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Review Article

Optical Properties of Uniaxial Trigonal Crystals of $3\bar{m}$ Point Group Symmetry under an Applied Electric Field along X Principal Axis

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ABSTRACT

Pockels activity in uniaxial trigonal crystals with $3\bar{m}$ point group symmetry was investigated under an electric field applied along the X principal axis. As the result, the uniaxial crystal deformed into a biaxial crystal exhibiting three distinct refractive indices. One principal axis was shown to lie approximately along the optical axis. The remaining two principal axes were rotated by approximately 45° in the plane transverse to the optical axis. Under application of the electric field, an X-polarized plane-wave light, rotates as it propagates along the optical axis. The length for 90° rotation was determined. Practical applications are discussed.

Keywords: Uniaxial crystal; Biaxial crystal; Pockels activity; Principal axes

INTRODUCTION

The subject of light polarization has been extensively investigated for a long time and its principles have been discussed in several text books [1,2]. The ability to control light polarization and its shape plays an important role in the performance of numerous optical systems and devices. For example, in guided wave optics, the direction of light polarization is maintained as characterized by the type of propagating mode inside an optical waveguide [3]. When anisotropic crystals are used in optical devices, their performance is mainly influenced by the shape and direction of light polarization [4].

Remembering that light behaviour at a boundary even between two isotropic media is influenced by the direction of its polarization, it is expected that the reflection and refraction coefficients become dependent on the direction of the incident light polarization. Under these circumstances, it is very important to be able to trace the shape and direction of light polarization and accurately follow it to characterize the performance of each optical system or device. In this regard, the treatment of the vector wave equation is a very powerful technique capable of determining the light polarization behaviour [3,4]. In simple cases where a monochromatic plane-wave light propagates in a specific medium, Jones vectors and matrices can handle the polarization behaviour. In such cases, a Jones vector will have two components and the Jones matrix will be a 2 × 2 matrix [2,4]. The Jones representation has been successfully applied to a variety of optical devices to characterize their performance. In opto-electronics where the optical properties of a material are influenced by an applied electric field, several useful devices have been devised [5].

Frequently, the way in which optical properties are influenced, depends significantly on the direction of light polarization. Numerous optical devices have been developed in which optical signals are controlled by commanding electronic signals. However, there is a high demand for high-speed light spatial switches. The performance of such spatial switches is based on the total reflection process [6,7], thermo-optic [8], etc. To the best of our knowledge, there have been no reports to date, on application of sufficiently high-speed switches in real systems. In this regard, a spatial light switch was recently patented [9]. The operation of such a switch is based on the polarization management of plane-wave light impinged onto an anisotropic uniaxial crystal of trigonal $3\bar{m}$ point group symmetry. The applied electric field was taken to be in the direction of the larger refractive index of the uniaxial crystal to activate the Pockels activity. Although the operation speed of such spatial switches is limited owing to device capacitance and light diffraction, they can operate in GHz range and are expected to find applications in a variety of fields such as medicine, image processing, surveillance, robotics, driver-less cars and components used in the education of optics. Therefore, it is necessary to investigate the behaviour of such crystals and trace the polarization shapes and directions associated with light propagation along their optical axis. The results presented here are believed to deepen the application scope of such crystals.

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LITERATURE REVIEW

Theory

The index ellipsoid of an anisotropic crystal takes the following form,

$$B_1^0 x^2 + B_2^0 y^2 + B_3^0 z^2 = 1 \quad (1)$$

Where,

$$B_1^0 = 1/n_x^2,\ B_2^0 = 1/n_y^2,\ B_3^0 = 1/n_z^2 \quad (2)$$

Superscript 0 stands for the initially unperturbed crystal state, x, y, z show the principal axes and n_x, n_y, n_z denote the refractive indices along x, y, z, respectively.

When an electric field is applied to the crystal, the perturbed index ellipsoid takes the form of-

$$B_1 x^2 + B_2 y^2 + B_3 z^2 + 2B_4 yz + 2B_5 xz + 2B_6 xy = 1 \quad (3)$$

Where, Bi (i = 1-6) represents the modified coefficients.

Assuming a linear relationship characterizing the Pockels effect,

$$\Delta B = B(E_a) - B(E_a = 0) = [r]E_a \quad (4)$$

Where, ΔB is the change in the coefficients, E_a represents the applied electric field and [r] denotes the tensor of the electro-optic coefficients. In the case of a uniaxial crystal with trigonal $3\bar{m}$ point group symmetry,

$$B_1^0 = B_2^0 = 1/n_o^2 \quad (5)$$

$$B_3^0 = 1/n_e^2 \quad (6)$$

Where, n_o and n_e represent the ordinary and extraordinary refractive indices, respectively and [r] is given in the form [4],

$$[r] = \{[0, -r_{22}, r_{13}], [0, r_{22}, r_{13}], [0, 0, r_{33}], [0, r_{51}, 0], [r_{51}, 0, 0], [-r_{22}, 0, 0]\} \quad (7)$$

Where, r_{ij} represents the electro-optic coefficient, E_a is taken to be in the direction of the principal x-axis of the crystal. Therefore,

$$E_a = E_x a_x \quad (8)$$

Where, the unit vector along x is denoted as a_x, from equation 4,

$$\begin{aligned} B_1 &= B_1^0 - r_{22}E_y + r_{13}E_z \\ B_2 &= B_2^0 + r_{22}E_y + r_{13}E_z \\ B_3 &= B_3^0 + r_{33}E_z \\ B_4 &= r_{51}E_y \\ B_5 &= r_{51}E_x \\ B_6 &= -r_{22}E_x \end{aligned} \quad (9)$$

And equations 3, 8 and 9 result as follows,

$$B_1^0 x^2 + B_2^0 y^2 + B_3^0 z^2 + 2r_{51}E_x xz - 2r_{22}E_x xy = 1 \quad (10)$$

In the x,y,z, coordinates,

$$[B_{ij}][x, y, z]^T . [x, y, z]^T = B_1 x^2 + B_{12}xy + B_{13}xz + B_{21}xy + B_2 y^2 + B_{23}yz + B_{31}xz + B_{32}yz + B_3 z^2 \quad (11)$$

Where, T represents the transposition and the dot indicates the inner product.

A comparison of equations 10 and 11 yields,

$$\begin{aligned} B_1 &= B_1^0 \\ B_2 &= B_2^0 \\ B_3 &= B_3^0 \\ B_{13} &= B_{31} = B_5 = r_{51}E_x \\ B_{12} &= B_{21} = -B_6 = -r_{22}E_x \\ B_{23} &= B_{32} = 0 \end{aligned} \quad (12)$$

The associated matrix can be put in the form of,

$$[M] = \{[B_1, -B_6, B_5], [-B_6, B_1, 0], [B_5, 0, B_3]\} \quad (13)$$

Where, equation 5 is used. The coefficients B_5 and B_6 are given in terms of Ex in equation 12.

The diagonalization of [M] yields the following eigen value equation

$$\lambda^3 - \lambda^2(2B_1 + B_3) - \lambda(B_6^2 - B_1^2 - 2B_1B_3 + B_5^2) - B_1^2B_3 + B_3B_6^2 + B_1B_5^2 = 0 \quad (14)$$

The solution gives three distinct Eigen values λ_1, λ_2 and λ_3 which determine the three refractive indices and signify that the initial uniaxial crystal of trigonal $3\bar{m}$ point group symmetry is converted to a biaxial one. The three corresponding Eigen vectors x', y', z', specify their principal axes.

As expected, if the applied electric field Ex vanishes, the Eigen value equation 14 is reduced to that of a uniaxial crystal.

DISCUSSION

Following the above theoretical procedure, numerical results of refractive indices n_x', n_y', n_z' and principal axes of the biaxial crystal designated by x', y', z', are presented.

As an example of a uniaxial crystal of trigonal $3\bar{m}$ point group symmetry, Lithium Niobate ($LiNbO_3$) was used in this study. This type of crystal is widely used in high-speed optical modulators applied in optical communication systems and networks. The applied electric field should be maintained below the break-down voltage. Numerous papers have been published in the literature with a few reports on applied electric fields as large as 63 V/μm or even 120 V/μm to lithium niobate [10-12]. Because the spatial light switches disclosed are expected to be capable of high-speed operation, the driving electric field must be maintained as small as possible [9]. Its operation is based on light polarization rotation in uniaxial trigonal $3\bar{m}$ point group symmetry crystals with an electric field applied in the direction of x principal axis which is the main objective of this study. If the voltage applied to a lithium niobate crystal exceeds ~20 V/μm, the high-speed spatial switching of light is seriously impeded because of the limitations of the driving electronic signal sources at high frequencies. On the other hand, fatigue-induced instabilities have been reported to be generated at applied electric fields of only ~0.5 V/μm in lithium niobate waveguides [13]. However, these results correspond to electric fields along the optical z axis, at raised temperatures with optical energy confined inside the waveguide. Along this orientation, the largest electro-optic coefficient r_{33} is used which characterizes the strongest Pockels activity. The objective of this work is to investigate lithium niobate behaviour when an electric field is applied along a different direction where smaller electro-optic coefficients come to play a role. Therefore, the instabilities reported may be relaxed [13].

Due to the above considerations, in what follows, the applied electric field is taken in the range of 0.1~20 V/μm along which practically useful numerical data will be presented. As the elctro-optic coefficients, high frequency measurements resulted in r_{51}=28 pm/V and r_{22}=3.4 pm/V at light wavelength of 0.6328 μm [14]. However, some lithium niobate manufacturers report slightly larger r_{22} indicating stronger electro-optic activity. However, to be safe, the above-mentioned electro-optic coefficients are retained in the present numerical results.

Figure 1, shows the variations in the refractive indices n_y' and n_z' *vs.* the applied electric field along the X direction where the principal axes of the perturbed crystal are represented by x', y', z'. As seen n_z' increases while n_y' decreases very rapidly with an increasing applied electric field. These variations indicate remarkable Pockels activity in response to an applied electric field along X-direction of a lithium niobate crystal. The following two remarks should be considered in the results depicted in Figure 1. First, the ordinary and extraordinary refractive indices were taken as n_o=2.2866 and n_e=2.2028 at a light wavelength of 0.6328 μm. These refractive indices can be found in many books and manufacturer's data sheet and are consistent up to the third decimal point. However, there were some differences in the fourth decimal point. The refractive index depends on the wavelength, temperature, lithium- concentration and growth conditions. Therefore, it might be possible to adjust some of these influential factors to compensate for the deviation, if any, at the fourth decimal point. Further investigations were carried out to determine the effects of variations in the fourth decimal point. Fortunately, the results showed similar trends in variations, except for a very small shift, allowing both the refractive indices n_y', n_z' to approach the initial no value. Second, at very small electric fields, refractive index variations can be observed only at the fifth or even sixth decimal points demanding for a very highly accurate numerical calculation. This is the case whereas, to begin with, the uniaxial crystal refractive indices were specified to only their fourth decimal point. In other words, the refractive indices n_y', n_z' are not reliable at applied electric fields smaller than approximately 0.1 V/μm. Their inclusion is intended to show that they converge to the initially assumed n_o value and are depicted as broken lines.

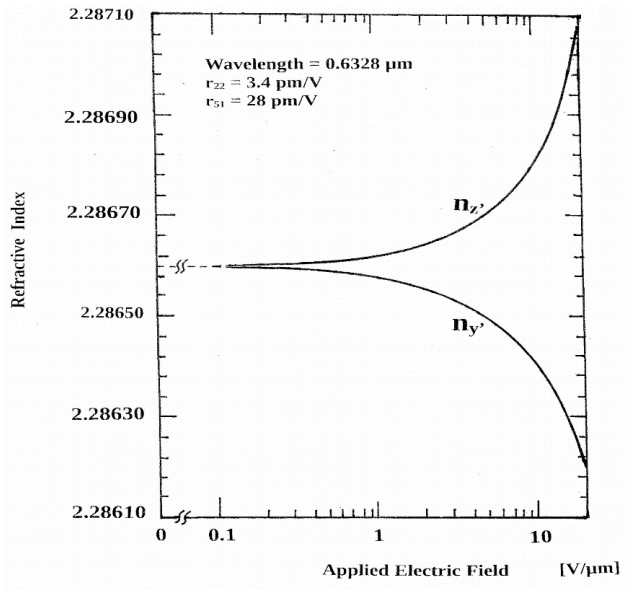

Figure 1: Variations of the refractive indices n_y' and n_z' of the biaxial crystal with applied electric field along the X direction.

Figure 2, shows the variations in the refractive index n_x' against the applied electric field. As shown, n_x' drops off following a trend similar to that of n_y'. Since the refractive indices n_x', n_y', n_z' depicted in Figures 1 and 2 are remarkably different, it is concluded that, as the consequence of application of an electric field along the x principal axis, the initially uniaxial lithium niobate crystal has been perturbed into a biaxial one.

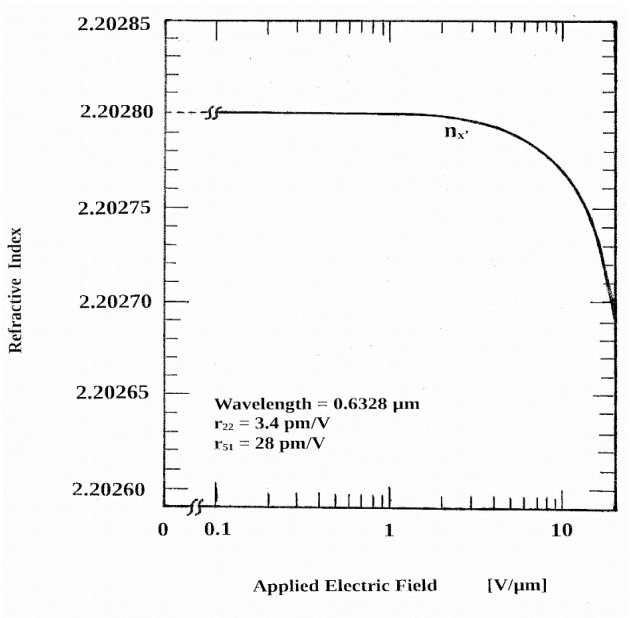

Figure 2: Variations of the refractive index n_x' of the biaxial crystal with applied electric field.

With respect to the principal axes of the perturbed biaxial lithium niobate crystal, variations in the inclination of the principal axis x' with respect to the optical z-axis of the uniaxial crystal shown by β is depicted in Figure 3. It can be concluded that these principal axes were rotated by approximately 45° over the entire range of applied electric fields. Such a rotation would be approximately δ=42° (in the transverse plane) at large applied electric fields. In the literature, a 45° rotation of the principal axes was reported for an applied electric field along the optical axis of a Potassium Dihydrogen Phosphate (KDP) [4]. Under these circumstances, light propagation along the optical axis of such crystals is impeded and the use of transparent electrodes becomes mandatory, together with the disappearance of the advantages of the form factor. In this regard, rotation of the principal axes by approximately 45° in a transverse plane accompanied by light propagation along the optical axis, is achieved for the first time.

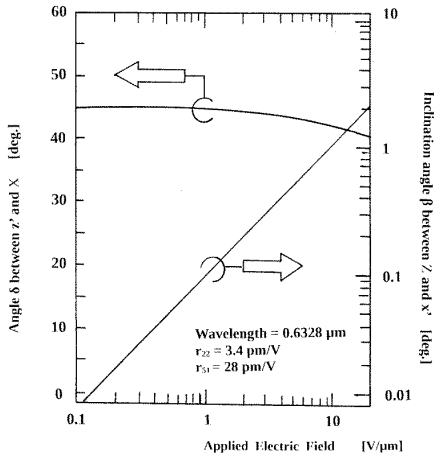

Figure 3: Rotation angle δ of Z' with respect to X and inclination angle β between Z and X'.

In lieu of the above explanations, one may look at the relative orientations of the principal axes x', y', z' with respect to the x, y, z, coordinates of the uniaxial crystal, as depicted in Figure 4.

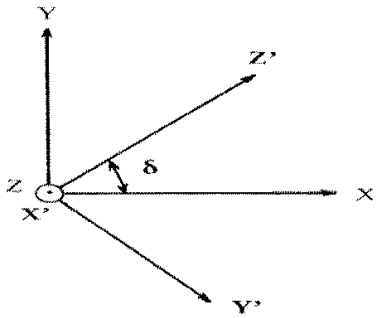

Figure 4: Approximate representation of principal axes X', Y', Z' in coordinate system X, Y, Z of uniaxial crystal.

As the inclination of x' with respect to z is, utmost 1° according to Figure 3, it is considered to be approximately perpendicular to the xy plane. The mutually perpendicular y' and z' principal axes are, to a good approximation, in the xy plane but rotated by the angle δ which is not significantly different from 45°. These achievements (depicted concisely in Figure 4) have significant applications in optical devices. The orientations of the principal axes suggest that the initial uniaxial crystal evolves into a biaxial crystal but with one principal axis maintained almost along the optical axis. The remaining two principal axes lie in a plane transverse to the optical axis but rotated by approximately 45°. This relative orientation is maintained over the entire range of applied electric fields. As an application, a fast spatial light switch is disclosed wherein an incident plane-wave light with a linear polarization along x is rotated to a new polarization along y, in association with propagation along the optical axis [9]. Typically, a 90° plane-wave polarization rotation is required in many applications. This can easily be achieved by adjusting the propagation length (along the optical axis) of a lithium niobate crystal, as shown in Figure 5, where the input light is considered to be a plane-wave.

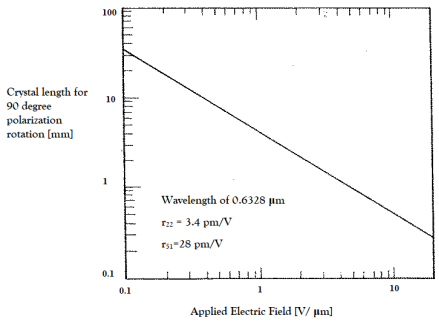

Figure 5: Required crystal length *vs.* applied electric field.

As shown, the variations in required crystal length with applied electric field follow a straight line. For example, at an applied electric field of 1 V/μm, a 4.2 mm long lithium niobate crystal can alter the light of linear polarization along x to that along y. The important point to note is that, because Pockels activity is a very fast phenomenon, such rotation can reach speeds in excess of several tens of GHz and possibly up to 100 GHz, depending on the propagation length and availability of high-voltage sources. Such fast 90° polarization rotation in the transverse plane is expected to find numerous applications in optical devices and systems. It is important to note that losses in quality lithium niobate crystals are small. Since propagation lengths less than ~1 cm are considered, the absorption losses have been neglected in the presented results.

CONCLUSION

In summary, when an electric field is applied along the x principal axis of a uniaxial crystal of trigonal $3\overline{m}$ point group symmetry, the crystal evolves into a biaxial crystal exhibiting 3 distinct refractive indices. One of the principal axes of the biaxial crystal lies approximately along the optical axis. The remaining two principal axes rotate, in the transverse plane, by approximately 45°. The polarization direction of an x-polarized plane wave input-light rotates when propagating along the principal axis. The required crystal length in achieving 90° polarization rotation is determined.

AVAILABILITY OF DATA

All relevant data are within the paper and its supporting file.

REFERENCES

1. Robson BA. The theory of polarization phenomena. Clarendon. 1974.
2. Yariv A, Yeh P. Optical waves in crystals. Wiley Interscience 1 ed, 2002.
3. Marcuse D. Light transmission optics (van Nostrand Reinhold electrical/computer science and engineering series). 1982.
4. Saleh BE, Teich MC. Fundamentals of photonics. Wiley. 1991:313.
5. Karim MA. Electro-optical devices and systems. 1990.
6. Nayyer J, Suematsu Y, Shimomura K. Analysis of reflection-type optical switches with intersecting waveguides. J Lightwave Technol. 1988;6(6):1146-1152.
7. Li B, Chua SJ. Reflection-type optical waveguide switch with bow-tie electrode. J Lightwave Technol. 2002;20(1):65.
8. Diemeer MB. Polymeric thermo-optic space switches for optical communications. Opt Mate. 1998;9(1-4):192-200.
9. Nayyer J, Ravanmehr V. Fast spatial light switches and their fabrication. United States Patent, US 10,816,871. 2020.
10. Hartwig U, Luennemann M, Buse K. OSA trends in optics and photonics. 2003.
11. Shuto Y, Amano M. Reflection measurement technique of electro-optic coefficients in lithium niobate crystals and poled polymer films. J Appl Phys. 1995;77(9):4632-4638.
12. Luennemann M, Buse K, Sturman B. Coupling effects for counterpropagating light beams in lithium niobate crystals studied by grating translation technique for extremely high external electric fields. J Appl Phys. 2003;94(10):6274-6279.
13. Nagata H, Mitsugi N, Sakamoto T, Kiuchi K, Ichikawa J. Applied-voltage induced fatigue of lithium niobate waveguide. Appl Phys Lett. 1996;68(3):301-303.
14. Turner EH. High-frequency electro-optic coefficients of lithium niobate. Appl Phys Lett. 1966;8(11):303-304.