Modeling of Inlet Zone in Hydrostatic Extrusion Process for Aluminum and Aluminum Alloys

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Abstract

The objective of this paper is to investigate the viscous shear heating effect on the formation of minimum film thickness in inlet zone of the hydrostatic extrusion process as function of various operating parameters. Minimum film thicknesses in the inlet zone has been evaluated at various extrusion speeds, material parameters, extrusion pressures, and die semi-angle for extrusion of low strength material. A strong numerical technique is used in the investigation of inlet zone of a hydrostatic extrusion process considering viscous shear heat generation and convection & conduction modes of heat removals for the study of the minimum film thickness.

Keywords: *Thermo-hydrodynamic lubrication, lubricant pressure, minimum film thickness, aluminum extrusion, Extrusion pressure, Material parameters*

1. Introduction

In hydrostatic extrusion method, billet is extruded through a die using pressurized fluid. The billet is supported over its entire length by the fluid pressure (Fig.1). The frictional force between the die and the billet happens to be low due to presence of high pressure fluid at the interface. Hydrostatic extrusion process is used in industries for production of items from difficult to deform materials. The aluminum extrusion industries have progressed as a global industry with increasing competition from other materials on the quality and price. A breakthrough in aluminum extrusion appeared with the manufacturing of airships and aircrafts [1] and difficult-to-extrude high strength aluminum alloys (aluminum-copper) were developed in 1930. The main challenge in extrusion processes is to achieve both dimensional and mechanical tolerances. Lubrication plays a vital role in extrusion process by preventing direct metal to metal contact at the die/billet interface, with the reduction of extrusion pressure and die wear and the enhancement of product quality and tool life.

The analysis of hot extrusion of aluminum is carried out [2-4], since aluminum has a strong tendency to adhere on steel die surface at elevated billed temperature and therefore development of adhesive layer on the die bearing surface occurs. The theoretical investigation and comparison with experimental work of lubricant film thickness on seizure phenomenon in commercially pure aluminum is analyzed [5] in cold extrusion process and seizure phenomenon is predominant near the die exit. The lubrication performance of palm oil is compared experimentally with paraffinic mineral oil in extrusion of pure aluminum [6] and proved that palm oil exhibits better lubrication (Reduction in extrusion load and better surface finishing) than paraffinic oil.

In order to numerically model the hydrostatic extrusion process, it is found that researchers [7-9] have given major focus on isothermal analyses of hydrostatic extrusion process either considering hydrodynamic lubrication regime or mixed lubrication mode. Few articles [10-11] have reported thermal considerations in hydrodynamic lubrication of hydrostatic extrusion process by considering approximate heat generating equation due to plastic deformation / omitting convection term in energy equation. It is necessary to mention here that at high extrusion speeds the role of convection terms should be accounted while computing the minimum film thickness in the inlet zone. Therefore, the objective of this work is to accurately compute the minimum film thickness by considering convection term in energy equation at elevated extrusion speeds. In order to compute minimum film thickness accurately at the exits of inlet zone in hydrostatic extrusion process; an efficient thermo hydrodynamic analysis of inlet zone has been carried out using a numerical technique developed by Elrod and Brewe [12] and further used by [13-15] in their research work, being used by the authors in the paper.

2. Mathematical formulation

Coupled solution of Reynolds equation, energy equation, and rheological relations is obtained for the minimum film thickness in hydrodynamically lubricated inlet zone of the hydrostatic extrusion process. Inlet zone has been indicated in a schematic diagram of the hydrostatic extrusion process as shown in Fig.1. It is assumed that inlet zone is formed by rigid billet and die.

2.1 Generalized Reynolds equation

Double integration of non-inertial momentum equation, $\partial p / \partial x = \partial / \partial y(\eta . \partial u / \partial y)$, for 1-D laminar flow with the boundary conditions (ζ = -1, $u = 0$, and ζ =+1, u = u_b), gives flow velocity of the lubricant in the computing domain. Thus, velocity expression for lubricant is;

$$u = A\int_{-1}^{\zeta} \xi \, d\zeta + B\int_{-1}^{\zeta} \xi \, \zeta \, d\zeta \tag{1}$$

Where,

$$A = \left(u_b - B\int_{-1}^{1} \xi \, \zeta \, d\zeta \right) / \int_{-1}^{1} \xi \, d\zeta \quad \text{And} \quad B = \left(h / 2 \right)^2 \nabla p$$

In eq. (1), the variation of viscosity has been modeled by Roelands' viscosity relation. This viscosity model is expressed as follows;

$$\eta = \eta_0 \exp[(\ln \eta_0 + 9.67)\left\{ -1 + (1 + 5.1\text{x}10^{-9} p)^{z'} \right\} - \gamma (T - T_0)] \tag{2}$$

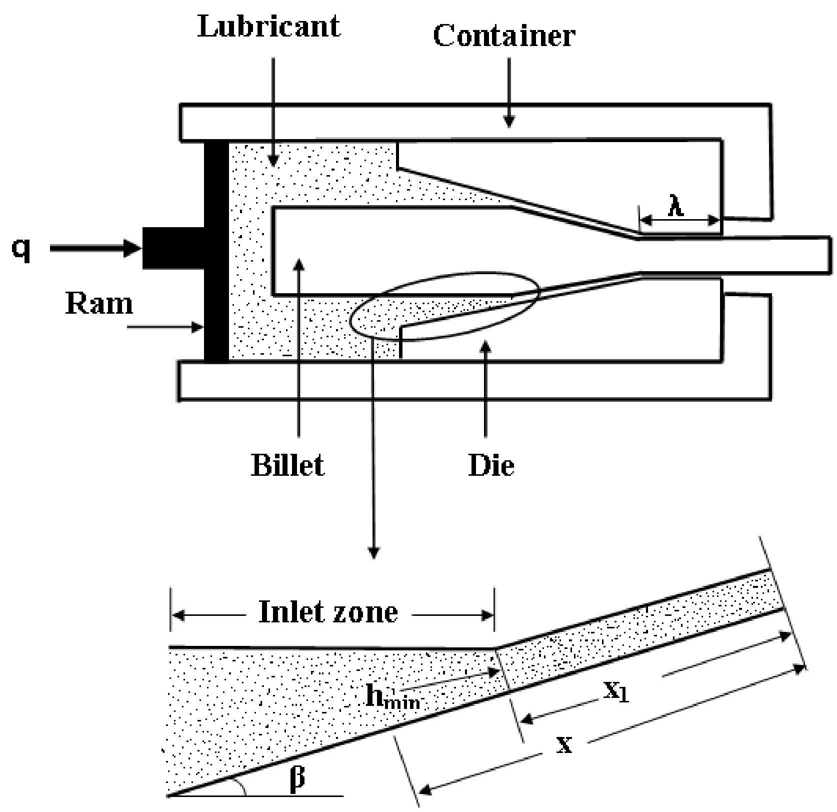

Figure 1 Schematic diagram of hydrostatic extrusion process

In the modeling, the viscosity has been expressed in terms of fluidity ($\xi = 1/\eta$) for the convenience of mathematical formulation. The expression for the fluidity variation across the film thickness is expressed by Legendre polynomial of order 2 as follows:

$$\xi(\zeta) = \sum_{k=0}^{3} \overline{\xi_k}\, P_k(\zeta) \tag{3}$$

The Legendre coefficients of fluidity, $\overline{\xi_k}$, are evaluated as;

$$\int_{-1}^{1} \xi(\zeta) P_k(\zeta)\, d\zeta = \left(2/(2k+1)\right)\overline{\xi_k}$$

OR

$$\overline{\xi_k} = \left((2k+1)/2\right)\sum_{i=0}^{3} w_i \xi_i P_k(\zeta_i) \tag{4}$$

The Legendre coefficients for fluidity have been written from equation (4) for four Lobatto locations as follows;

$$\overline{\xi}_0 = \frac{1}{12}\left\{\xi_d + 5\xi_{-1} + 5\xi_{+1} + \xi_b\right\} \tag{5a}$$

$$\overline{\xi}_1 = \frac{1}{4}\left\{\xi_b - \xi_d + \sqrt{5}\left(\xi_{+1} + \xi_{-1}\right)\right\} \tag{5b}$$

$$\overline{\xi}_2 = \frac{5}{12}\left\{\xi_d + \xi_b - \left(\xi_{+1} + \xi_{-1}\right)\right\} \tag{5c}$$

$$\overline{\xi}_3 = \frac{7}{12}\left\{\xi_b - \xi_d + \sqrt{5}\left(\xi_{+1} - \xi_{-1}\right)\right\} \tag{5d}$$

In order to develop generalized Reynolds equation, the expression for lineal mass flux has been developed as follows;

$$\dot{m} = \int_{-h/2}^{+h/2} \rho\, u\, dy \tag{6}$$

Using Eqs. (1) and (6), the following expression has been evolved;

$$\frac{\dot{m}}{\rho} = \left(u_b\right)\frac{h}{2} - \frac{h}{3}\overline{\xi}_1 A - \frac{h}{3}\left(\overline{\xi}_0 + \frac{2}{5}\overline{\xi}_2\right)B \tag{7}$$

Divergence of mass flux (from eq.(7)) leads to the generalized Reynolds equation, which is as follows:

$$\nabla.(\dot{m}/\rho) = 0$$

$$Or$$

$$\nabla \cdot \overline{\xi}_p h^3\, \nabla p = 6\, u_b\, \nabla h - 2\, \nabla \cdot (\overline{\xi}_1 / \overline{\xi}_0) h (u_b - u_d) \tag{8}$$

Where, $\overline{\xi}_p = \overline{\xi}_0 + 0.4\overline{\xi}_2 - (\overline{\xi}_1^2 / 3\overline{\xi}_0)$

2.2 Energy equation

The temperature variation across the lubricating film is represented by Legendre polynomial of order 2 and is expressed as;

$$T(\zeta) = \sum_{k=0}^{3} \overline{T}_k\, P_k(\zeta) \tag{9}$$

The Legendre coefficients of temperature, $\overline{T}_k$, are evaluated as;

$$\int_{-1}^{1} T(\zeta) P_k(\zeta)\, d\zeta = \frac{2}{2k+1}\overline{T}_k$$
$$Or \qquad (10)$$
$$\overline{T}_k = \frac{2k+1}{2}\sum_{i=0}^{3} w_i T_i P_k(\zeta_i)$$

The expressions for Legendre coefficients of temperature i.e. $\overline{T_0}$, $\overline{T_1}$, $\overline{T_2}$, and $\overline{T_3}$, are written as below using eq.(10).

$$\overline{T_0} = \frac{1}{12}\left\{T_d + 5T_{-1} + 5T_{+1} + T_b\right\} \qquad (11a)$$

$$\overline{T_1} = \frac{1}{4}\left\{T_b - T_d + \sqrt{5}(T_{+1} + T_{-1})\right\} \qquad (11b)$$

$$\overline{T_2} = \frac{5}{12}\left\{T_d + T_b - (T_{+1} + T_{-1})\right\} \qquad (11c)$$

$$\overline{T_3} = \frac{7}{12}\left\{T_b - T_d + \sqrt{5}(T_{+1} - T_{-1})\right\} \qquad (11d)$$

Using eqs. (11a) to (11d), the following two equations have been obtained;

$$\overline{T_2} = (T_b + T_d)/2 - \overline{T_0} \qquad (12)$$

$$\overline{T_3} = (7/3)\left[\left(T_b - T_d\right)/2 - \overline{T_1}\right] \qquad (13)$$

In the eqs. (11a) to (11d), surface temperatures of billet and die are considered constant (i.e. bounding solids are treated as adiabatic)

$$T_b\left(x, +h/2\right) = T_0 \qquad (14)$$

$$T_d\left(x, -h/2\right) = T_0 \qquad (15)$$

The energy equation used in the present analysis (without dilatational viscosity) is;

$$\rho\, u\, C_p . \partial T / \partial x = \partial / \partial y\left(k . \partial T / \partial y\right) + \eta\, (\partial u / \partial y)^2 \qquad (16)$$

In order to compute temperatures 'T_{-1}' and 'T_{+1}', two equations involving 'T_{-1}' and 'T_{+1}' are required. Therefore, zeroth and first moments of energy equation (16) have been taken across the film thickness as follows;

Zeroth moment:

$$\int_{-h/2}^{+h/2} \frac{h}{2} u \frac{\partial T}{\partial x}\, d\zeta = \chi\left\{\left[\frac{\partial T}{\partial y}\right]_{+2} - \left[\frac{\partial T}{\partial y}\right]_{-2}\right\} + \frac{1}{\rho C_p}\int_{-h/2}^{+h/2} \frac{h}{2}\phi\, d\zeta \qquad (17)$$

First moment;

$$\int_{-h/2}^{+h/2}\frac{h^2}{4}u\frac{\partial T}{\partial x}\zeta\,d\zeta=\chi\frac{h}{2}\left\{\left[\frac{\partial T}{\partial y}\right]_{+2}-\left[\frac{\partial T}{\partial y}\right]_{-2}\right\}+\frac{1}{\rho C_p}\int_{-h/2}^{+h/2}\frac{h^2}{4}\phi\zeta\,d\zeta \tag{18}$$

The subscripts '+2' and '-2' are used to denote the upper (billet) and lower (die) surfaces respectively.

Simplifications of equations (17) and (18) yield:

$$C_0\left(C_1\frac{\partial\overline{T_0}}{\partial x}+C_2\frac{\partial\overline{T_1}}{\partial x}+C_3\frac{\partial\overline{T_2}}{\partial x}+C_4\frac{\partial\overline{T_3}}{\partial x}\right)=C_5\left(\left(\frac{2}{h}\right)^2\left(\frac{T_2+T_{-2}}{2}-\overline{T_0}\right)\right)+C_6\left(\frac{2}{h}\right)^2 \tag{19}$$

$$C_0\left(C_7\frac{\partial\overline{T_0}}{\partial x}+C_8\frac{\partial\overline{T_1}}{\partial x}+C_9\frac{\partial\overline{T_2}}{\partial x}+C_{10}\frac{\partial\overline{T_3}}{\partial x}\right)=C_{11}\left(\left(\frac{2}{h}\right)^2\left(\frac{T_{+2}-T_{-2}}{2}-\overline{T_1}\right)\right)+C_{12}\left(\frac{2}{h}\right)^2 \tag{20}$$

3. Computational procedure

The solution of the present model starts, for the inlet zone as shown in Fig.1, with the known isothermal pressure distribution within the domain as obtained from the converged coupled solution of eqs. (21) and (22). The equations are;

$$\partial/\partial x\left(h^3.\partial p/\partial x\right)=6\eta_0\,u_b.\partial h/\partial x \tag{21}$$

Where, h is the lubricant film thickness in the inlet zone and is given by;

$$h=h_{\min}+(x-x_1)\tan(\beta) \tag{22}$$

Eq. (21) is subjected to boundary conditions as follows;

$$p=\sigma_y+q,\quad dp/dx=0\quad at\ x=x_1 \qquad (i)$$
$$p=q\quad at\ x=1.01x_1 \qquad (ii)$$

The convergence criterion for pressure and temperature are;

For pressure;

$$|\left(\sum p_i\right)_{N\text{-}1}-\left(\sum p_i\right)_N|/|\left(\sum p_i\right)_N|\quad<10^{-3}$$

For temperature;

$$\left|\left(\sum T_i\right)_{N-1} - \left(\sum T_i\right)_N\right| / \left|\left(\sum T_i\right)_N\right| \quad < 10^{-3}$$

Where 'N' is the iteration number and 'i' denotes the nodal position in domain. The converged solution for the pressure and temperature are obtained when the convergence criteria for both pressure and temperature are satisfied simultaneously. The input data is taken from Table 1.

4. Results and discussion

Table 1 : Input data (*Snidle et al. 1973)*

1. Billet:	
-Density (ρ_b), kg/m^3	271
-Initial diameter (D_i), cm	11.43
-Yield strength (σ_y), MPa	138
2. Lubricant;	
-Density (ρ_o), kg/m^3	890
-Exponent in Roelands' viscosity model ($Z^{'}$)	0.63
-Inlet oil temperature (T_o), K	303
-Pressure-viscosity coefficient (α), Pa^{-1}	$1.639\text{x}10^{-8}$
-Specific heat (c_p), J/kg-K	2000
-Temperature viscosity coefficient (γ),K^{-1}	0.054
-Thermal conductivity (k), W/m-K	0.173
-Viscosity (η_0), Pa-s	0.1
3. Die:	
-Die semi-angle (β)	15^0
-Length of land (λ), m	0.004

A Galerkin type approach has been made of temperature effects in continuous lubricating film in the inlet zone of extrusion process. In this process four points two at the boundaries and two at intermediate locations known as lobatto's points enables satisfactory prediction of lubricant across film temperature distribution for substantial viscosity distribution. The above method yields two partial differential equations, one for the local space-mean temperature, and one for the first transverse moment of the lubricant temperature distribution. The coupled solution of these partial differential equations with generalized Reynolds equation has been presented for steady state condition in the inlet zone of hydrostatic extrusion process. Input parameters used in the present mathematical modelling are provided in Table 1. Additional input data are also attached in the presented figures.The variation of thermal minimum film thickness with semi-die angle (10 degree to 25 degree) is plotted in Fig. 2 for various values of billet speeds (1 m/s to 15 m/s) and it is little bit decreasing with semi-die angle for a constant billet speed. The variation of thermal minimum film thickness with billet speed is plotted in Fig. 3 for various value of semi-die angle. The variation of thermal minimum film thickness with billet speed is plotted in Fig. 4 for various value of extrusion pressure (100 MPa to 300 MPa) and thermal minimum film thickness with billet speed is plotted in Fig. 5 for various value of material parameter (2 to 3). The thermal minimum film thickness is significantly decreasing in Fig. [4-6] with increase of billet speed because of inclusion of convection term in the energy equation and it is responsible for the significant heat removal from the lubricant domain at elevated speeds.

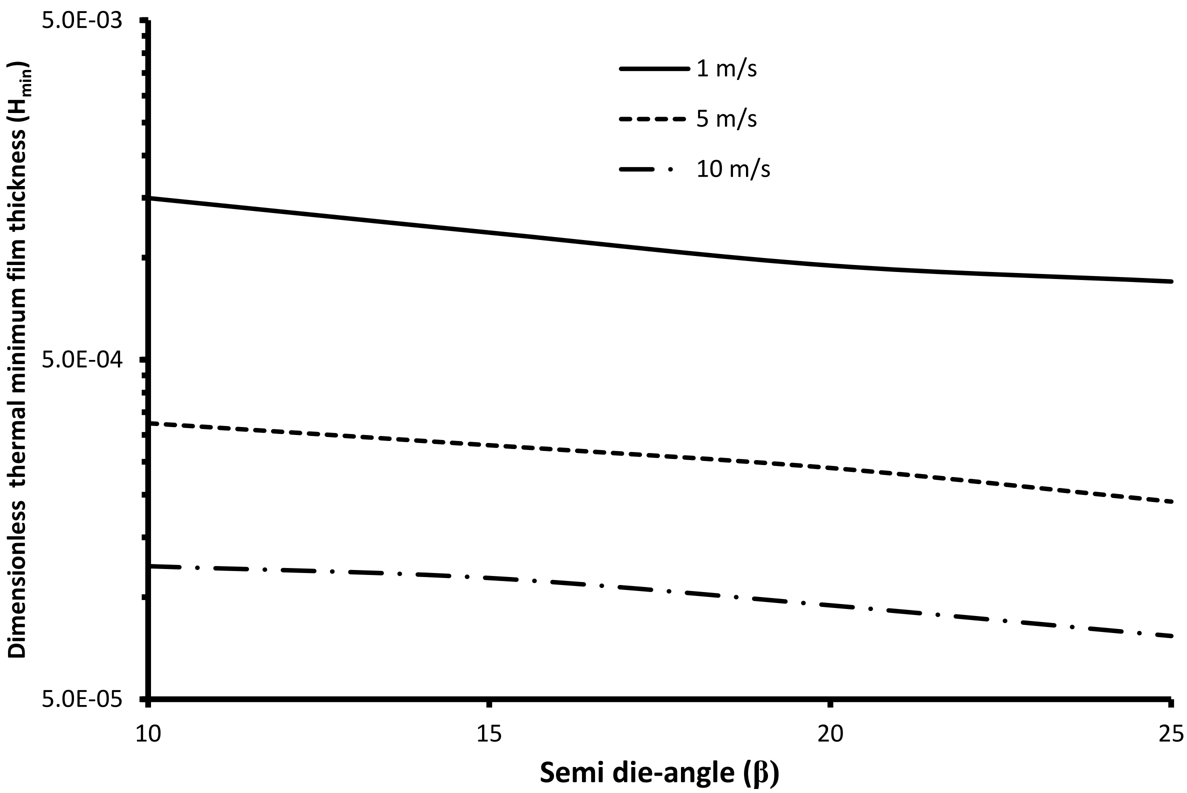

Fig.2 Variation of dimensionless thermal minimum film thickness with die semi angle (β) at various extrusion speeds, [G=2.26, q=100 MPa]

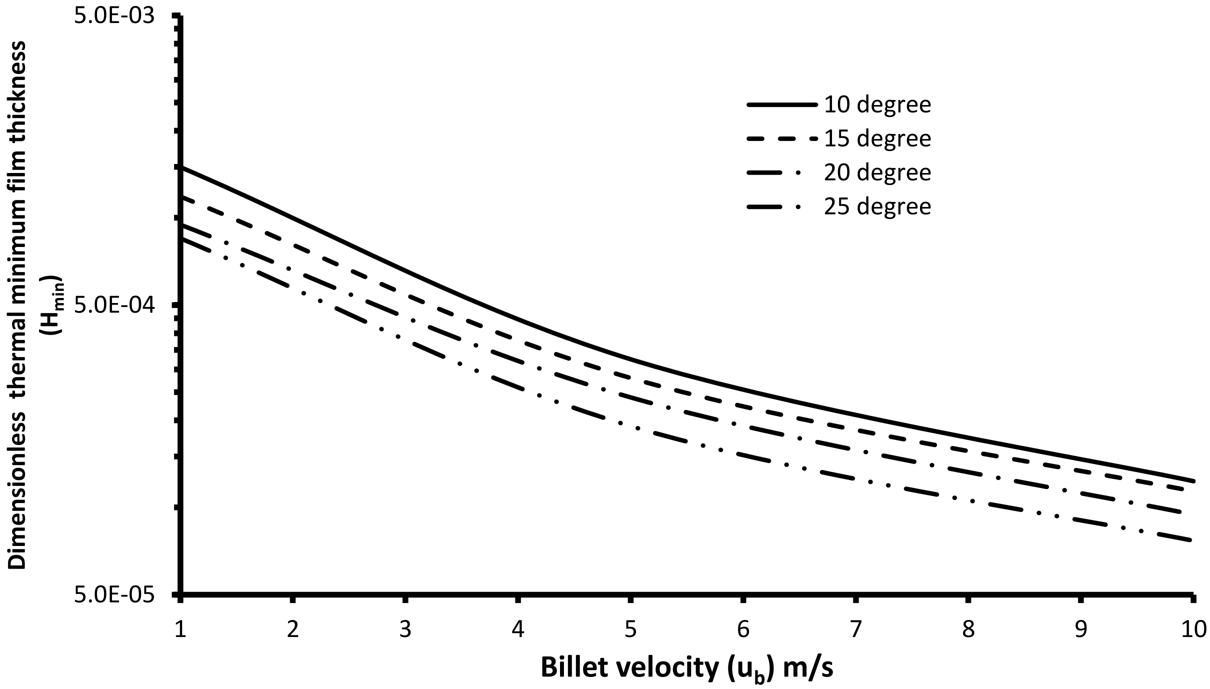

Fig.3 Variation of dimensionless thermal minimum film thickness with billet speeds at various die semi angle (β), [G=2.26, q=100 MPa]

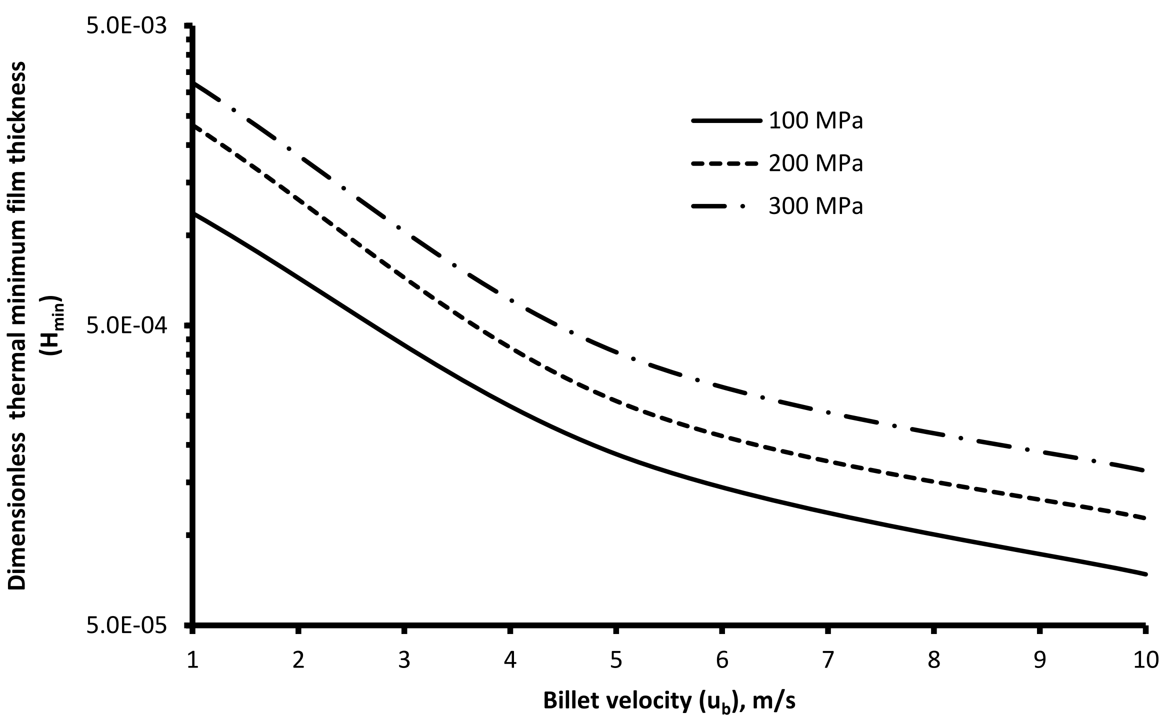

Fig.4 Variation of dimensionless thermal minimum film thickness with billet speeds at various extrusion pressures [G=2.26, β=15 degree]

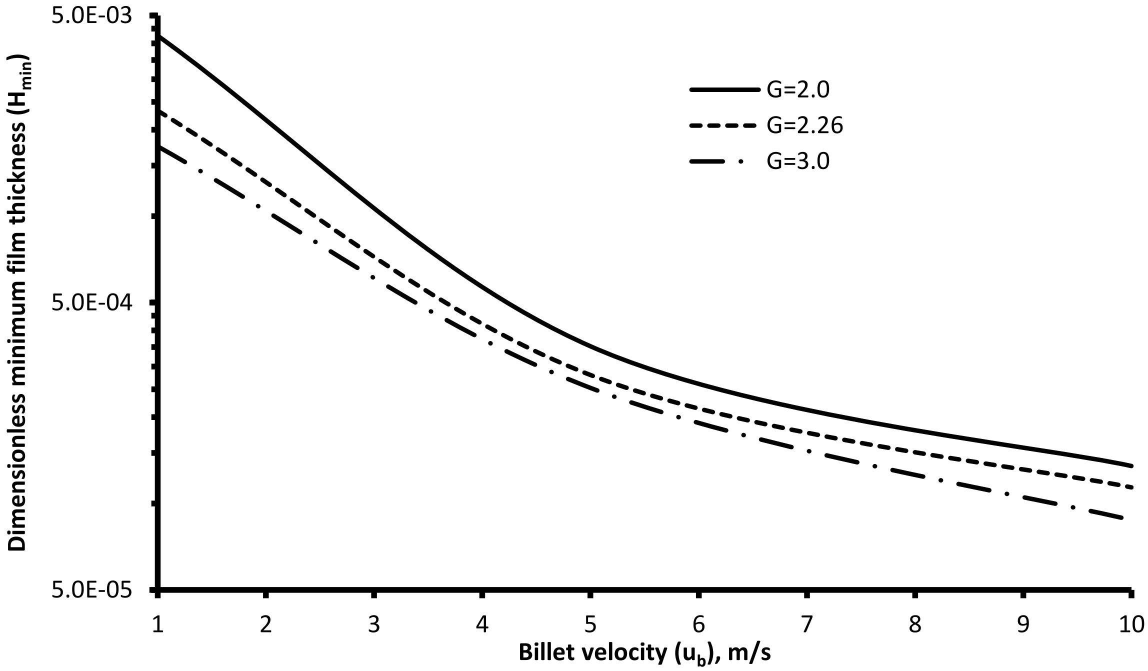

Fig.5 Variation of dimensionless thermal minimum film thickness with billet speeds at various material parameters (G) [β=15 degree, q=200 MPa]

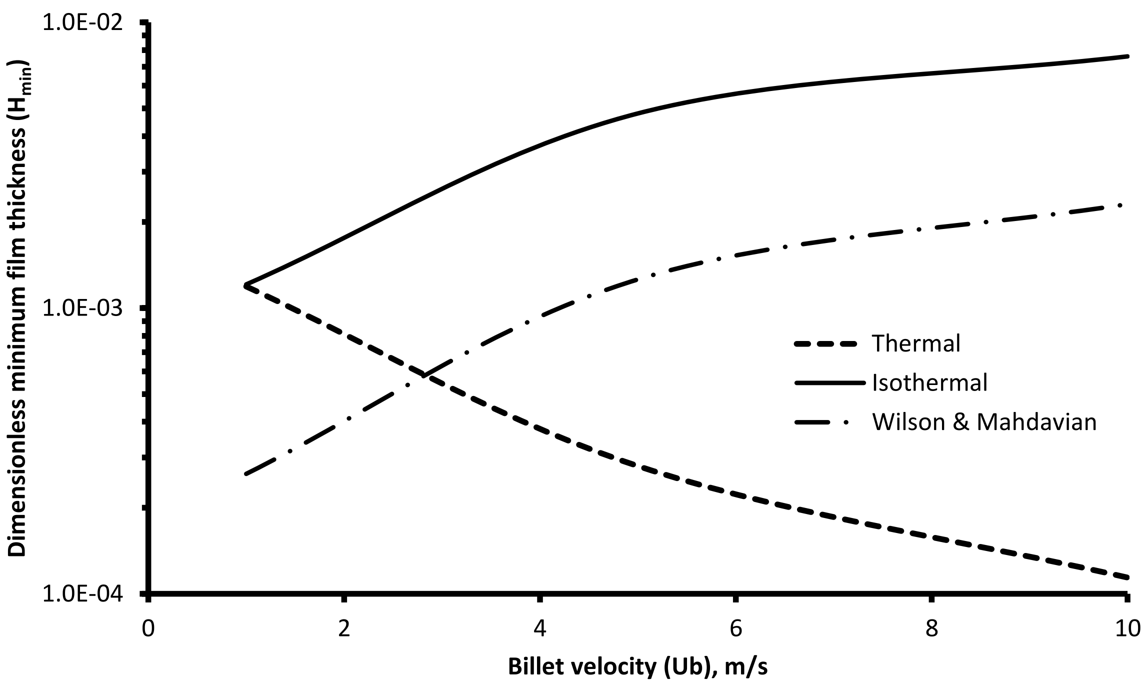

Fig.6 Variation of dimensionless minimum film thickness with billet speeds [β=15 degree, q=100 MPa, G=2.26]

5. Conclusion

The thermal minimum film thickness in the inlet zone of hydrostatic extrusion process using aluminum as billet material is evaluated numerically using Lobatto quardature technique. The viscous heating effect is found to be significant at elevated billet speed as a result thermal minimum film thickness is a decreasing function of billet velocity, semi-die angle and material parameters. The thermal minimum film thickness is an increasing function of extrusion pressure as additional quantity of lubricant is pumped at billet die interface when extrusion pressure is increased and lubricant film thickness becomes thicker.

Nomenclature

D	Diameter of billet
D_i	Initial diameter of billet
D_f	Final diameter of billet
C_p	specific heat of lubricant, J/kg-K
G	Material parameter ($\alpha\sigma_y$)
h_{iso}	isothermal minimum film thickness at $x=1.01x_1$, m
h_{min}	minimum film thickness, m
h_{th}	thermal minimum film thickness at $x=1.01x_1$,
H_{min}	dimensionless minimum film thickness, (h_{min}/λ)
H_{th}	dimensionless thermal minimum film thicknees, (h_{th}/λ)
k	thermal conductivity of lubricant , $Wm^{-1}K^{-1}$
$\dot{m}_x$	lineal mass flux, kg/m-sec
p	lubricant pressure, N/m^2
P_k	Legendre polynomial
T	film temperature, K
$\overline{T}_k$	Legendre coefficients for temperature
T_0	ambient temperature, K
u	lubricant velocity, m/s
u_b	billet velocity, m/s
x	coordinate along extrusion direction, m
x_1	length of work zone, m
y	coordinate perpendicular to extrusion direction, m
z'	exponent in Roeland's viscosity model
$+1$	subscript for Lobatto location, $(\zeta = 1/\sqrt{5})$
$+2, b$	subscript for die surface
-1	subscript for Lobatto location, $(\zeta = -1/\sqrt{5})$
$-2, d$	subscript for billet surface

Greeks

α	pressure viscosity coefficient, m^2/N

β die semi angle
γ temperature coefficient of viscosity, K^{-1}
ζ dimensionless parameter, $2y/h$
η viscosity of lubricant, Pa-s
η_0 viscosity of lubricant at T_0, Pa-s
ξ fluidity function, $(1/\eta)$
$\overline{\xi_k}$ Legendre coefficients for fluidity
ρ density, kg/m^3
χ thermal diffusivity, $k/(\rho C_p)$
σ_y yield strength of billet material, MPa
λ length of land

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