Generation, analysis and evaluation of bi-phase complementary pairs

Ashish Kumar [1,2], Abhishek Shrivastava [2], Samaresh Bhattacharjee [1], Phani Kumar [1], Manish Naja [1]

[1] Aryabhatta Research Institute of Observational Sciences (ARIES),
Department of Science & Technology, Govt. of India,
Manora Peak, Nainital, Uttarkhand, India
[2] Nims Institute of Engineering and Technology,
Nims University, Jaipur, India

Corresponding Author Email: ashish@aries.res.in

Abstract

Bi-phase complementary codes are commonly used in radar application to achieve high resolution spatial and temporal measurements with suppressed range sidelobes. In this paper, a user friendly interactive graphical user interface (GUI) based tool capable of generating and analyzing a pair of complementary sequences of length upto 512 is presented. The effect of code length on key code parameters like merit factor, discrimination etc were evaluated and presented. In addition, ninety-six basic sets of 16-bit complementary pair searched through Monte-Carlo based search routine are tabulated. An empirical relation between code length and maximum possible set of basic complementary pairs is also established.

Keywords: *radar, complementary, code, bi-phase, Monte-Carlo, GUI, LabVIEW.*

1. Introduction

The concept of bi-phase coding in pulse radar system has a long history and becoming a favorable choice for designing a radar system to probe longer range with high resolution by utilizing maximum transmitted average power [1]. The basic bi-phase codes that are widely being used are: Barker [2] and complementary [3, 4] codes. Barker codes are mainly used in defence radar system while the bi-phase complementary codes are popular in atmospheric radar system. The complementary codes are often used in pairs to suppress the range sidelobes considerably in order to improve the detectability of weak signals like atmospheric echoes [5]. In pulse radar system, N-bit bi-phase coding is accomplished by dividing radio frequency (RF) transmit pulse in N sub-pulses of equal duration with 0° and 180° phase set according to bit value transition in a given code.

2. Complementary Sequences

Consider N-bit binary sequences represented as $A_T = \{a_1, a_2, a_3, ..., a_N\}$ and $B_T = \{b_1, b_2, b_3, ..., b_N\}$, where $a_i, b_i \in \{-1, 1\}$. The aperiodic autocorrelation function (AACF) with lag k of A_T and B_T are defined respectively as [6]:

$$R_A(k) = \begin{cases} \sum_{i=1}^{N-k} a_i a_{i+k} \ , 0 \le k < N \\ R_A(-k) \ , -N < k < 0 \end{cases} \quad (1)$$

and,

$$R_B(k) = \begin{cases} \sum_{i=1}^{N-k} b_i b_{i+k} \ , 0 \le k < N \\ R_B(-k) \ , -N < k < 0 \end{cases} \quad (2)$$

According to Golay [3, 4], the pair A_T and B_T is called complementary pair if:

(i) R_A and R_B has sidelobes equal in magnitude but opposite in sign,
(ii) $R_A(k) + R_B(k) = 0$ for $k \neq 0$, and
(iii) $R_A(k) + R_B(k) = 2N$ at k=0 i.e. a single peak of twice the original length at zero lag.

For clarity, consider an example of two 16-bit complementary sequences A_T ={1, 1, 1, -1, 1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1}and B_T = {1, 1, 1, -1, 1, 1,-1, 1, -1, -1, -1, 1, 1, 1, -1, 1}. Their AACFs are given as R_A = {-1, 0, -1, 0, -5, 0, 3, 0, 1, 0, 1, 0, 1, 0, 1, 16, 1, 0, 1, 0, 1, 0, 1, 0, 3, 0, -5, 0, -1, 0, -1} and R_B = {1, 0, 1, 0, 5, 0, -3, 0, -1, 0, -1, 0, -1, 0, -1, 16, -1, 0, -1, 0, -1, 0, -1, 0, -3, 0, 5, 0, 1, 0, 1} respectively. The result when R_A and R_B are summed is {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 32, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}. The example is illustrated in Fig. 1.

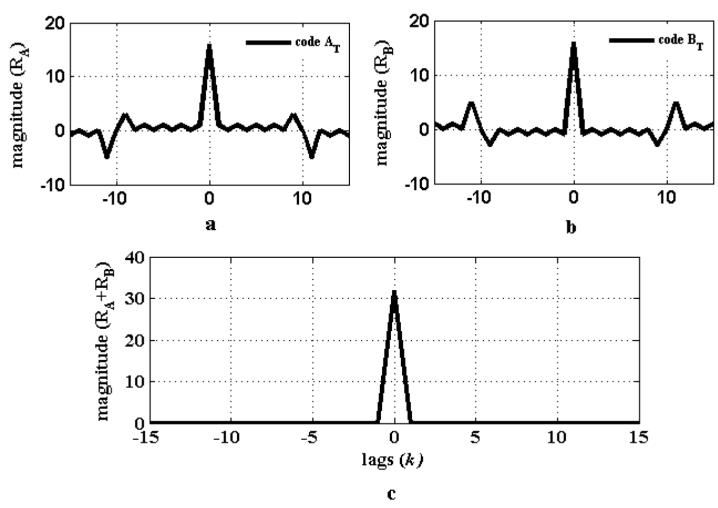

Fig. 1: AACF of sequences (a) A_T, (b) B_T and (c) $R_A + R_B$

2. Results and Discussion

From a given 2-bit complementary pair, 2^n bit complementary pair (where, n is an integer ≥ 2) can be generated based on the methods described in Golay [4] and Turyn [7].

In this context, a potential GUI tool developed in LabView environment for generation and analysis of 2^n bit complementary pair (where, $2 \leq n \leq 9$). The front panel of the tool in depicted in Fig. 2. The framework of the tool comprises of two sections namely complementary pair generation (CPG) and complementary pair analysis (CPA). The CPG section accepts 2-bit complementary pair as an input to generate a complementary pair of the specified code length. CPA section calculates AACF of each code of the generated pair and summed them thereafter. The performance parameters generally be required to investigate the quality of any binary sequence are - energy function, merit factor, discrimination, energy efficiency, quality factor, integrated sidelobe level (ISL) and peak sidelobe level (PSL). In radar application, merit factor and discrimination of a complementary pair should be as high as possible, whereas ISL and PSL of the same complementary pair should be as low as possible [8–9]. The CPA section of the tool also calculates all the mentioned seven performance parameters of the generated complementary pair to examine the code quality.

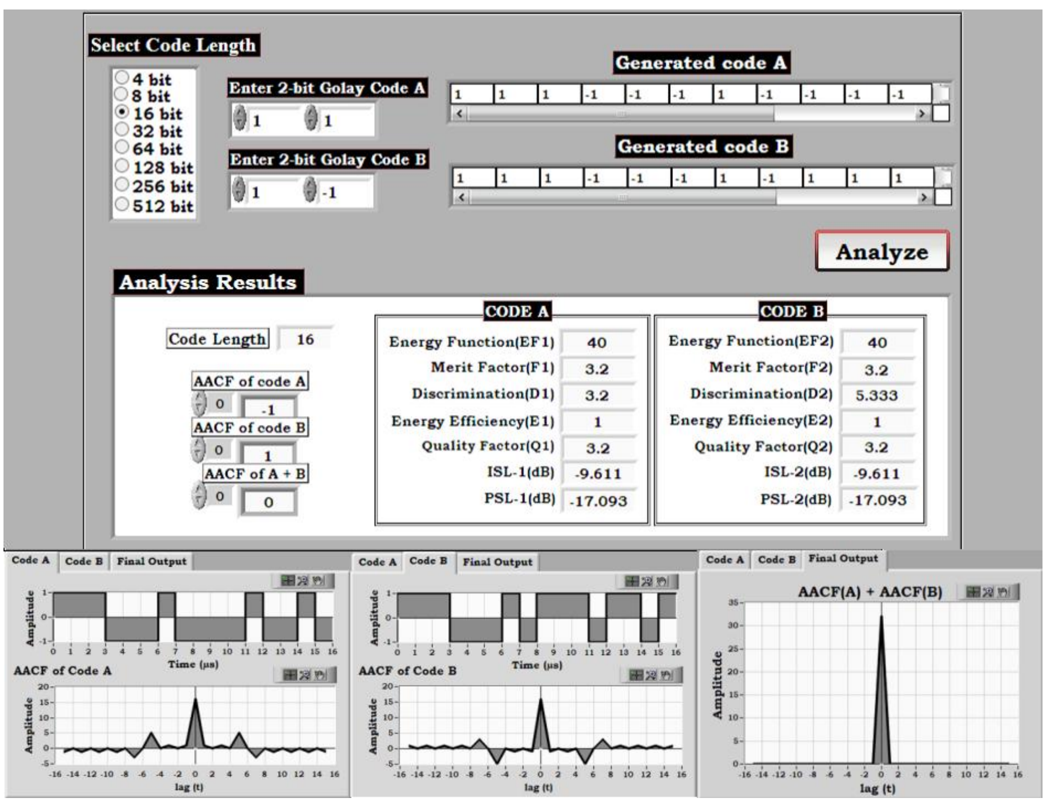

Fig. 2: GUI for complementary sequence generator and analysis

In the present work, out of the seven parameters, the analysis results obtained from the tool for four important parameters (merit factor, discrimination, ISL and PSL) upto code length 512 are plotted and presented in Fig. 3.

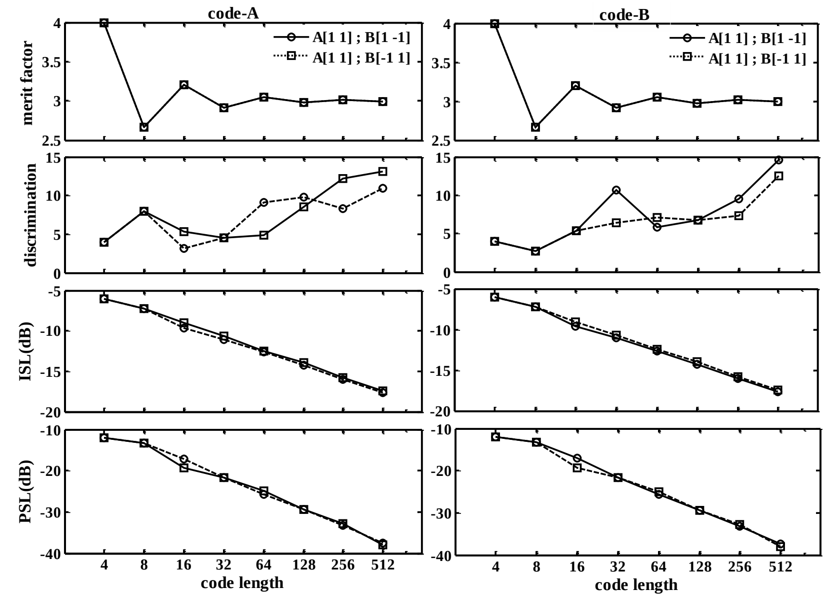

Fig. 3: Plots of performance parameters for code A and code B upto code length 512

It is clearly evident from Fig. 2 that all parameters are showing better performance with increase in code length except merit factor which settles around 3.2 at higher code lengths.

Based on the Monte Carlo technique [10], a search routine is also incorporated in the present tool to generate and find out the basic sets of possible complementary pair for a given code length N. All pairs strictly obey the properties mentioned in the preceding section. Ninety-six basic sets of 16-bit complementary pair with merit factor 3.2 are generated through the search routine after performing 10,000 iterations. The hexadecimal form of the obtained result is tabulated in Table 1. The PSL, ISL and discrimination (D_A and D_B) values of each complementary pair are also computed and presented in Table 1 as organized reference. The reported ninety-six basic sets can be used further for generating additional pairs of 16-bit code length by following the Golay properties [4].

Similarly, basic sets of complementary pair upto code length 128 are produced using the search routine. The maximum possible basic set of complementary pair (M_{BCP}) for the code length ($N = 2^n$ where, n is an integer) is given as:

$$M_{BCP} = \frac{N}{4}[(\log_2 N)!] \quad (3)$$

Table 1**:** Basic sets of 16-bit complementary pair (merit factor = 3.2)

code pair	code pair	code pair	performance parameters
[2E21, 2EDE] [A390, A36F] [90A3, 905C] [747B, 7484] [C5F6, C509] [21D1, 212E]	[6FA3, 6F5C] [8B84, 8B7B] [5C6F, 5C90] [D121, D1DE] [3AF6, 3A09] [DED1, DE2E]	[093A, 09C5] [F63A, F6C5] [848B, 8474] [7B8B, 7B74] – –	PSL= -19.3dB ISL= -9.03dB $D_A = D_B = 5.3$
[095C, 09A3] [A309, A3F6] [BEB1, BE4E] [EB27, EBD8] [7D8D, 7D72] [B1BE, B141] [5C09, 5CF6] [4E41, 4EBE] [DE74, DE8B] [AC06, ACF9] [48E2, 481D] [EB1B, EBE4] [7421, 74DE] [3560, 359F] [727D, 7282] [D7D8, D727] [9F35, 9FCA] [8B21, 8BDE] [6F3A, 6FC5] [903A, 90C5] [41B1, 414E] [BE8D, BE72]	[7241, 72BE] [1247, 12B8] [3A90, 3A6F] [C56F, C590] [418D, 4172] [7BD1, 7B2E] [53F9, 5306] [EDB8, ED47] [06AC, 0653] [B7E2, B71D] [D8D7, D828] [828D, 8272] [281B, 28E4] [B812, B8ED] [2714, 27EB] [824E, 82B1] [8D82, 8D7D] [14D8, 1427] [D814, D8EB] [2728, 27D7] [1BD7, 1B28] [28D8, 2827]	[D184, D17B] [842E, 84D1] [CA9F, CA60] [B17D, B182] [F6A3, F65C] [4E7D, 4E82] [D7E4, D71B] [6035, 60CA] [E4EB, E414] [E4D7, E428] [4712, 47ED] [1BEB, 1B14] [8D41, 8DBE] [1D48, 1DB7] [7D4E, 7DB1] [E2B7, E248] [2E84, 2E7B] [218B, 2174] [F9AC, F953] [141B, 14E4] – –	PSL= -19.3dB ISL= -8.1dB $D_A = D_B = 5.3$
[CAF9,CA06] [4847,48B8] [B7B8,B747] [F9CA,F935]	[3506,35F9] [AC60,AC9F] [0635,06CA] [9F53,9FAC]	[539F,5360] – – –	PSL= -17.1dB ISL= -9.6 dB $D_A = 3.2$ $D_B = 5.3$
[47B7,4748] [6053,60AC] [121D,12E2] [EDE2,ED1D]	[1D12,1DED] [B848,B8B7] [E2ED,E212] –	– – – –	PSL= -17.1dB ISL= -9.6 dB $D_A = 5.3$ $D_B = 3.2$

5. Conclusion

A potential GUI tool in LabView is developed for generation and analysis of 2^n bit complementary pair (where, $2 \leq n \leq 9$). The quality of code pairs are analyzed by calculating different performance parameters like merit factor, discrimination, ISL and PSL. From the analyzed result it is understood that parameters improves with code length at the expense of processing time. Further, ninety-six basic sets of 16-bit complementary code pair having merit factor 3.2 are generated through the Monte-Carlo based search routine. The reported sets can be used further for generating additional sets of 16-bit complementary pair for custom applications. In addition, the relation is established to calculate the maximum possible sets of basic complementary pair for a given code length.

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