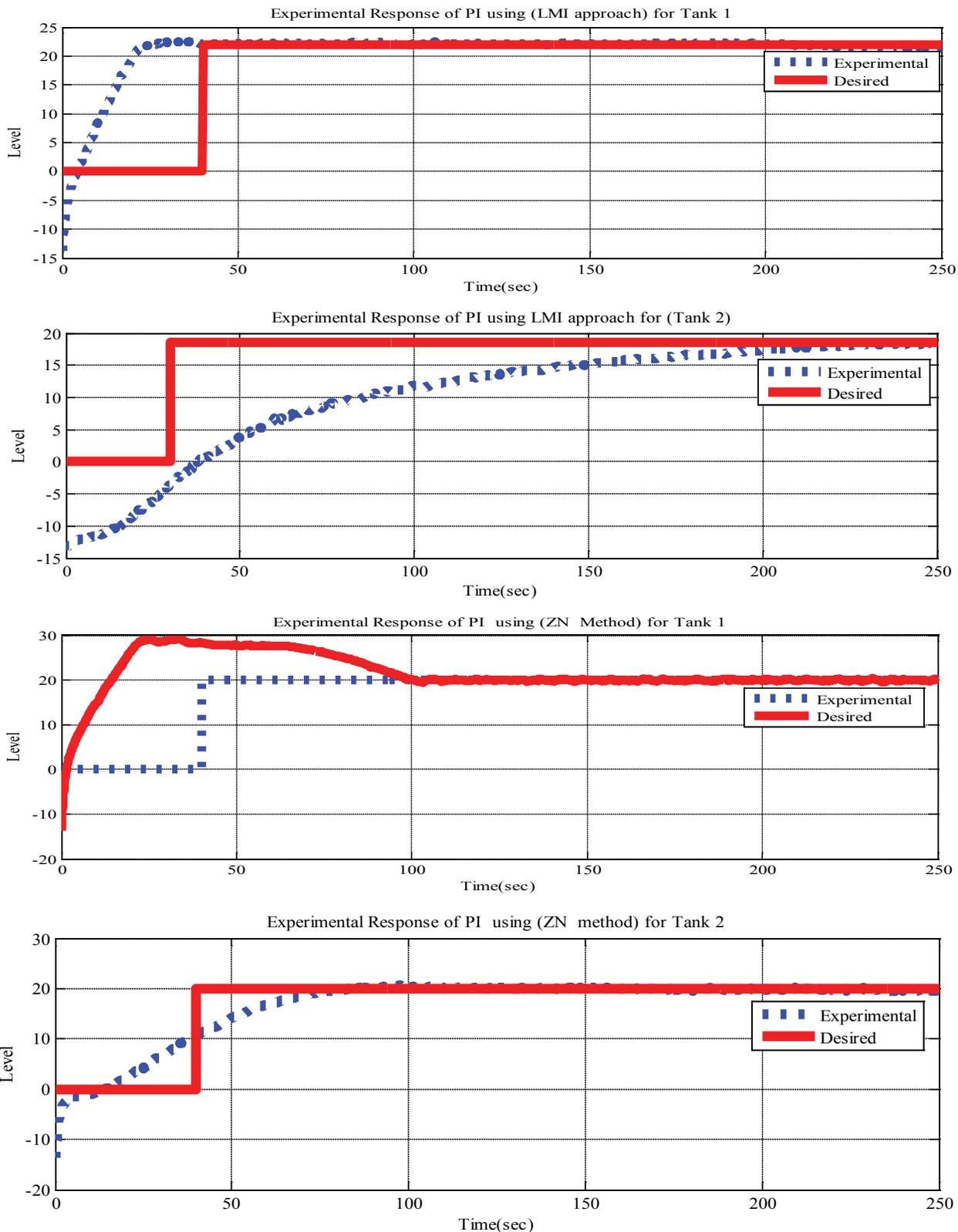
Figure 15: Experimental Responses LMI Tuned PI and Ziegler Nicholas Tuned PI.

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