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Short Communication

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Controlling Chaos in a Food Chain Model through Threshold Harvesting

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Abstract

In this paper, we propose a new harvesting strategy namely the harvesting for controlling chaotic population in a food chain model. In particular, we have taken the three species Hastings and Powell food chain model for demonstration. We have shown threshold harvesting strategy can be effectively employed to obtain a steady or cyclic behaviour from chaotic fish population by varying either the frequency of harvesting or the amount of harvesting of fish population. Numerical simulation results are presented to show the effectiveness of the scheme. We obtain steady state; limit cycle, period-2 and period-4 behaviour from chaotic Hastings and Powell model. This threshold harvesting strategy will be very useful for species conservation and fishery management.

Keywords: Chaos; Chaos control; Threshold mechanism; Hastings and Powell model; Maximum sustainable yield

Introduction

An extremely common phenomenon in nonlinear dynamical systems arising from a variety of disciplines is chaos. Chaotic dynamics is interesting to analyse but for technological processes like optimization of the production in a production farm chaos is highly unwanted or even harmful. Therefore, strategies are required to devise control algorithms capable of achieving the desired type of behaviour from a chaotic system. There has been many techniques for designing effective control of chaotic systems but very few of these methods are applicable to control a biological systems like food chain models [1-4]. Many authors proposed harvesting model for ecological systems [5-8]. But none of these are very useful from applied point of view because harvesting at every time is not realistic and meaningful for many biological systems. Those methods also required knowledge of functional response and knowledge of system parameters.

Now, we shall discuss a new harvesting mechanism. Consider a general N-dimensional food chain model

$$\frac{dX}{dt} = F(X)$$

Where X=(x_1, x_2,…….., x_N) are the state variables. Let the variable x_i is chosen for harvesting. The threshold harvesting strategy is as follows. We shall check the population size represented by the state variable x_i at regular interval of time. At the time of checking if the population of variable x_i exceeds a critical population x_i^*, then harvesting will be done and collect (x_i- x_i^*) number of fishes otherwise do not collect any fish. This assumption is natural because in fishery because harvesting of fish takes place at regular interval of time. Hastings and Powell [1] introduced a continuous time model of a food chain incorporating nonlinear functional responses and shown that model exhibits chaotic dynamics in long term behaviour when biologically reasonable parameter values are chosen. Wilson et al. [4] had obtained chaotic dynamics in a multi-species fishery. Managing such a chaotic fishery system, demands different approach for controlling chaos. Chattopadhyay et al. [5] interpret the population of first, second and third species of the Hastings and Powell [1] model as the population of the toxin producing phytoplankton (TPP), zooplankton and fish respectively. Then according to Hastings and Powell [1], the fish population will vary chaotically for some biologically significant parameter region. In this work, we have shown that threshold harvesting strategy can be applied for controlling chaotic fish population and to obtain regular fish population dynamics e.g., steady state, limit cycle, period-2, period-4 etc. Here the thresholding variable is chosen as the fish population variable of the system. In section-2, the threshold harvesting mechanism is discussed. In section-3, application of threshold harvesting for controlling chaotic dynamics of fish population is demonstrated for chaotic Hastings and Powell [1] model. In section-4, numerical simulation results are discussed. Finally a conclusion is drawn in section-5.

Threshold Harvesting Mechanism

Consider a general N dimensional dynamical system, described by the following evolution equations

$$\frac{dx_1}{dt} = f_1\ (x_1, x_2, x_3, \ldots\ldots, x_N; t),$$
$$\frac{dx_2}{dt} = f_2\ (x_1, x_2, x_3, \ldots\ldots, x_N; t),$$
$$\frac{dx_N}{dt} = f_N\ (x_1, x_2, x_3, \ldots\ldots, x_N; t),$$

Where X=(x_1, x_2, x_3,……, x_N)are the state variables. Let the variable x_i, $i \in 1, 2, \ldots, N$ is chosen as the monitored variable which we want to control. The mechanism for threshold action in this system is as follows. Control will be triggered after a finite time interval. Whenever the value of the monitored variable exceeds a critical threshold x^* and the variable x_i will then be reset to x^*, i.e.

if $x_i \le x^*$ then no harvesting

if $x_i > x^*$ then $x_i \rightarrow x^*$.

The dynamics continues undisturbed until x_i exceeding the threshold value. When it exceeds then control resets its value to x^* again. As the system parameters are left invariant by this method therefore it acts

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only on state variable. In fact the method requires no knowledge of the parameters, which is advantageous for controlling chaos in biological systems. The moment thresholding is removed the system is back to its original dynamics. The threshold action is necessarily stroboscopic, as the threshold condition can be checked only at finite intervals. Here we will study the interesting effects of implementing the threshold action at varying intervals. We will show that changing the frequency of thresholding leads to many different regular temporal patterns. In fact very infrequent thresholding is capable of yielding amazingly simple and regular behaviour of a chaotic system.

Hastings and Powell Model

In this section, we shall first discuss briefly the Hastings and Powell [1] three species food chain model. Hastings and Powell [1] assume X as the number of species at the lowest level of the food chain, Y the number of species that preys upon X, and Z the number of the species that preys upon Y . The model takes the form

$$\frac{dX}{dT} = R_0X(1 - \frac{X}{K_0}) - C_1F_1(X)Y,$$
$$\frac{dY}{dT} = F_1(X)Y - F_2(Y)Z - D_1Y,$$
$$\frac{dZ}{dT} = C_2F_2(Y)Z - D_2Z,$$

With

$$F_i(U) = \frac{A_iU}{(B_i + U)} \text{ for i=1,2,}$$

representing the functional response. Here T is time. The constant R_0 is the 'intrinsic growth rate' and the constant K_0 is the 'carrying capacity' of species X. The constants C_1^{-1} and C_2 are conversion rates of prey to predator for species Y and Z respectively, D_1 and D_2 are constant death rates for species Y and Z respectively. The constants A_i and B_i for i = 1, 2 parametrize the saturating functional response, B_i is the prey population level where the predation rate per unit prey is half its maximum value. With the following dimensionless variables

$$x = \frac{X}{K_0}, y = \frac{C_1Y}{K_0}, z = \frac{C_1Z}{C_2K_0}, t = R_0T,$$

the model takes the form

$$\frac{dx}{dt} = x(1-x) - f_1(x)y,$$
$$\frac{dy}{dt} = f_1(x)y - f_2(y)z - d_1y,$$
$$\frac{dz}{dt} = f_2(y)z - d_2z - hz,$$
$$f_i(U) = \frac{a_iu}{(1+b_iu)} \text{ for } i = 1,2.$$

where d_2=d'_2+ h. where h represents the rate of harvesting. Chattopadhyay et al. [5] interpret the variables x, y, z as the toxin producing plankton (TPP), zooplankton and fish population respectively. They have interpreted the parameters a_1, a_2, b_1, b_2, d_1 and d'_2 as intrinsic birth rate of prey, intrinsic death rate of predator population and h is the rate of harvesting. In this work we interpret the variables x, y, z as x as TPP, y as Zooplankton and z as fish population.

Numerical Simulation Results

We apply threshold harvesting technique on Hastings and Powell model. We impose threshold condition on z variable here. Because in fish population model it is possible to threshold fish population by harvesting fish at a regular interval. In the Hastings and Powell model we interpret the z variable as fish population. The parameters of the model are chosen a_1=5.0, a_2=0.1, b_1=2.8, b_2=2.0, d_1=0.4, d'_2=0.01. For these set of parameter values the model have chaotic behaviour. We use Runge Kutta 4th order scheme for solving the system with time step 0.005. Since thresholding, the variable z is biologically meaningful; we choose different threshold values for z together with different time interval of control. We start with choosing threshold value (MSY effort)h=h_{MSY} =0.5 with control acts at interval δt=0.10, and obtain the time evolution of Hastings and Powell model is shown in figure 1. And phase diagram in y-z plane is depicted in figure 2. From figures 1 and 2 it is clear that Hastings and Powell model is chaotic for the threshold value (MSY effort) h=h_{MSY}=0.5 with control acts at interval δt=0.10. Therefore MSY

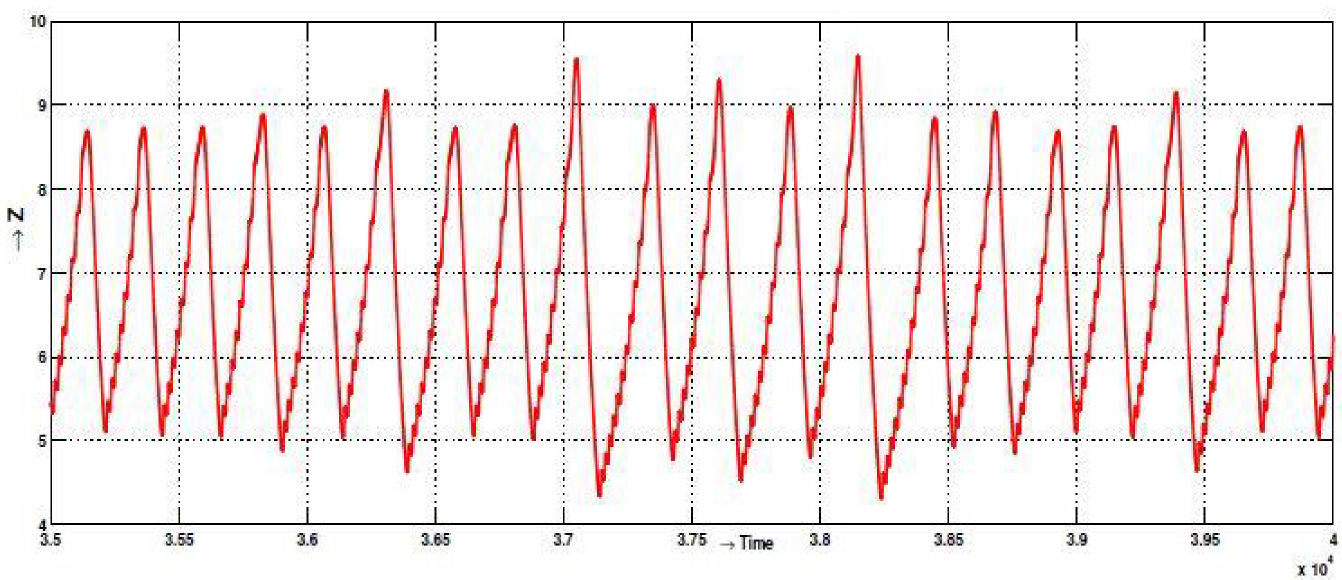

Figure 1: The time evolution of Hastings and Powell model under threshold control variable z, with threshold value h=0:5 with control acts at interval t=0:10, the chaotic behaviour is shown for the parameters a1=5:0; a2=0:1; b1=2:8; b2=2:0; d1=0:4; d02=0:01.

is not a stable for the effort $h=h_{MSY}=0.5$. The chaotic Hastings and Powell model under threshold control of variable z, with threshold value (MSY effort) $h=h_{MSY}=0.13$, the control acts at interval $\delta t=0.275$, the controlled steady state behaviour which is shown in figure 3. From figure 3, it is clear that chaotic behaviour of fish population is controlled and steady state dynamics is obtained under the threshold harvesting. In this case MSY is stable steady state behaviour for the effort $h=h_{MSY}=0.13$. The time evolution of chaotic Hastings and Powell model under threshold control of variable z, with threshold value (MSY effort) $h=h_{MSY}=0.175$, the control acts at interval $\delta t=0.15$ is shown in figure 4, From figure 4 it is clear that chaotic MSY is replaced by limit cycle MSY under the threshold control. The same chaotic model under threshold control of variable z, with threshold value (MSY effort) $h=h_{MSY}=0.18$, the control acts at an interval $\delta t=0.075$, obtain period-2 behaviour of the threshold control system which is shown in figure 5. The model under threshold control of variable z, with threshold value (MSY effort) $h=h_{MSY}=0.2$, the control acts at an interval $\delta t=0.095$, obtain period-4 behaviour of the threshold control system which is shown in figure 5. Therefore by different choice of MSY effort we can obtain any periodic behaviour. We have employed successfully the threshold mechanism to Hastings and Powell model figure 6 and obtain steady state limit cycle, period-2 and period-4 behaviour of the system.

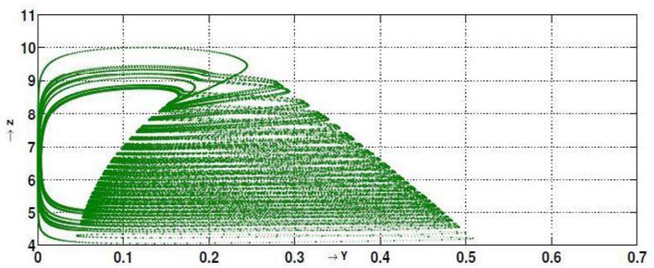

Figure 2: The Hastings and Powell model under threshold control variable z, with threshold value h=0:5 with control acts at interval t=0:10, the chaotic behaviour is shown for the parameters a1=5:0; a2=0:1; b1=2:8; b2=2:0; d1=0:4; d02=0:01.

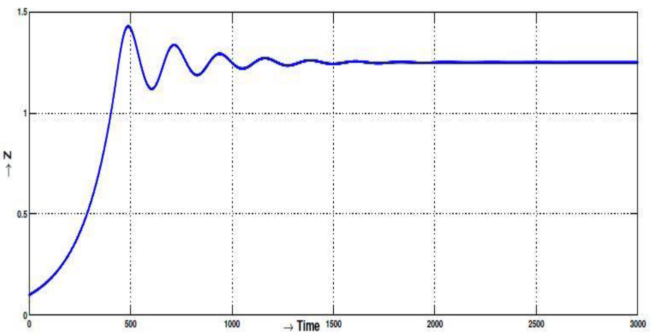

Figure 3: The Hastings and Powell model under threshold control variable z, with threshold value h=0:13 with control acts at interval _t=0:275, the steady state behaviour is shown for the parameters a1=5:0; a2=0:1; b1=2:8; b2=2:0; d1=0:4; d02=0:01.

Conclusion

We define the threshold harvesting as the catching of fish after an interval of time provided fish population is above some critical value. The knowledge of functional response is required for applying almost all existing chaos control methods. However, in threshold harvesting strategy no knowledge of system parameters and functional responses are required. Therefore this method is suitable for controlling chaos of many biological or real world systems. A hyper chaotic food chain can also be controlled to obtain steady state or low periodic behaviour choosing suitable threshold value. The threshold harvesting method of chaos control is suitable for any controlling chaotic dynamics of a particular state variable of any biological model. Therefore by choice of threshold value (MSY effort) we may harvests the fish population to MSY. This control strategy is really very useful for fishery management of marine ecological systems. This method is also applicable for biological conservation of species in real world biological systems.

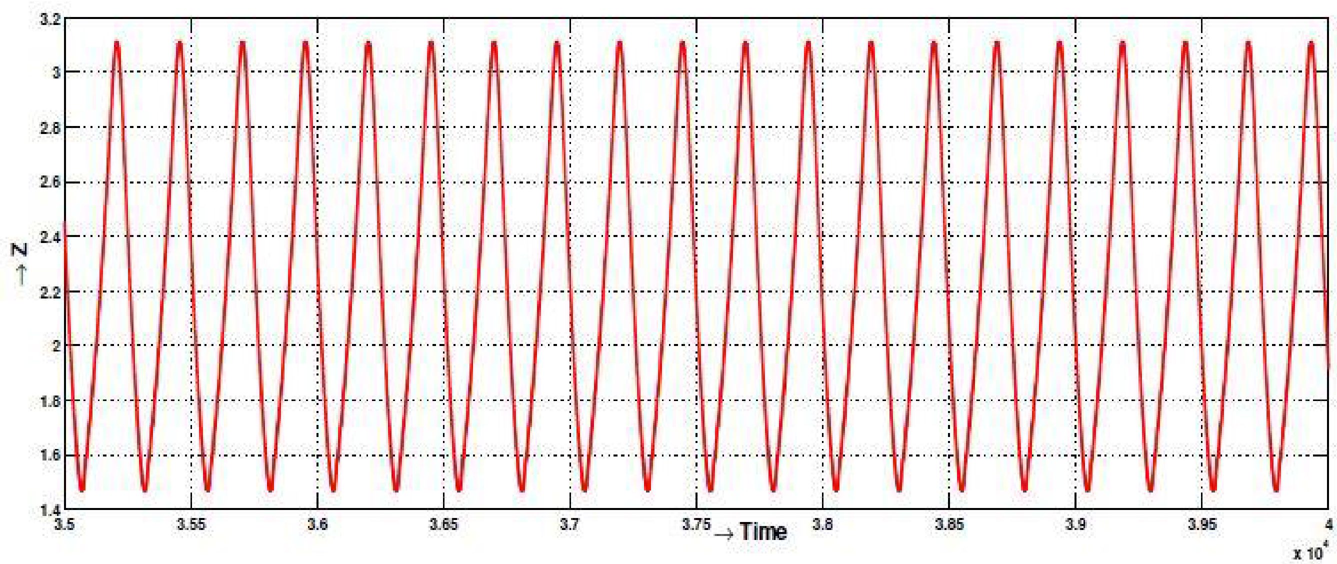

Figure 4: The time evolution of Hastings and Powell model under threshold control variable z, with threshold value h=0:175 with control acts at interval t=0:15, the limit cycle behaviour is shown for the parameters a1=5:0; a2=0:1; b1=2:8; b2=2:0; d1=0:4; d02=0:01.

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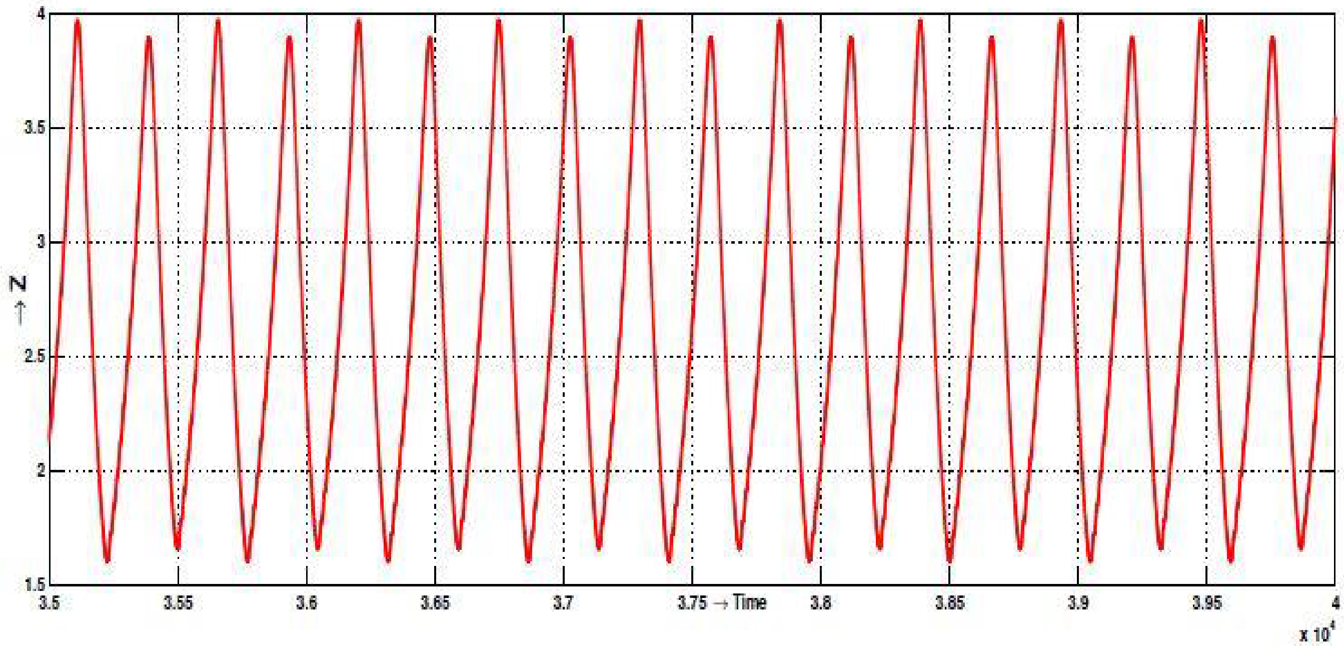

Figure 5: The time evolution of Hastings and Powell model under threshold control variable z, with threshold value h=0:2 with control acts at interval t=0:075, the period-2behaviour is shown for the parameters a1=5:0; a2=0:1; b1=2:8; b2=2:0; d1= 0:4; d02=0:01.

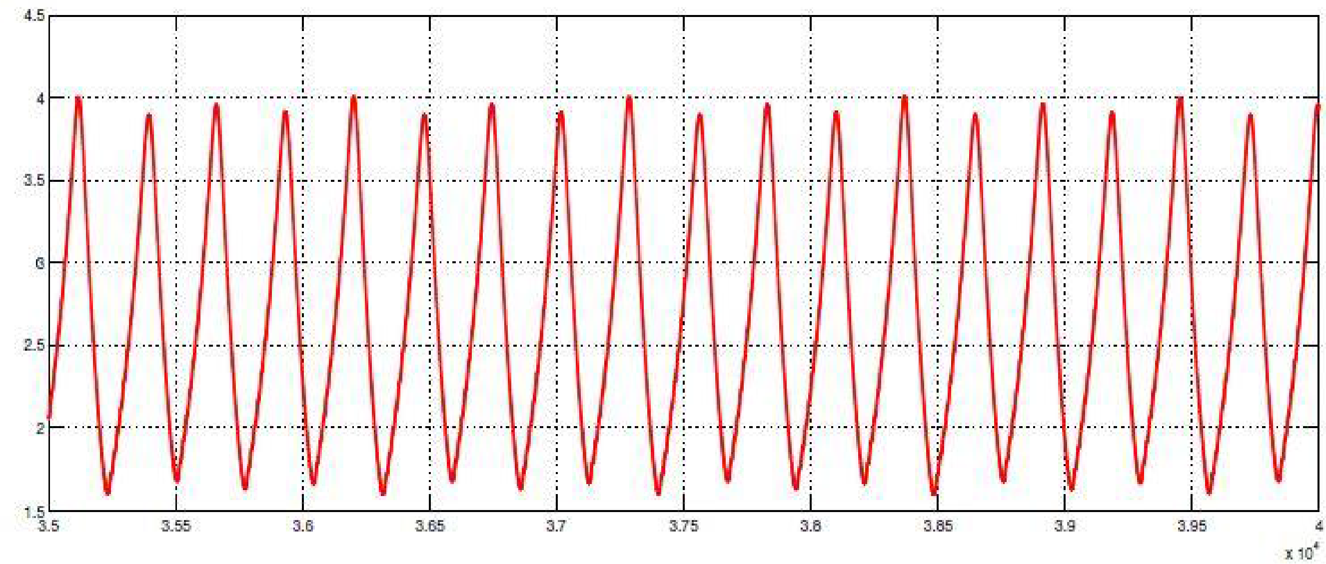

Figure 6: The time evolution of Hastings and Powell model under threshold control variable z, with threshold value h=0:2 with control acts at interval t=0:095, period-4 is shown for the parameters a1=5:0; a2=0:1; b1=2:8; b2=2:0; d1=0:4; d02=0:01.

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