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Empirical Performance Study of Alternative Option Pricing Models: An Application to the French Option Market

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Abstract

The mispricing of the deep-in-the money and deep-out-the-money generated by the Black and Scholes model is now well documented in the literature. In this paper, we discuss different option valuation models on the basis of empirical tests carry out on the French option market. We examine methods that account for non-normal skewness and kurtosis, relax the martingale restriction, mix two log-normal distributions, and allows either for jump diffusion process or for stochastic volatility. We find that the use of a jump diffusion and stochastic volatility model performs as well as the inclusion of non normal skewness and kurtosis in terms of precision in the option valuation.

Keywords: Implied Volatility; Stochastic Volatility Model; Jump Diffusion Model; Skewness; Kurtosis

Introduction

The failure of the Black and Scholes [1] model to provide correct valuation is attributable to the parsimonious assumptions used to derive the model. One of the strong assumptions of the model is that security prices follow a constant variance diffusion process with log-returns normally distributed whereas the constant variance assumption has always been rejected by many studies.

The ARCH models literature set up by Engle [2] and Bollerslev [3] has been devoted to the volatile behavior of stock return variances. This empirical literature has proved the stochastic nature of stock return variances and their correlation with security price levels shoing that returns are both skewed and leptokurtic. The systematic deviations from the observed prices correspond to a "gap" in the option valuation as noticed Black (1975) when stating that "one possible explanation for this pattern is that we have left something out of the formula".

As Rubinstein [4] notice, the Black and Scholes [1] model tends to systematically misprice in-the-money and out-the-money options. Different studies lead to different conclusions knowing whether this model underprices deep-out-of-the-money and overprices deep-in-of-the-money options or the contrary. Nevertheless, Rubinstein [4] concluded that the bias direction can change across different periods. The biases are not in the same direction for all markets and they are not constant over time. Observed moneyness biases constitute evidences against the hypothesis that asset returns are homoskedastic and normally distributed; this gave an impulse to the development of option pricing models for alternative processes.

The Constant Elasticity of Variance option pricing model of Cox and Ross [5] relax the constant volatility hypothesis and enabled the instantaneous conditional volatility of asset returns to depend deterministically upon the level of asset price. The implied binomial tree models of Dupire (1994), Derman and Kani (1994), and Rubinstein (1994) can be considered as flexible generalizations of the CEV model while the stochastic volatility option pricing models of Scott [6], Wiggins [7], Johnson and Shanno (1987) use numerical methods as the Monte Carlo simulations to price options when the variance varies.

Hull and White [8] solve explicitly for the options price by using Taylor expansion. But to produce analytical results they imposed the correlation between increments to be null. They solve the case for non zero correlation through Monte Carlo simulations. Hull and White [9] derive a model in series expansion form allowing for instantaneous correlation between stochastic volatility and the stock price.

Stein and Stein [10] derive a closed-form solution when volatility is driven by an Ornstein-Uhlenbeck (AR1) process but with zero correlation between both increments. Heston [11] proposes a closed-form solution, but with arbitrary correlation between volatility and spot returns.

Jarrow and Rudd [12] approximate the log-normal probability distribution by an arbitrary distribution in terms of a series expansion. The idea is to derive an option pricing model expressed as the sum of the [1] formula plus adjustment terms permitting to capture the impact on the option price of the third and fourth moments of the underlying security stochastic process. In the same manner, Corrado and Su [13] derive and test empirically a European option pricing model that extends the [1] model to take into account for non-normal skewness and kurtosis in the distribution of stock returns.

The skewness and kurtosis coefficients are estimated simultaneously along with the implied standard deviation. They find that the adjustments for skewness and kurtosis are effective in removing systematic strike price biases from Black-Scholes [1] model for S&P500.

The recognition that asset returns are leptokurtic, especially for short periods, incited [14] to look for option pricing through jump-diffusion processes. Ball and Torous [15,16] propose a simplified version of the jump-diffusion model known as the as the Bernoulli Jump process. Later on, Maltz [17] fitted this model to market option price data in order to estimate the ex-ante probability distribution of exchange rates. Bates [18] fit a jump diffusion option pricing model to option data extracting implicit volatility, skewness and kurtosis.

Duan [19] develops a GARCH option pricing model capable of reflecting the changes in the conditional volatility using numerical simulation. Ritchken and Trevor [20] developed also an option pricing

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model using a lattice algorithm to price both American and European options under discrete time GARCH processes. We recall that [21] showed that the GARCH model can be written as an approximation to certain diffusion equations which has been postulated in the option pricing literature. Duan, Gautier and Simonato [22] propose a series approximation to value American options for GARCH processes with one lag in the variance dynamics. Heston and Nandi [23] develop a closed-form option valuation formula for an asset whose variance follows a NGARCH process.

In this paper, we adopt Whaley's [24] simultaneous equations procedure to estimate the implied parameters serving for the calculation of the option prices. This paper is organized as follows. Section 2 presents the models that are implemented. Section 3 gives details about sampling methodology. Section 4 displays the empirical results. Section 5 assesses the statistical performance of the models. Section 6 summarizes and concludes.

Option Pricing Models

We compare the out-of-sample performances of different option pricing models using the Black-Scholes [1] model as a benchmark.

Skewness and kurtosis adjusted model

The expansion methods are now largely used in quantitative finance. The idea is to start with an expansion formula for the risk neutral density considered as a general probability distribution. The first term of the expansion corresponds either to log-normal or normal distribution. The following terms can be therefore considered as successive corrections to the log-normal or normal approximations. The series is truncated at a finite order fixed empirically, which gives a parametric approximation of the risk neutral distribution. From this expression, we can derive an option pricing formula. The only drawback of this method is that when the infinite sum in the expansion represents a probability distribution, finite order approximations of it may become negative, leading to negative probabilities far enough in the tails, which generates severe mispricing for options that are far enough from the money.

We follow the approach of Corrado and Su [13] in the use of a semi-parametric option pricing formula. They use the Gram-Charlier series expansion to model the distribution of stock log-prices. This method focuses on the skewness and kurtosis deviations from normality. Stuart and Ord (1987) discussed the distinction between Edgeworth expansion and Gram-Charlier expansion. To obtain an option pricing formula that correct the bias of the Black-Scholes [1] model, they add to this formula, adjustment terms accounting for non normal skewness and kurtosis. For a density function $f(x)$, the Gram-Charlier series expansions are defined as:

$$f(x) = \sum_{n=0}^{\infty} c_n H_n \varphi(x)$$

$\varphi(x)$ is a normal density function and $H(x)$ are Hermite polynomials derived from successively higher derivatives of $\varphi(x)$ while the coefficient Cn are determined by moments of the distribution function $F(x)$. The Gram-Charlier series, which are infinite series, are here truncated to eliminate terms after the fourth moment. This turns out to be a good proxy for option pricing due to the consideration of non-normal skewness and kurtosis. A truncated series that take into account skewness and kurtosis gives the following density function with centered mean and reduced variance for the following density:

$$g(z) = n(z)\left[1 - \frac{\mu_3}{3!}\left(z^3 - 3z\right) + \frac{\mu_4 - 3}{4!}\left(z^4 - 6z^2 + 3\right)\right]$$

with

$$n(z) = \frac{1}{\sqrt{2\pi}}\exp\left(-z^2/2\right)$$

Under risk-neutral measure, one can apply the density function $g(z)$ to derive a formula for a European call as being the present value of an expected payoff at expiration. This call price is derived from the following expression:

$$C_{GC} = \exp[-r\tau]\int_k^{\infty}\left(S_t - K\right)\, g\left(z(S_t)\right)\, dz(S_t)$$

where

$$z(S_t) = \frac{(\ln S_t - \mu)}{\sigma\sqrt{\tau}} \text{ and } \mu = \ln S_0 + (r - \frac{\sigma^2}{2})\tau$$

We obtain an option pricing call formula based on the Gram-Charlier series as being:

$$C_{GC} = C_{BS} + \mu_3 Q_3 + (\mu_4 - 3)Q_4$$

where

$$Q_3 = \frac{1}{3!}S_0\sigma\sqrt{\tau}\left((2\sigma\sqrt{\tau} - d)n(d) + \sigma^2\tau\, N(d)\right)$$

$$Q_4 = \frac{1}{4!}S_0\sigma\sqrt{\tau}\left((d^2 - 1 - 3\sigma\sqrt{\tau}(d - \sigma\sqrt{\tau}))n(d) + \sigma^3\tau^{3/2}N(d)\right)$$

with

$$d = \frac{\ln(S_0/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}$$

Implicit stock price adjusted model

The principle of the no-arbitrage approach is that the price of an option is obtained by taking the expectation of its discounting payoffs relative to a risk-neutral density. Arbitrage is not feasible in the condition that the risk-neutral density mean satisfy the martingale restriction. This restriction states that the price of the underlying asset implied by the option price must be the same that its observed market price, otherwise its should means the existence of arbitrage opportunities within a market with no frictions as shown by Harrisson and Kreps [25].

In the situation of frictions in the market, the option prices are determined by equilibrium and not by no-arbitrage reasoning, thus, the martingale restriction need not to be met. Among the frictions that can be encountered in the market, such as transaction costs or illiquidity, we can add information costs. The idea of estimating an implicit stock price is not recent. Manaster and Rendleman [26] is the first to invert [1] formula to estimate daily implied stock prices and daily implied volatility parameters. They concentrate their work around the idea of forecasting returns using the implied stock price.

They find that the implied stock price exceeds the observed price and affirmed that implied prices contain information regarding equilibrium stock prices that is not totally reflected in observed stock prices. One other reason to explain the difference between both stock prices was that transactions between both markets were not synchronous and thus, the difference represents more a recent rather than better information.

However, if they find that the second hypothesis explains a large part theses differences, "it is still possible that a significant fraction of implied prices represents more recent information than observed stock prices". Later on, Longstaff [27] carries out a similar test on what he called "the martingale restriction". He uses S&P100 index options data from 1988 to 1989, with American call options for which the American exercise effect should be alleviated. He focuses on a five minute window from 2:00 PM to 2:05 PM using index values during this window.

He find that the implied S&P100 index cost is superior to the actual index cost for 442 cases out of 444 daily estimates by an amount of 1.004. He explaines that since an option can be seen as a levered position in the underlying asset, which means that purchasing a stock through the option market costs more than on the spot market. We intuitively understand that accessing to a derivative market implies additive costs such as transaction or information costs.

Comparing the Black-Scholes [1] model with an equilibrium version of the model in which the martingale restriction is relaxed, he obtaines that, more than a half of the pricing errors occurring with the [1] model, is eliminated. We note that Patilea, Ravoteur and Renault (1995) propose an econometric approach based on the concept of implicit stock prices applied to the Hull and White [8] model and Renault [28] explains that introducing an implicit stock price may explain the skewness observed in the smile.

In this paper, we don't test the martingale restriction following the very same methodology of Longstaff [27], we just want to check whether relaxing the martingale restriction, i.e, estimating an implicit stock price, will improve in our context the option pricing. We don't limit ourselves to a small time period window since we don't have enough observations per window.

Mixture of lognormal distributions

Melick and Thomas [29] estimate option pricing as a mixture of log-normal densities. They consider that the risk-neutral density can be adjusted correctly as a mixture of various log-normal densitites, representing different views of future reality. This method integers a number M of log-normal distributions but we limit our case to the simple case M=2. We suppose the risk-neutral density being equal to:

$$h^Q(S_t) = \sum_{i=1}^{M} \alpha_i LN\left(S_t; m_i, \sigma_i\sqrt{t}\right)$$

with

$$\sum_{i=1}^{M} \alpha_i = 1 \text{ and } m_i = \ln\left(S_0\right) + \left(\mu_i - \sigma_i^2/2\right)\, t$$

and $LN(S_t;m^Q,\sigma^2 t)$ is the log-normal density function under risk-neutral measure. Within this configuration, the investors can imagine a situation for the future where, for instance, two configurations are expected, for which they affect two probabilities α_1 and α_2 and where the asset growth rate will be subject to two drift dynamics (μ_1 and μ_2) and two volatilities (σ_1 and σ_2). The call price of a mixture of log-normal densities can be written as:

$$C_{2LN}\left(K,\tau,\alpha_i,m_i,\sigma_i\right) = e^{-rT}\sum_{i=1}^{M}\alpha_i \left[\begin{array}{l}\exp\left(m_i+\frac{1}{2}\sigma_i^2\tau\right) \\ \left[1-N\left(\frac{\ln(K)-m_i-\sigma_i^2\tau}{\sigma_i\sqrt{\tau}}\right)\right]-K\left[1-N\left(\frac{\ln(K)-m_i}{\sigma_i\sqrt{\tau}}\right)\right]\end{array}\right]$$

Under the risk-neutral probability, we impose the following constraint:

$$\sum_{i=1}^{M}\alpha_i \exp\left(m_i+\frac{1}{2}\sigma_i^2\tau\right) = S_0\, e^{r\tau}$$

The advantage of the procedure is that the option prices are given as an average of the Black-Scholes [1] prices for different volatilities weighted by the respective weights of each distribution in the mixture. By construction, this mixture of log-normals has thin tails unless one allows high values of variance. As recall Campa, Chang and Reider [30], probability in the tails declines monotonically and always decays quickly enough for preventing unrealistic kurtosis. The only drawback of this procedure is that it is not supported by any theoretical economic background, which could lead to a lack of economic meaning.

Stochastic volatility model

Hull and White [9] consider the following stochastic processes for the returns and for the volatility through the risk-neutral data generating process given by:

$$\frac{dS}{S} = rdt + \sqrt{V}\, d\tilde{Z}_S$$

$$dV = (a+bV)dt + \xi\sqrt{V}\, d\tilde{Z}_V$$

where a, b, and ξ are constant and $d\tilde{Z}_S$ and $d\tilde{Z}_V$ are Wiener processes under risk-neutral probability and r is the instantaneous interest rate supposed constant. V is the asset's instantaneous variance rate. In order to ensure that the drift rate of V will not be negative, it is required that $a \geq 0$. From the dynamic of the variance, we can obtain a constant drift with b=0, a constant proportional drift with a=0 or a mean reverting process with a>0 and b=0 from which V will tend to revert to a long-run level $-a/b$ with a mean reversion rate $-b$. The time required for the expected deviation to be halved, the half-life is given by $-\ln 2/b$.

Hull and White [9] set up a power series expansion procedure based on the security price distribution conditional on the average value of the stochastic variance. This technique remains one of the most tractable one since if one compare it to the analytical approach proposed by Heston [11] based on Fourier inversion method. As in Corrado and Su [13], we set in the series expansion, the variance being equal to its long run reversion value since it is a credible hypothesis; however, running the model as it appears in the Hull and White [9] article is not more difficult, it simply requires an additional parameter to be estimated. Under this expansion, they showed how to correct the bias in the [1] model using a precise approximation from a second order Taylor series expansion:

$$\text{Bias correction} = Q1\rho\xi + \left(Q2 + Q3\rho^2\right)\xi^2 + o\left(\xi^2\right)$$

The terms of the series expansions are defined as:

$$Q1 = -\frac{1}{b^2\tau}V\left(1+b\tau-e^{b\tau}\right) S\frac{\partial C}{\partial S\partial V}$$

$$Q2 = \frac{1}{4b^3\tau^2}V\left(3+2b\tau+e^{2b\tau}-e^{b\tau}\right)\frac{\partial^2 C}{\partial V^2}$$

$$Q3 = -\frac{1}{b^3\tau}V\left[e^{b\tau}(2-b\tau)-(2+b\tau)\right] S\frac{\partial^2 C}{\partial S\partial V} - \frac{2}{b^3\tau^2}V\left[e^{b\tau}(2-b\tau)-(2+b\tau)\right]\frac{\partial^2 C}{\partial V^2}$$

$$+\frac{1}{2b^4\tau^2}V^2\left(1+b\tau-e^{b\tau}\right)^2 S\frac{\partial^3 C}{\partial S\partial V^2} + \frac{1}{b^4\tau^3}V^2\left(1+b\tau-e^{b\tau}\right)^2\frac{\partial^3 C}{\partial V^3}$$

We obtain a stochastic-adjusted call price when adding the bias correction to the Black and Scholes [1] call price, which yields:

$$C_{HW} = C_{BS}(V) + Q1\rho\xi + Q2\rho\xi^2 + Q3\rho^2\xi^2$$

If we assume a constant process variance, i.e, $\xi = 0$, we collapse in the Black and Scholes [1] formula since the pricing bias is null.

Jump diffusion model

The research of a distribution that fit the best the behavior of stock returns still continue to be a dominant issue in finance. After the

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introduction of arithmetic and geometric Brownian motions, much attention was devoted to Poisson distributions as a valid specification of stock returns. In addition to empirical evidence, these models succeeded in capturing the "abnormal" components of the total change in stock price as recalled Ball and Torous [15], while the "normal" component, considered as marginal changes, is captured by a standard log-normal diffusion process.

We know that large values of returns occur too often to be consistent with normality and also, positive and negative returns of a given size are not equally likely. We observe kurtosis if jumps in either direction are equally likely and we observe in addition skewness, if jumps in one direction are larger or more frequent. Both skewness and kurtosis are captured by the Poisson distribution as noticed Jorion [31]. However, as recalled Renault [28], the expansion of these models are limited to the fact that there are few cases where closed-form solutions are given, specifically when there is a non zero probability of early exercise, or when the distribution of jumps is neither lognormal nor discrete. We follow Maltz [17] for pricing options with Jump Diffusion Model. In this section we assume that follows a log-normal jump diffusion, i.e., the addition of a geometric Brownian motion and a Poisson jump process. This price process under the risk-neutral probability can be shown to be:

$$\frac{dS}{S} = \left(r - \lambda E(k)\right) dt + \sigma d\tilde{Z}_S + k dq_\tau$$

with q_τ a Poisson counter with average rate of jump occurrence λ (prob(dq = 1) = λdt) and k the jump size. [15-17] suppose as a simplification that during the life of the option there will occur at most one jump of constant size. If no events occur in the option life, the associated probability is ($1-\lambda\tau$) and will be $\lambda\tau$ if one event occurs during this time interval. When such event occurs, there is an instantaneous jump in the stock price. This simplified version is called by Ball and Torous [15,16] as the Bernoulli distribution version of the jump-diffusion model:

$$C_{JUMP} = (1-\lambda\tau)\left[\frac{S_t}{1+\lambda k\tau} N\left(d_1 + \sigma\sqrt{\tau}\right) - K\exp(-r\tau) N\left(d_1\right)\right] + \lambda\tau\left[\frac{S_t}{1+\lambda k\tau}(1+k) N\left(d_2 + \sigma\sqrt{\tau}\right) - K\exp(-r\tau) N\left(d_2\right)\right]$$

where:

$$d_1 = \frac{\ln\left(S_t/K\right) - \ln(1+\lambda k\tau) + \left(r - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

and:

$$d_2 = \frac{\ln\left(S_t/K\right) - \ln(1+\lambda k\tau) + \ln(1+k) + \left(r - \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}$$

This formula corresponds to the Black and Scholes [1] call option value weighted by the probability of a jump and by the probability of no jump with the stock price divided by the expected value of a jump ($1-\lambda k\tau$).

Data Description and Sampling Methodology

The description of the data

Option prices are extracted from SBF-BOURSE DE PARIS. The data set contains the PXL option quotations, the strike prices, the maturity and the CAC 40 quotations during October 1998. We use intra-day observations for all quotations. The PXL options are European style options that are written on the CAC 40 Index. Our study is based on call options. Their maturity lasts 6 month. The interest rate is taken from DATASTREAM. We chose the PIBOR 6 months as the risk-free interest rate. For the dividend yield, we use ex-post dividend rate downloaded from DATASTREAM.

The sampling methodology

We have in total 1402 intra-daily observations that represent option premium and all the features linked to these options, such as, the strike price, the underlying index price, the maturity, the minimum number of purchasable options, the risk-free interest rate. The maturity chosen is March 1999 and the minimum number of purchasable options is 50. The PIBOR interest rate is a daily rate.

To each option quote, correspond two intra-minutes CAC 40 quotes, since the index is quoted every 30 seconds, which means that there is a quotation for the first 30 seconds of a minute and another quotation for the last 30 seconds. We retain only the quotation pertaining to the first 30 seconds of the minute as underlying quote. We delete each quotation resembling exactly to the following.

There are 600 out-of-the-money calls and 28 in-the-money calls, i.e. 628 calls. There are 554 out-of-the-money puts and 220 in-the-money puts, i.e. 774 puts. For our study, we consider the 600 out-of-the-money calls and the 554 out-of-the-money puts due to the fact they are more traded; they represent 1154 observations. We transform the out-of-the-money puts into theoretical observed in-the-money calls. Using the Black and Scholes [1] formula as a function, we extract the volatility by equating the function with the corresponding premium. Using this extracted volatility and the other corresponding parameters of the out-of-the-money puts, we re-compute through the Black and Scholes [1] formula for a call, a theoretical price of a supposed in-the-money call.

Empirical Tests

We don't display the results showing the out-of-sample performance of the Black and Scholes [1] model as a benchmark. We first estimate daily implied standard deviation (ISD) of call options written on the CAC 40 index from intra-day data sample. We calculate a prior-day ISD and we use it as an input to compute the current-day option price. Theoretical Black and Scholes [1] prices based on an out-of-sample ISD are then compared to their corresponding observed prices. The next models follow the same procedure.

First estimation procedure

We compute the Skewness and Kurtosis-adjusted Black and Scholes [1] option price model. We estimate the *ISD*, the option implicit skewness *SK* and the option implicit kurtosis *KU*, through the Simultaneous Equations Procedure, which gives us implied values of our parameters by minimizing the following sum of squares:

$$\min_{ISD,SK,KU} \sum_{j=1}^{N} [C_{OBS\ j} - C_{GC\ j}(ISD, SK, KU)]^2$$

$C_{GC}(ISD,SK,KU)$ is the call price of the proposed model computed with the *ISD* parameter and which includes the implied third and four moments *SK, KU*. This call price is calculated for any option in a given current day's sample.

Column 7 gives the details about the results obtained by the comparison between the skewness and kurtosis-adjusted [1] model and the corresponding observed prices in the market. The average deviation spread in is computed as follows:

$$\frac{C_{OBS} - C_{ISD,SK,KU}}{C_{OBS}} \times 100$$

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Table 1 gives the results obtained from the first estimation procedure.

We notice that the average implied volatility is about 42.86% with a spike observed the second day reaching 49.41%. The average percentage obtained for the option implied skewness is -1.22. The average implied kurtosis is 3.05. We see in column six that in average, 71.2% of the theoretical prices are outside the ± 1% spread applied in the observed prices. Using the Black and Scholes [1] model, we obtain for the calls an average of 97.1%, which signifies a gain of about 26 %. The average deviation is around 16.22 FF (column 8) for an average call price of 451.16 FF (column 9). The average spread between the observed price and the corresponding theoretical price is about 7.68%, which indicates that the mispricing generally observed in the different models is significantly reduced with this model. Figure 1 shows the effects of allowing non-normal skewness and kurtosis in the theoretical ISD Black and Scholes [1] formula on October, 2nd 1998. We recall that positive moneyness signifies out-of-the-money options and negative moneyness implies in-the-money options. We see that this model reduces sharply the price deviation or bias effect observed in figure 1. This constitutes a clear improvement in terms of precision in the option valuation.

Second estimation procedure

We compute the implicit stock price in addition to the implied volatility from the Black and Scholes [1] model. We estimate the ISD, the option implied volatility and *IS*, the implicit underlying asset price, through the Simultaneous Equations Procedure, which gives us implied values of our parameters by minimizing the following sum of squares:

$$\min_{ISD,IS} \sum_{j=1}^{N} [C_{OBS\ j} - C_{M\ j}(ISD, IS)]^2$$

$C_M(ISD,IS)$ is the call price of the second model computed with the ISD parameter and IS, the implied stock price. This call price is calculated for any option in a given current day's sample (Table 2).

We notice that the average implied volatility is about 33.03% with a spike observed the second day reaching 38.16%. The average percentage obtained for the option implied stock price is 3339.77. We see in column five that in average, 93.1% of the theoretical prices are outside the ± 1% spread applied in the observed prices. Three of them are completely outside, which indicates a very slight improvement, but that is not significant relative to our benchmark, i.e., the Black and Scholes [1] model. The average deviation is around 43.09 FF in column 7 for a average call price of 451.16 FF. The average spread between the observed price and the corresponding theoretical price is about -18.56%, which indicates that the mispricing generally observed in the benchmark is reduced by two. Figure 2 shows the effects of estimating implicitly the stock price in the theoretical Black and Scholes [1]

Trading date	Number of data	ISD (%)	Option Implied Skewness ISK	Option Implied Kurtosis IKU	Proportion of Theoretical Prices Different from the Observed Prices at ± 1 %	Average Spread between Observed and Theoretical Price (%)	Average Deviation of Theoretical Price from Observed Prices (FF)	Average Call Price (FF)
02/10/98	153	49.416	-1.083	3.151	0.810	28.702	20.378	419.250
05/10/98	69	47.436	-1.062	2.831	0.696	1.909	17.839	528.873
06/10/98	53	45.694	-1.025	2.815	0.698	2.564	16.868	506.300
07/10/98	49	45.580	-1.195	3.712	0.673	19.961	20.444	406.673
08/10/98	56	46.754	-1.027	2.821	0.804	31.224	19.376	441.110
09/10/98	41	48.173	-1.447	4.203	0.659	-7.466	19.913	338.436
12/10/98	68	43.203	-1.196	2.883	0.985	-3.986	23.798	500.436
13/10/98	55	42.325	-1.253	2.894	0.400	1.082	18.696	626.313
14/10/98	86	42.859	-1.366	3.789	0.651	0.435	12.718	345.208
15/10/98	59	40.170	-1.195	2.907	0.864	11.065	16.228	378.210
16/10/98	60	39.765	-1.371	2.903	0.700	-5.325	14.172	385.422
19/10/98	32	39.941	-1.366	2.852	0.719	3.327	9.551	395.641
20/10/98	88	39.165	-1.289	2.916	0.580	7.274	11.190	526.652
21/10/98	57	40.007	-1.299	2.885	0.667	6.378	11.160	564.885
22/10/98	47	39.291	-1.330	2.893	0.532	-1.952	10.054	566.498
23/10/98	31	40.482	-1.269	2.827	0.903	24.462	15.179	391.574
26/10/98	25	38.303	-1.148	2.817	0.760	11.033	18.150	341.497
Average	64.111	42.857	-1.222	3.054	0.712	7.688	16.219	451.166

All the observed prices correspond to call options traded in October 1998 in the MONEP.

Table 1: Comparison of non normal skewness and kurtosis model prices and observed prices of PXL call options.

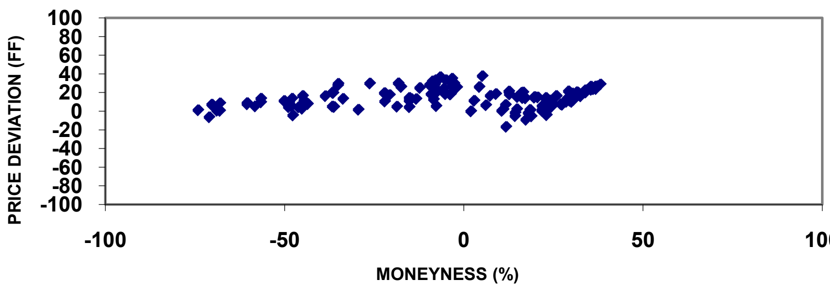

Figure 1: Adjusted skewness and kurtosis model - October, 2nd 1998.

Citation: Aboura S (2013) Empirical Performance Study of Alternative Option Pricing Models: An Application to the French Option Market. J Stock Forex Trad 2: 108. doi:10.4172/2168-9458.1000108

formula on October, 2nd 1998. We globally see that options around the money are generally all over-estimated. The estimation failed to provide an improvement in terms of price deviation. 16 out of 18 daily implicit stock prices are superior to the observed stock prices and the average difference between them is about 2.27% with a median value at 2.54% and about 2.50% when compared to the average of observed values in the day.

Third estimation procedure

In this procedure, we compute an option price as a mixture of two log-normal distributions. We estimate two implied volatilities and m_1, m_2, α_1, α_2 through the Simultaneous Equations Procedure, which gives us implied values of the parameters. The Simultaneous Equations Procedure minimizes the following sum of squares:

$$\min_{ISD,m,\alpha} \sum_{j=1}^{N} [C_{OBS\ j} - C_{2LN\ j}(ISD_1, ISD_2, m_1, m_2, \alpha_1, \alpha_2)]^2$$

$C_{2LN}(ISD_1, ISD_2, m_1, m_2, \alpha_1, \alpha_2)$ is the call price of the proposed model computed for any option in a given current day's sample (Table 3).

We notice that the average implied volatility is about 50.50% for ISD_1 and three times less for ISD_2 with 16%. We see in column seven that in average, 91.7% of the theoretical prices are outside the ±1% spread applied to the observed prices. Four of them are completely outside, which indicates a slight improvement, but that is not significant relative to our benchmark.

The average deviation is around 37.81 FF in column 9 for an average call price of 451.16 FF. The average spread between the observed prices and the corresponding theoretical prices is about -8.66%, this is an improvement in pricing compared to the benchmark (Figure 3) shows the effects of considering a mixture of two log-normal distributions on October, 2nd 1998.

This form of estimation failed to provide an improvement in terms of price deviation and we observe that the prices generated by this mixture are globally the same than those generated by the implicit stock price model.

Fourth estimation procedure

We estimate implicitly the parameters of the Hull and White [9] model. We estimate the correlation parameter ρ, the volatility of volatility parameter ξ and the coefficient of mean reversion b and the ISD through the Simultaneous Equations Procedure, which gives us implied values of our parameters by minimizing the following sum of squares:

$$\min_{ISD,\rho,\xi,b} \sum_{j=1}^{N} [C_{OBS\ j} - C_{HW\ j}(ISD, \rho, \xi, b)]^2$$

$C_{HW}(\rho, \xi, b)$ is the call price of the Hull and White [9] model. This call price is calculated for any option in a given current day's sample (Table 4).

Trading date	Number of data	ISD (%)	Option Implied Stock Price IS	Proportion of Theoretical Prices Different from the Observed Prices at ± 1 %	Average Spread between Observed and Theoretical Price (%)	Average Deviation of Theoretical Price from Observed Prices (FF)	Average Call Price (FF)
02/10/98	153	38.158	3036.741	1.000	-60.031	76.532	419.250
05/10/98	69	37.506	3097.794	0.942	-19.283	45.895	528.873
06/10/98	53	37.433	3150.394	0.830	-16.927	50.194	506.300
07/10/98	49	34.885	3203.877	0.959	-44.953	33.987	406.673
08/10/98	56	37.989	3045.192	1.000	-68.087	79.396	441.110
09/10/98	41	30.986	3213.468	0.976	-11.808	54.859	338.436
12/10/98	68	32.368	3316.699	1.000	14.847	72.750	500.436
13/10/98	55	33.166	3339.479	0.836	-1.775	39.135	626.313
14/10/98	86	28.806	3422.876	0.919	-8.973	32.703	345.208
15/10/98	59	30.550	3457.335	0.847	-6.200	33.062	378.210
16/10/98	60	29.257	3496.891	0.900	-2.293	24.723	385.422
19/10/98	32	29.516	3492.022	0.938	-9.126	22.979	395.641
20/10/98	88	32.620	3477.614	0.898	-3.893	32.953	526.652
21/10/98	57	32.520	3492.155	0.947	-12.402	31.770	564.885
22/10/98	47	33.129	3503.112	0.915	-8.276	36.129	566.498
23/10/98	31	28.902	3500.633	0.968	-49.638	36.298	391.574
26/10/98	25	30.023	3529.909	0.960	-6.669	29.141	341.497
Average	64.111	33.035	3339.776	0.931	-18.558	43.089	451.166

All the observed prices correspond to call options traded in October 1998 in the MONEP.

Table 2: Comparison of implicit stock price model and observed values of PXL call options.

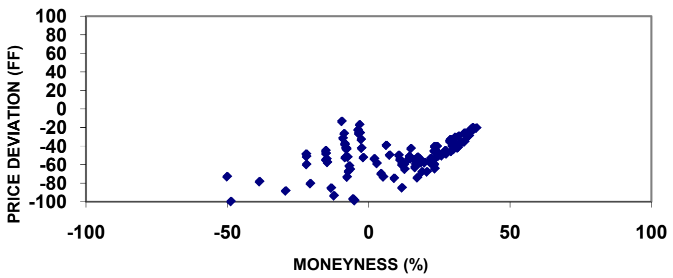

Figure 2: Implicit stock price -model -October, 2nd 1998.

Trading date	ISD1 (%)	ISD2 (%)	m_1	m_2	α_1	Proportion of Theoretical Prices Different from the Observed Prices at ± 1 %	Average Spread between Observed and Theoretical Price (%)	Average Deviation of Theoretical Price from Observed Prices (FF)t
02/10/98	36.503	39.046	8.047	8.042	0.848	1.000	-60.557	76.632
05/10/98	46.494	10.486	7.841	8.236	0.685	0.986	5.865	39.746
06/10/98	26.979	41.575	8.211	7.854	0.404	1.000	-18.158	41.204
07/10/98	19.579	68.446	8.187	7.595	0.606	0.918	-18.724	27.639
08/10/98	34.642	33.996	8.070	8.013	0.834	1.000	-66.727	79.382
09/10/98	41.005	11.800	7.889	8.250	0.725	0.976	10.982	40.351
12/10/98	30.449	32.399	8.079	8.024	0.827	0.971	15.293	75.205
13/10/98	23.065	45.590	8.196	7.776	0.685	0.945	3.711	33.933
14/10/98	31.404	43.655	8.113	8.045	0.862	0.930	-8.326	31.660
15/10/98	26.341	45.359	8.157	7.968	0.837	0.864	-6.018	32.795
16/10/98	17.729	55.596	8.257	7.736	0.652	0.817	5.180	15.740
19/10/98	44.034	18.006	7.947	8.260	0.441	0.875	-4.066	14.355
20/10/98	45.498	16.593	7.898	8.271	0.442	0.955	7.003	30.265
21/10/98	51.874	16.550	7.890	8.269	0.419	0.526	-1.264	15.680
22/10/98	45.592	17.366	7.916	8.275	0.424	0.872	4.011	22.267
23/10/98	47.876	19.694	7.909	8.266	0.390	1.000	-26.642	42.918
26/10/98	19.857	17.260	7.950	8.286	0.411	0.960	11.267	23.085
Average	50.495	15.995	7.961	8.256	0.425	0.917	-8.657	37.815

All the observed prices correspond to call options traded in October 1998 in the MONEP.

Table 3: Comparison of mixture of lognormal distribution model and observed values of PXL call options.

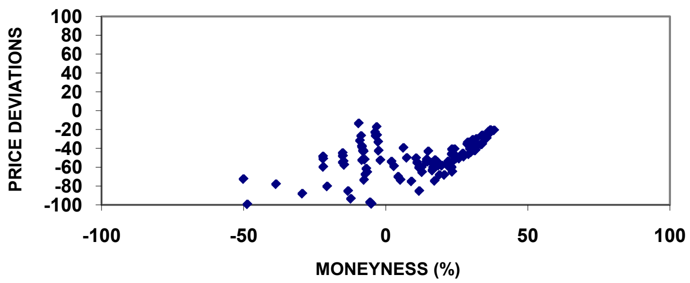

Figure 3: Mixture of Lognormal Distribution model - October, 2nd 1998.

The average percentage obtained for the implied correlation parameter is −0.60. We see in column five that in average, 93.1% of the theoretical prices are outside the ± 1% spread applied to the observed prices. Two of them are completely outside, which indicates a very slight improvement, but that is not significant relative to our benchmark. The average deviation is around 23.32 FF in column 9 for a average call price of 451.16 FF. The average spread between the observed price and the corresponding theoretical price is about 17.89%, which shows that the mispricing generally observed in the benchmark is reduced two times. (Figure 4) shows the effects of estimating option prices with a stochastic volatility model on October, 2nd 1998.

The Hull and White [9] model provides significant improvements in terms of price deviations for far from the money call options.

Fifth estimation procedure

We estimate implicitly the parameters of the jump diffusion model. We estimate the jump occurrence parameter λ, the jump size parameter k and the implied volatility through the Simultaneous Equations Procedure, which gives us implied values of three parameters by minimizing the following sum of squares:

$$\min_{\lambda,k,ISD} \sum_{j=1}^{N} [C_{OBS\ j} - C_{JUMP\ j}(\lambda, k, ISD)]^2$$

$C_{JUMP}(\lambda, k, ISD)$ is the call price given by the jump diffusion model. This call price is calculated for any option in a given current day's sample (Table 5).

We notice that the average implied volatility is about 24 %. The average percentage obtained for the implied average rate of jump occurrence is 0.297 and the implied average jump size is about −0.53. About 74.62% of the theoretical prices are lying outside the ±1% spread applied to the observed prices. No one is totally outside, showing that the improvements that brings this jump diffusion model.

The total gain relative to the benchmark is about 23 %. The average deviation is around 15.17 FF for an average call price of 451.16 FF. The average spread between the observed prices and the corresponding theoretical prices is about −5.14 %, which indicates a considerable decrease in the mispricing. Figure 2 shows the effects of allowing one jump of constant size in the theoretical [1] framework on October, 2nd 1998 (Figure 5).

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Trading date	Number of Data	ISD (%)	Implied Volatility of Volaility	Implied Correlation	Half-Life Of Volatility (Days)	Proportion of Theoretical Prices Different from the Observed Prices at ± 1 %	Average Spread between Observed and Theoretical Price (%)	Average Deviation of Theoretical Price from Observed Prices (FF)
02/10/98	153	44.397	2.655	-0.560	21.357	0.974	35.785	30.525
05/10/98	69	43.575	2.101	-0.696	21.229	0.913	16.722	25.472
06/10/98	53	41.858	2.994	-0.520	21.443	0.943	13.139	19.630
07/10/98	49	41.391	2.241	-0.624	21.253	0.898	33.809	21.958
08/10/98	56	42.251	3.156	-0.478	21.456	1.000	52.034	28.610
09/10/98	41	41.497	1.856	-0.717	21.165	1.000	0.751	20.252
12/10/98	68	40.347	2.531	-0.629	21.333	0.897	3.734	25.566
13/10/98	55	40.843	3.549	-0.460	21.563	0.818	5.202	22.714
14/10/98	86	37.122	4.496	-0.421	21.795	0.860	6.620	15.132
15/10/98	59	35.363	2.082	-0.715	21.224	0.966	44.622	25.641
16/10/98	60	35.043	1.699	-0.655	21.127	0.983	0.208	21.515
19/10/98	32	36.232	2.337	-0.652	21.278	0.938	10.951	23.923
20/10/98	88	35.031	2.936	-0.526	21.415	0.932	12.970	22.693
21/10/98	57	36.609	2.577	-0.516	21.316	0.947	12.752	27.162
22/10/98	47	35.676	3.063	-0.495	21.437	0.872	2.533	18.458
23/10/98	31	37.675	2.873	-0.512	21.392	0.968	34.398	26.174
26/10/98	25	34.912	3.101	-0.486	21.445	0.920	17.998	20.984
Average	64.111	39.038	1.858	-0.606	21.157	0.931	17.896	23.318

All the observed prices correspond to call options traded in October 1998 in the MONEP.

Table 4: Comparison of implicit Hull and White (1988) model and observed values of PXL call options.

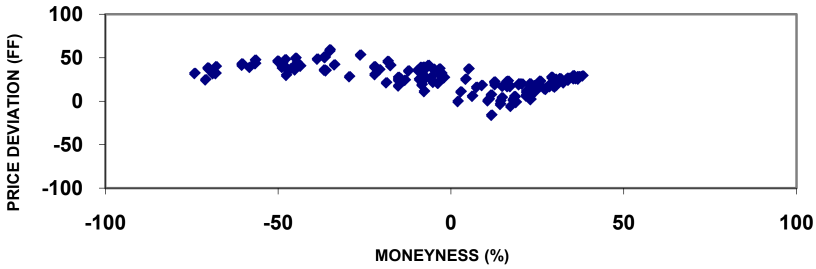

Figure 4: Hull and White-model - October, 2nd 1998.

Trading date	Number of Data	ISD	Option Implied Jump Occurrence	Option Implied Jump Size	Proportion of Theoretical Prices Different from the Observed Prices at ± 1 %	Average Spread between Observed and Theoretical Price (%)	Average Deviation of Theoretical Price from Observed Prices (FF)	Average Call Price (FF)
02/10/98	153	0.273	0.377	-0.537	0.778	4.490	17.148	419.250
05/10/98	69	0.279	0.421	-0.537	0.768	-13.107	16.414	528.873
06/10/98	53	0.269	0.387	-0.544	0.642	-6.343	14.829	506.300
07/10/98	49	0.272	0.359	-0.542	0.857	-17.044	16.482	406.673
08/10/98	56	0.259	0.389	-0.509	0.875	6.844	17.806	441.110
09/10/98	41	0.265	0.397	-0.538	0.805	-9.838	15.616	338.436
12/10/98	68	0.237	0.517	-0.473	0.926	-6.087	19.867	500.436
13/10/98	55	0.254	0.382	-0.498	0.582	-3.704	17.429	626.313
14/10/98	86	0.254	0.341	-0.510	0.709	-5.438	10.354	345.208
15/10/98	59	0.233	0.421	-0.471	0.763	-12.297	11.351	378.210
16/10/98	60	0.225	0.415	-0.467	0.767	-3.076	10.705	385.422
19/10/98	32	0.227	0.397	-0.463	0.563	-1.906	7.855	395.641
20/10/98	88	0.211	0.477	-0.440	0.693	-1.053	19.192	526.652
21/10/98	57	0.237	0.335	-0.492	0.596	-1.349	17.138	564.885
22/10/98	47	0.254	0.270	-0.536	0.574	-5.698	15.451	566.498
23/10/98	31	0.249	0.268	-0.533	0.871	-5.348	14.571	391.574
26/10/98	25	0.214	0.516	-0.427	0.920	-6.417	15.680	341.497
Average	64.111	0.239	0.297	-0.529	0.746	-5.139	15.170	451.166

All the observed prices correspond to call options traded in October 1998 in the MONEP.

Table 5: Comparison of jump diffusion model prices and observed prices of PXL call options.

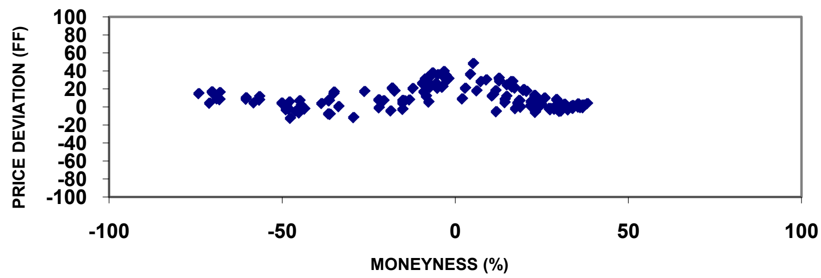

Figure 5: Jump diffusion model - October, 2nd 1998.

Models	MAE	MAPE	PE
Benchmark	38.827	0.209	1.206
I - Series expansions	11.248	0.413	-1.668
II - Martingale restriction	40.760	0.171	-1.312
III - Mixture of log-normals	36.818	0.146	0.030
IV - Stochastic volatility	18.589	0.628	10.497
V - Jump diffusion	10.007	0.0632	-0.899

The pricing errors are computed as the difference between the observed call price and the theoretical price.

Table 6: Comparison between alternative models.

Models	MAE	MAPE	PE			
	ITM	OTM	ITM	OTM	ITM	OTM
Benchmark	41.673	36.271	0.061	0.342	42.806	-36.349
I - Series expansions	9.785	12.563	0.014	0.772	-0.514	-4.783
II – Martingale restriction	60.235	23.262	0.0869	0.247	28.734	-12.489
III – Mixture of log-normals	55.285	20.225	0.074	0.211	20.146	-1.045
IV – Stochastic volatility	22.418	15.149	0.03	1.166	21.717	0.336
V - Jump diffusion	12.24	8	0.0165	0.105	-0.907	-4.389

Table 7: Comparison between alternative models for ITM and OTM call options.

The reduction in the price deviation is clear comparing to the benchmark and is very comparable to model 2 and model 5.

The Statistical Performance of the Models

We want to evaluate the statistical significance of the improvement in terms of performance from out-of-sample adjustments brought by each model and for that, we use the following Z-statistic that represents the difference between the two proportions, P_1 and P_2 Hoel (1984):

$$Z = \frac{P_1 - P_2}{\sqrt{P_1(1-P_1)/N_1 + P_2(1-P_2)/N_2}}$$

P_1 and P_2 are sample proportions, while N_1 and N_2 are corresponding sample sizes.

We obtain after computation:

For the first model: Z=18.218 - P_1=0.971 and P_2=0.712; N_1=N_2= 1154

For the second and fourth model: Z=4.470 - P_1=0.971 and P_2=0.93; N_1=N_2=1154

For the third model: Z=5.680 - P_1=0.971 and P_2=0.917; N_1=N_2=1154

For the fifth model: Z=16.383 - P_1=0.971 and P_2=0.746; N_1=N_2=1154

The Z-statistics of 4.470, 5.680 and 18.218 are statistically significant at more than 99.99% confident level, which corresponds to a fractile of 2.32.

We want to go further into the analysis of the out-of-sample performance of our various models. We denote C, the observed price of the option and $\hat{C}$, the theoretical price of the same option. We use the Mean of Absolute forecast Error (MAE) and the Mean Absolute Percentage forecast Error (MAPE) (Lauterbach and Schultz (1990)). For n being the number of options, the value of the MAE is equal to the mean of the difference in absolute value between the observed prices and the theoretical prices:

$$MAE = \sum_{i=1}^{n} \left| C_i - \hat{C}_i \right| \Big/ n$$

For each option the valuation error percentage is given by:

$$\frac{C_i - \hat{C}_i}{\hat{C}_i}$$

The $MAPE$ value is:

$$MAPE = \sum_{i=1}^{n} \left| \frac{C_i - \hat{C}_i}{\hat{C}_i} \right| \Bigg/ n$$

The value of MAE and MAPE are actually complementary because if the first one gives a classical estimation of the difference, the second one gives the difference relative to the theoretical price. Tables 6 and 7 give the results obtained using the PE, MAE and MAPE measures (Table 6).

The jump diffusion model holds the best MAPE and MAE and the second best pricing errors while the stochastic volatility model has surprinsingly the worst MAPE and pricing errors. (Table 7) presents the forecast errors distinguishing in-the-money options and out-the-money options.

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We observe that the series in expansion model produces the best forecasts concerning ITM options for PE, MAE and MAPE measures. However, the best MAE and MAPE measures concerning OTM options, are obtained by the jump diffusion model, which has also the second best pricing error measure. The martingale restriction model has the worst performance among the five other models. The two models (1 and 5) that overestimate the ITM options have the best PE; the only model (4) that underestimates the OTM options has also the best PE. We globally note that the stochastic volatility model and the series in expansions model suffer form severe mispricing of OTM options.

Conclusion

In this paper, we carry out empirical tests of different models likely to correct the pricing bias that occurred in the classical Black and Scholes [1] framework and we also test the statistical performance of the given models. We observed that the Black and Scholes [1] model undervalues out-the-money calls and overvalues in-the-money calls. The jump diffusion model performs better than the other four models and overall has the best results concerning the out-the-money call options while the series in expansion model holds the best forecasts concerning the in-the-money call options. The stochastic volatility model arrives in third position with no outstanding performances. As for the mixing two log-normal distibutions and the implicit stock price model, they didn't offer the expected precision. In further research, we will extend this work to propose an option valuation model that incorporates both transaction costs and information costs in order to correct the bias due to the frictions in the market.

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