ASHDIN
PUBLISHING

Research Article

Mathematical Model for Computing Maximum Power Output of a PV Solar Module and Experimental Validation[⋆]

V. P. Sethi,[1] K. Sumathy,[1] S. Yuvarajan,[2] and D. S. Pal[3]

[1]*Department of Mechanical Engineering, North Dakota State University, Fargo, ND 58102, USA*
[2]*Department of Electrical and Computer Engineering, North Dakota State University, Fargo, ND 58108, USA*
[3]*Department of Mathematics, Statistics and Physics, Punjab Agricultural University, Ludhiana, Punjab 141004, India*
Address correspondence to V. P. Sethi, vpsethi68@yahoo.co.in

Abstract A solar simulator is used for testing the panel at various irradiance and temperature ranges. Testing is done at North Dakota State University, Fargo, USA in March 2011.

1 Introduction

The conversion of solar radiation into electricity has been extensively studied by many authors [2,4,5,11]. In particular, photovoltaic (PV) cells allow the energy transported by electromagnetic waves (i.e., photons) to be directly converted into electricity. Predicting the performance of PV panels is essential for design engineers. From the characteristic I–V curve of a given PV cell, three key physical quantities are defined: the short-circuit current, the open-circuit voltage, and the values of current and voltage that permit the maximum power to be obtained. These variables correspond to well-defined points in the I–V plane. The determination of these points is essential for the development of appropriate PV cell models. The nonlinear and implicit relationships that exist between them, however, necessitate using iterative numerical calculations [9]. Furthermore, most of these parameters depend on both the cell temperature and the solar irradiance; therefore, the knowledge of their behavior is crucial to correctly predict the performance of PV cells and arrays.

In terrestrial applications, solar cells are generally exposed to temperatures varying from 10 °C to 50 °C. The performance of a solar cell is influenced by temperature as its performance parameters, namely, open-circuit voltage (V_{oc}), short-circuit current (I_{sc}), curve factor (CF), and efficiency, are temperature dependent [7]. Different models

based on the current and voltage (I–V) characteristic curve of a P-N junction are used to describe the behavior of PV cells. In these models, a photocurrent is associated with the generation of electron-hole pairs, while a recombination current accounts for diffusion of electrons and holes across the junction. An ideal PV cell model considers only photocurrent and recombination current, where effects of electrical resistances are neglected [9]. It is apparent that practical applications must include the effects of both of them. In addition, the P-N junction itself is modeled as double-diode without resistance for polycrystalline silicon cells, and single-diode ones are used for amorphous silicon cells [3]. It is obvious that the application of the final model must correspond to the best compromise between simplicity and accuracy.

Many mathematical models have been developed to estimate PV efficiency, power, short-circuit current, and open-circuit voltage. An excellent review of different correlations is used to determine these quantities as a function of irradiance and cell temperature [8]. An explicit model was developed in [1] to determine currents and voltages for PV points by using manufacturers' data, where the electrical current changes linearly with irradiance and temperature, while voltages are expressed by temperature coefficients and a correlation term for the irradiance. An analytical model based on manufacturers' data was presented in [6]. The authors have explicitly expressed the PV current and power as a function of voltage by using a shading linear factor expressed for open-circuit voltage loses for irradiances from 1,000 W/m^2 to 200 W/m^2; I–V and P–V curves can thus be produced without calculating explicit expressions of current and voltage at key operational points. It must be pointed out that most of the aforementioned works deal with models that are essentially based on linear temperature and irradiance relationships that require parameters which are not available from manufacturers' datasheets. In the current study, a

solar simulator is developed and used to test an amorphous silicon PV solar module (15 watt peak, 12 volt, amorphous silicon solar panel used for battery charging) at various irradiance levels of $400\,W/m^2$, $700\,W/m^2$, and $1{,}000\,W/m^2$ and operation temperatures from $40\,°C$ to $100\,°C$.

2 Methodology

2.1 Theoretical

The effect of operating temperature on the electrical efficiency of a PV cell/module can be traced to the temperature's influence upon the current, I, and the voltage, V, as the maximum power is given by

$$P_{mp} = V_{mp}I_{mp} = (FF)V_{oc}I_{sc}. \tag{1}$$

It turns out that both the open-circuit voltage and the fill factor decrease substantially with temperature (as the thermally excited electrons begin to dominate the electrical properties of the semi-conductor), while the short-circuit current increases, but only slightly for c-Si solar cells [12].

2.1.1 I–V characteristics and maximum power output

From the measured solar cell parameters (I_{sc}, V_{oc}, T), the maximum power (P_{mp}) can be calculated. The maximum power of a solar cell is the point on the I–V characteristic curve, at which the product $P = I \times V$ is at its maximum value. The relation between I and V for an illuminated solar cell is given by Wagner [10] in the form of (2) and (3) as below:

$$I = I_{il} - I_0\left(e^{\frac{V+IR_s}{V_T}} - 1\right) \tag{2}$$

or expressed by V:

$$V = V_T \cdot \ln\left(\frac{I_{il} - I + I_0}{I_0}\right) - IR_s. \tag{3}$$

In the above equation, I_{il} denotes the electric current generated by illumination, I_0 is the diode current, R_s is the inner resistance ($0.03\,\Omega$, commonly used value in literature), and V_T is the thermal voltage. In order to employ (2) or (3), the diode current, the inner resistance, as well as the thermal voltage had to be known. V_T was calculated from the measured cell temperatures (T) by using the formula

$$V_T = \frac{mkT}{q}, \tag{4}$$

where m is the diode factor (1.6), q is the charge (1.602×10^{-19}), and k is the Boltzmann constant ($1.38{\times}10^{-23}\,J\,K^{-1}$). The diode current ($I_0$) was determined from the measured open-circuit voltage (V_{oc}) by employing the following equation:

$$I_0 = I_{il}e^{-\frac{V_{oc}}{V_T}}. \tag{5}$$

Neglecting the resistance and with $V = 0$, it follows from (2) (short circuit) that $I_{il} \sim I_{sc}$. To compute P_{mp}, a computer program is developed using Newton-Raphson's method. The present calculations referred to P_{mp} based on that scheme. After some rearrangements and substitutions by using (2) and (3), the following equation is obtained:

$$I_{mp} + \frac{(I_{mp} - I_{sc} - I_0)\left[\ln\left(\frac{I_{sc}-I_{mp}}{I_0}+1\right) - \frac{I_{mp}R_s}{V_T}\right]}{1 + (I_{sc} - I_{mp} + I_0)\frac{R_s}{V_T}} = 0, \tag{6}$$

I_{mp} denotes the electric current at its maximum point. Applying Newton-Raphson's method on the previous expression yields I_{mp}:

$$I_{mp+1} = I_{mpi} - \frac{f(I_{mpi})}{f'(I_{mpi})}. \tag{7}$$

Equation (7) clarifies Newton-Raphson's method to calculate I_{mp} in an iterative form. The subscript i denotes the ith iteration, and f and f' are denoted in (8) and its derivative in (9), respectively. Once I_{mp} is determined, V_{mp} and P_{mp} can be calculated as given in (10) and (11), respectively, as follows:

$$\begin{aligned} f(I_{mp_i}) = &\, I_{mp} + 2(I_{sc} + I_o - I_{mp})I_{mp}\frac{R_s}{V_t} \\ &+ (I_{mp} - I_{sc} - I_o)\left[\ln(I_{sc}+I_o-I_{mp}) - \ln(I_o)\right] \\ = &\, 0, \end{aligned} \tag{8}$$

$$\begin{aligned} f'(I_{mp_i}) = &\, 2 - 4I_{mp}\frac{R_s}{V_t} + 2(I_{sc} + I_o)\frac{R_s}{V_t} \\ &+ \left[\ln(I_{sc} + I_o - I_{mp}) - \ln(I_o)\right], \end{aligned} \tag{9}$$

$$V_{mp} = V_T \ln\left(\frac{I_{sc} - I_{mp}}{I_0} + 1\right) - I_{mp}R_s, \tag{10}$$

$$P_{mp} = I_{mp} \cdot U_{mp}. \tag{11}$$

2.2 Experimental

Testing of solar panel was done in the research laboratory of the Department of Computer Sciences and Electronics, North Dakota State University, Fargo, USA, during the month of March 2011. Experimental setup consisted of a solar simulator fitted with two light sources (metal halide HID lamp of 1 kW each) hung from top of the simulator roof, facing a horizontal platform which was modified to move up and down in the vertical direction (Figure 1) using a steel rope and pulley arrangement.

An amorphous silicon solar module capable of producing 15 W (maximum power output and used as battery charger) at standard test conditions was placed centrally on the movable platform in a horizontal position. Three variable resistances (rheostats) were used to apply load on the PV module for generating I–V characteristics and power conditioning curves at selected irradiance and
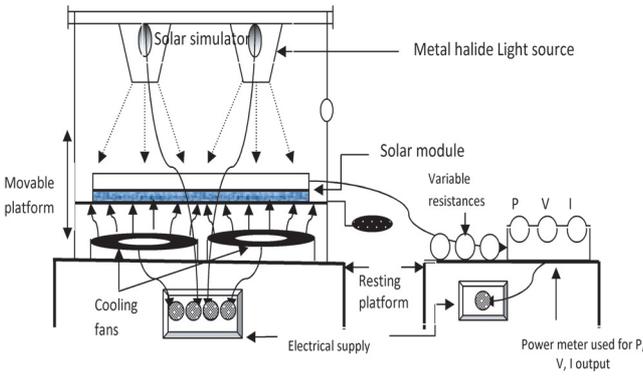
Figure 1: Experimental setup for testing of solar PV module using solar simulator.

Figure 2: I–V characteristics of solar PV module at a horizontal position measured at various solar radiation levels.

Figure 3: Power conditioning curves of solar PV module at a horizontal position measured at various solar radiation levels.

operation temperatures. In order to control the operation temperature of the PV module, two high-speed fans were fitted and operated from below the wired mesh platform on which the PV module was placed for selective cooling of the panel. The irradiance levels falling on the PV module were varied by altering the vertical distance between the light source and the movable platform on which the module was placed. The power, voltage, and current output were recorded at each load using WT 2030 model digital power meter. Cell temperature of the module (surface temperature beneath the glass cover) was recorded using a non-contact-type handheld infrared thermometer having an operating range of $-20\,°C$ to $320\,°C$. The thermometer was pointed toward the module from $30\,cm$ distance at three different locations on the module, and then the average was taken. Solar radiation was measured using a digital solarimeter (SL-100, KIMO, Italy).

3 Results and discussion

3.1 Solar irradiance dependence on the I–V characteristics and maximum power produced

The pronounced effect of solar radiation variation on the I–V characteristics and power conditioning curves of the amorphous silicon PV module is shown in Figures 2 and 3, respectively. The solar irradiance was varied as $400\,W/m^2$, $700\,W/m^2$, and $1,000\,W/m^2$. The measured values of I_{sc}, V_{oc}, I_{mp}, and V_{mp} at $1,000\,W/m^2$ irradiance level are $1.321\,A$, $21.14\,V$, $0.936\,A$, and $14.75\,V$, respectively. At $700\,W/m^2$ irradiance level, these values are $0.886\,A$, $21.28\,V$, $0.664\,A$, $14.57\,V$, respectively; and at $400\,W/m^2$ irradiance level, they are $0.593\,A$, $21.63\,V$, $0.433\,A$, and $15.239\,V$, respectively. The corresponding values of P_{mp} shown in Figure 3 are $13.806\,W_p$ at $1,000\,W/m^2$, $9.674\,W_p$ at $700\,W/m^2$ (29.92% less as compared to P_{mp} generated at $1,000\,W/m^2$), and $6.598\,W_p$ at $400\,W/m^2$ (31.79% further lesser as compared to P_{mp} generated at $700\,W/m^2$), respectively.
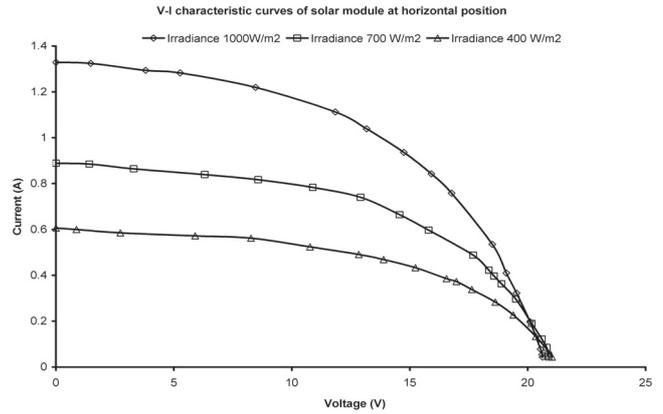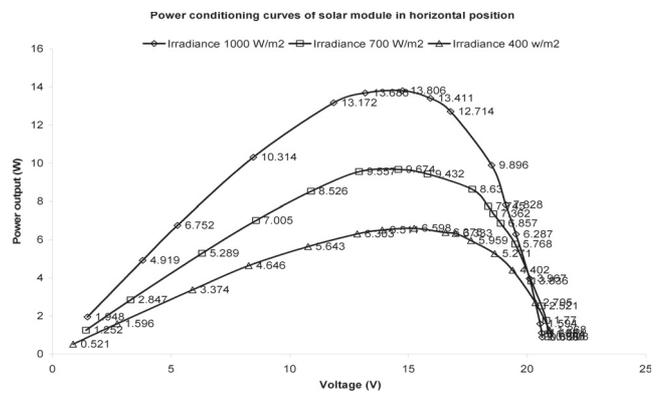
3.2 Operation temperature dependence on the I–V characteristics and maximum power output

It is known that the cell operation temperature has some bearing on the I–V characteristics and maximum power output (P_{mp}) of a solar cell. In order to exactly quantify the effect of temperature on P_{mp} at the same radiation level for an amorphous silicon solar PV module, simulations were performed by varying the module operating temperature from $40\,°C$ to $100\,°C$ to generate the I–V characteristic curves as shown in Figure 4 and the maximum power output (P_{mp}) as shown in Figure 5. It was observed that at $1,000\,W/m^2$ radiation level, P_{mp} was $12.531\,W_p$ at $60\,°C$ and $10.775\,W_p$ at $100\,°C$ (14% lower). At $700\,W/m^2$ radiation level, P_{mp} was $8.606\,W_p$ at $45\,°C$ and $7.761\,W_p$ at $75\,°C$ (9.8% lower). At $400\,W/m^2$ radiation level, P_{mp} was $6.068\,W_p$ at $40\,°C$ and $5.697\,W_p$ at $60\,°C$ (6.11% lower).

3.3 Validation of the proposed mathematical model

The measured and computed values of P_{mp} at various selected irradiance levels are shown in Figure 6, which
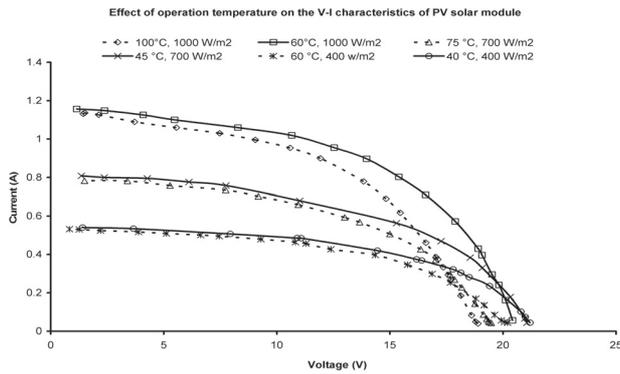
Figure 4: Effect of operation temperature on the I–V characteristics of PV solar module.
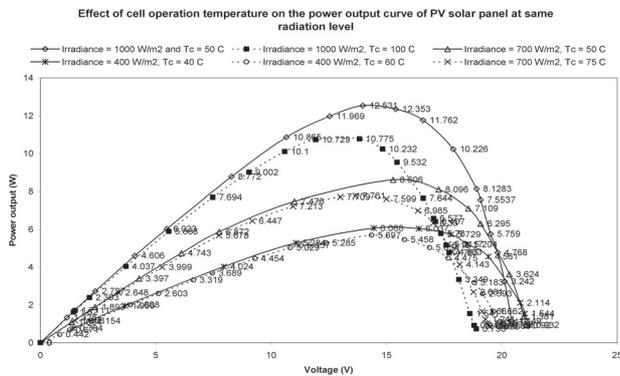
Figure 5: Effect of cell operation temperature on the power output of the PV solar panel at the same irradiance level.
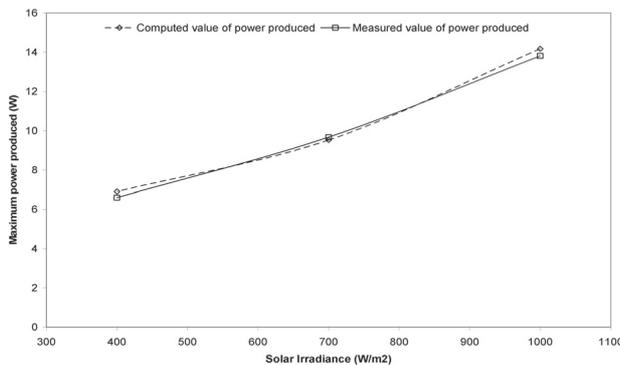
Figure 6: A comparison of computed and measured values of maximum power produced at various selected irradiance levels.

clearly indicate that there is a complete matching of the two values within less than 5% deviation, thus proving the accuracy of the developed model.

4 Conclusions

(a) The maximum power produced decreases by about 30% when solar irradiance decreases from 1,000 W/m^2 to

700 W/m^2 level and further lowers by about 31% when solar irradiance decreases from 700 W/m^2 to 400 W/m^2 level.

(b) The effect of operating temperature is also studied on the P_{mp}, which shows that at the same irradiance of 1,000 W/m^2, P_{mp} decreases by about 14% when the cell operation temperature is lowered from 100 °C to 50 °C.

(c) The developed model can accurately predict the performance of the PV solar panels.

Nomenclature

P_{mp} Maximum power produced [W$_p$]

V_{mp} Voltage at maximum power point [V]

I_{mp} Current at maximum power point [A]

V_{oc} Open-circuit voltage [V]

I_{sc} Short-circuit current [A]

T_c Panel operation temperature [°C]

I Current at any point on the I–V curve [A]

I_{il} Electric current generated by illumination [A]

I_o Diode current [A]

R_s Inner resistance [Ω]

V_T Thermal voltage [V]

m Diode factor [dim]

q Electron charge $= 1.602 \times 10^{-19}$ [C]

k Boltzmann constant $= 1.38 \times 10^{-23}$ [J K^{-1}]

References

[1] A. Bellini, S. Bifaretti, V. Lacovone, and C. Cornaro, *Simplified model of a photovoltaic module*, in International Conference on Applied Electronics, Pilsen, Czech Republic, 2009, 47–51.

[2] A. De Vos, *Thermodynamics of Solar Energy Conversion*, Wiley-VCH, Berlin, 2008.

[3] J. A. Gow and C. D. Manning, *Development of a photovoltaic array model for use in power-electronics simulation studies*, IEE Proceedings – Electric Power Applications, 146 (1999), 193–200.

[4] M. A. Green, *Third Generation Photovoltaics: Advanced Solar Energy Conversion*, Springer-Verlag, Berlin, 2003.

[5] T. Markvart, ed., *Solar Electricity*, John Wiley & Sons, Chichester, 2nd ed., 2000.

[6] E. I. Ortiz-Rivera and F. Z. Peng, *Analytical model for a photovoltaic module using the electrical characteristics provided by the manufacturer data sheet*, in 36th IEEE Power Electronics Specialists Conference (PESC '05), Recife, Brazil, 2005, 2087–2091.

[7] P. Singh, S. N. Singh, M. Lal, and M. Husain, *Temperature dependence of IV characteristics and performance parameters of silicon solar cells*, Solar Energy Materials and Solar Cells, 92 (2008), 1611–1616.

[8] E. Skoplaki and J. A. Palyvos, *On the temperature dependence of photovoltaic module electrical performance: A review of efficiency/power correlations*, Solar Energy, 83 (2009), 614–624.

[9] M. G. Villalva, J. R. Gazoli, and E. R. Filho, *Comprehensive approach to modeling and simulation of photovoltaic arrays*, IEEE Transactions on Power Electronics, 24 (2009), 1198–1208.

[10] A. Wagner, *Photovoltaic Engineering*, Springer-Verlag, Berlin, zweite ed., 2006.

[11] P. Würfel, *Physics of Solar Cells: From Basic Principles to Advanced Concepts*, Wiley-VCH, Berlin, 2nd ed., 2009.

[12] H. A. Zondag, *Flat-plate PV-Thermal collectors and systems: A review*, Renewable and Sustainable Energy Reviews, 12 (2008), 891–959.