Collinear Libration Points in the Perturbed Circular Restricted Three-Body Problem with Heterogeneous and Luminous Primaries Surrounded by a Belt

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ABSTRACT

This paper studies a modified type of the classical restricted three-body problem where both primaries are radiating and are heterogeneous oblate spheroids with three layers of different densities, encompassed by circular cluster of material points centred at the mass center of the system (belt) together with additional effects of small perturbations in the Coriolis and centrifugal forces. Using semi analytical and numerical approach, the collinear points are found to be unstable. In addition to the three collinear libration points in the classical case, it appears that there is one more collinear point which resulted from the effects of heterogeneity of the participating primaries together with potential from a belt, though the additional points are found linearly unstable. A practical application of this model could be the study of motion of a dust grain near the heterogeneous and luminous star surrounded by a belt.

Keywords: CR3BP, Perturbation; Radiation; Heterogeneous Spheroid and Potential from a belt.

INTRODUCTION

The restricted three body- problem (R3BP) consist three bodies: two primary bodies having finite mass (and such that >) moving under their mutual gravitational attraction, and the third with a negligible mass with (infinitesimal) body, whose motion is influenced by the primaries. If the primaries move on circular orbits about their common centre of mass, it is referred as Circular restricted three body problem(CR3BP).It is to determine the motion of the infinitesimal mass.Szebehely,v.(1967) and Valtonen,V. and Karttunen,H.(2006) gave the solution of CR3BP.They showed that if the primary bodies were fixed in a rotating coordinate system, five Libration points existed. Three of the points are on the line linking the primaries, while the two are in equilateral triangular with the primaries. The collinear points are linearly unstable, whereas the triangular points are linearly stable for the mass ratio of the primaries. Stability of the collinear points of CR3BP with perturbations have achieved in recent times.Poynting (1903) indicated that small particles were equally influenced by the gravitation and light radiation force as they moved toward luminous celestial bodies. Radzievskii (1950,1953) established that the presence of direct solar radiation pressure caused a variation in the sites of the libration points of the CR3BP. Radzievskii called CR3BP photo gravitational when one or both of the primaries were discharges of radiation.

Jiang and Yeh (2004) have studied a modified CR3BP by considering the influence from a belt (circular cluster of material points) for a planetary systems, and they found out that the likelihood to get libration points around the inner part of the belt was greater the one nearly the outer part. Jiang and Yeh (2006).and Singh and Taura (2013) consequently transformed and changed the configuration of the dynamical system , such that new libration points existed.

Considering the small perturbations in the Coriolis and centrifugal forces, different researches (Szebehely 1967 b; Bhatnagar and Hallan 1978; AbdulRaheem and Singh 2006; Singh 2009; Singh and Begah 2011; Singh 2013 and Singh and Haruna 2014) have described their effects on the motion of the third body. AbdulRaheem and Singh 2006; Singh 2009; Singh and Begah 2011; Singh 2013 and Singh and Haruna 2014 in their studies of collinear libration points found that they are all unstable.

Suraj et al (2014) studied the stationary solution of the planar restricted three body problem when the primaries are heterogeneous oblate spheroids with three layers of different densities and sources of radiation. They observed that the triangular points are stable while the collinear libration points are unstable (Figures 1).

Table 1: Numerical computations of the Coordinates of the collinear equilibrium points and for difference cases.

Cases	xL_1	xL_2	xL_3	xL_4
1	-0.2491	-0.3272	0.1071	0.0213
2	-0.2301	-0.3303	0.1164	-
3	-0.2303	-0.3303	0.1165	-
4	-0.2302	-0.3301	0.1165	-
5	-0.2304	-0.3274	0.1067	0.0213
6	-0.2491	-0.3271	0.6813	-
7	-0.2489	-0.3272	0.1069	0.0217
8	-0.2487	-0.3274	0.1069	0.0213
9	-0.2488	-0.3272	0.1069	0.0213
10	-0.2304	-0.3303	0.8952	-
11	-0.2304	-0.3301	0.1165	-
12	-0.2301	-0.3303	0.1163	-
13	-0.2487	-0.3274	0.1067	-
14	-0.2304	-0.3303	0.1164	-

This study aims to examine the stability of collinear point of the infinitesimal body in the perturbed CR3BP when both primaries are sources of radiation, with heterogeneity surrounded by a belt.

This paper is organised as: sect. 1, which is introduction; sect.2 mathematical formulation of the problem, and sect.3 is dedicated to the determination of the sites of the collinear libration points. The linear stability of collinear points and the conclusion are presented in sect. 4 and 5 respectively.

MATHEMATICAL FORMULATION OF MODEL

The problem

Let and be the masses of the primaries with > , let the mass of the infinitesimal body moving in the plane of motion of the primaries. The positions of the primaries are defined with respect to a rotating coordinate from oxyz where x-axis overlaps with line joining them and whose origin coincides with the center of mass of and . The y - axis is perpendicular to the x-axis ,then z-axis is normal to the orbital plane of the primaries. Let be the distance between and , the distance between and ; and R the distance between and . The coordinates of , and are (,0), (,0) and ( ) respectively.

Location of Collinear libration points

Equilibrium points are those points at which no net force on the infinitesimal mass, therefore in effect at any points if the third body is placed with zero velocity, it will stay there. The libration points satisfying all the derivatives of the coordinates with respect to the time are zero at these points.

The coordinates of the collinear points for different cases as classified in the following order are presented,thus:

1. The present problem (perturbation in the Coriolis and centrifugal force together with heterogeneity , radiation pressure of both primaries and potential from a belt) .

2. Heterogeneity of the bigger primary only.

3. Radiation pressure of both primaries only.

4. Radiation pressure of the bigger primary only.

5. Heterogeneity of the bigger primary with potential from a belt only.

6. perturbation in centrifugal force together with radiation pressure of both primaries and potential from a belt.

7. Heterogeneity of the bigger primary , radiation pressure of both primaries and potential from a belt.

8. Potential from a belt only.

9. Radiation of the bigger primaries and Potential from a belt only.

10. The classical case (Absence of the perturbations).

11. Heterogeneity of both primaries and Radiation.

12. Heterogeneity of both primaries.

13. Heterogeneity of both primaries with potential from a belt.

14. Heterogeneity of both primaries with perturbation in the centrifugal force.

15. Linear Stability of the collinear points.

16. We examine the stability of an equilibrium point, which its ability to restrain the body motion in its vicinity, to do so we displace the infinitesimal body a little from an equilibrium point with small velocity. If its motion is rapid departure from the vicinity of the point, we call such a position of equilibrium an unstable .If the body oscillates about the point, it is said to be a stable position.

CONCLUSION

A study of the effects of heterogeneous and luminous primaries surrounded by a belt (circular cluster of material point) in the perturbed circular restricted three-body problem on the locations and stability of collinear points is carried out. Equation (5) gives the motion equations of the infinitesimal body under the influence of the aforementioned parameters .The equations are affected by the all perturbations. Analytically and numerically, we have determined the positions of the collinear libration points and seen the effects of the aforesaid perturbations on them. It is discover that in counting to the three L1, L2, L3 collinear libration points in the classical problem, it appears that there is one more collinear point, which results from the effects of heterogeneity of the primaries together with potential from a belt. It is noticed that in

spite of the influence of heterogeneous and luminous primaries surrounded by a belt (circular cluster of material point) in the perturbed CR3BP, the instability characteristics behaviour of the collinear points remain unchanged as in the classical case, together with the additional new collinear point. This is confirmed from numerical approach as it reveals existence of at least a positive root. As a result the motion is unbounded and we conclude that the collinear libration points are unstable.

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