

## Application of the Nash Nonlinear Grey Bernoulli Model for Forecasting Foreign Exchange Rate of Taiwan's Top Two Trading Partners

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### Abstract

The precise prediction of foreign exchange rate is very important for international traders and investors. This study adopts nonlinear grey Bernoulli model (NGBM) and Nash NGBM (NNGBM) to predict the currency exchange rate of Taiwan's two top trading partners, America and China. The simulation results show that Taiwan's currency will appreciate against USD and CNY from the fourth quarter of 2015 to the second quarter of 2016. The conclusions can act as reference for international traders and investors.

**Keywords:** Grey forecasting; Nonlinear Grey Bernoulli model; Exchange rate; Currency; Nash

### Introduction

Grey forecasting [1] is one topic of grey theory proposed by Deng [2]. When the data are few, most forecasting models are restricted. Fortunately, the grey forecasting merely needs four data to construct model and its forecasting performance is satisfactory. They are applied in many fields, including Economics [3,4] finance [5], agriculture [6], air transportation [7], electric load [8], industry [9], and industrial wastewater [10] and so on.

The evolution of the grey forecasting still continues to develop. The researchers try to improve the original models to get higher forecasting performance. They develop different types of grey forecasting models, including Grey-Markov model [11], Grey-Fuzzy model, Grey-Taguchi model [12], Grey Verhulst model [13], the nonlinear grey Bernoulli model (NGBM) [14,15], Nash NGBM [16] and so on.

Besides, some researchers use different algorithm methods to solve the optimization problem. For example, the particle swarm optimization algorithm (PSO) [17] and genetic algorithm (GA) [18,19] are applied to seek for the optimal solution. In addition, the nonlinear optimized model can be solved by computer software [3,10].

Export and import are very important to Taiwan. Currency exchange rate affects exports and imports. Thus, this study adopts NGBM and Nash NGBM to predict foreign exchange rate for Taiwan's two major trading partners, America and China.

This paper is organized as follows. Section 4 introduces the mathematics of NGBM, and defines the forecasting relative percentage error. In section 3, the case study is to forecast Taiwan's currency against USD and CNY. Finally, section 6 presents conclusions.

### Mathematical Methodology

This procedure of deriving Nash NGBM (1,1) are described below [16]:

**Step 1:** Assume that the original series of data with  $m$  entries is:

$$X^{(0)}(1,m) = \{x^{(0)}(k) \mid x^{(0)}(k) \geq 0, k = 1, 2, \dots, m\} \quad (1)$$

where raw matrix  $X^{(0)}(1,m)$  represents the non-negative original time series data.

**Step 2:** Create  $X^{(1)}(1,m)$  using a one-time accumulated generation operation (1-AGO)

$$X^{(1)}(1,m) = \left\{ x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), \mid x^{(1)}(k) \geq 0, k = 1, 2, \dots, m \right\} \quad (2)$$

**Step 3:** The nonlinear grey Bernoulli differential equation has following form,

$$\frac{d\hat{x}^{(1)}}{dt} + \alpha \hat{x}^{(1)} = \beta [x^{(1)}]^n, \quad n \neq 1, \quad (3)$$

where  $n$  is any real number but unity. The background value is  $\hat{x}^{(1)}(t) \equiv px^{(1)}(k) + (1-p)x^{(1)}(k+1) = z^{(1)}(k)$ , where  $p \in [0,1]$ .

**Step 4:** A discrete form of (3) is described as:

$$x^{(0)}(k) + \alpha z^{(1)}(k) = \beta [z^{(1)}(k)]^n, \quad k = 2, 3, 4, \dots, n \neq 1 \quad (4)$$

By using the least square method, we can obtain the above model parameters  $\alpha$  and  $\beta$ . They can be written as

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = (Z^T Z)^{-1} Z^T X, \quad (5)$$

where  $Z$  and  $X$  are defined as follows.

$$Z = \begin{bmatrix} -z^{(1)}(2) & [z^{(1)}(2)]^n \\ -z^{(1)}(3) & [z^{(1)}(3)]^n \\ \vdots & \vdots \\ -z^{(1)}(m) & [z^{(1)}(m)]^n \end{bmatrix}, \quad X = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(m) \end{bmatrix}, \quad (6)$$

**Step 5:** Suppose that  $x^{(0)}(1) = x^{(1)}(1)$ . Thus, the corresponding particular solution of (3) is

$$\hat{x}^{(1)}(k+1) = \left[ \left( x^{(0)}(1)^{1-n} - \frac{\beta}{\alpha} \right) e^{-\alpha(1-n)k} + \frac{\beta}{\alpha} \right]^{1/(1-n)}, \quad n \neq 1, \quad k = 1, 2, 3, \dots, \quad (7)$$

**Step 6:** Calculate  $\hat{x}_i^{(0)}(k+1)$  which is defined as

$$\hat{x}^{(0)}(k+1) = x^{(1)}(k+1) - x^{(1)}(k) \quad (8)$$

**Step 7:** In the grey model, the main criteria for assessing forecasting accuracy are relative percentage error (RPE) and the average relative percentage error (ARPE).

The RPE is defined as

$$\xi(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \quad (9)$$

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	2014/ Q2	2014/ Q3	2014/ Q4	2015/ Q1	2015/ Q2	2015/ Q3	2015/ Q4	2016/ Q1	2016/ Q2	n	p	(κ)%
	30.1459	30.0613	30.8555	31.5436	30.8413	32.011						
NGBM	30.1459	29.8724	30.8744	31.3433	31.5379	31.5646	31.4786	31.3127	31.0877	0.1	0.5	0.9955
ξ (κ)%	0	0.6283	-0.0611	0.6349	-2.2585	1.39456						
NNGBM	30.14598	29.99	30.961	31.3941	31.5501	31.5375	31.4123	31.2074	30.9442	0.1	0.6	0.9663
ξ (κ)%	0	0.2368	-0.3437	0.47404	-2.298	1.47923						

**Table 1:** The forecast performance of NGBM (1,1) and NNGBM (1,1) for forecasting NTD against USD.

	2014/Q2	2014/ Q3	2014/ Q4	2015/ Q1	2015/ Q2	2015/ Q3	2015/ Q4	2016/ Q1	2016/ Q2	n	p	ξ(k)%
Actual	4.8373	4.8763	5.0187	5.0578	4.9725	5.0778						
NGBM	4.8373	4.8619	4.9992	5.0474	5.0504	5.0262	4.9841	4.9297	4.8664	0.1	0.5	0.6947
ξ (κ)%	0	0.296	0.389	0.2056	-1.5656	1.0174						
NNGBM	4.8373	4.8785	5.0105	5.0526	5.0493	5.0187	4.9704	4.9099	4.8407	0.1	0.6	0.6033
ξ (κ)%	0	-0.043	0.1634	0.102	-1.5439	1.1634						

**Table 2:** The forecast performance of NGBM (1,1) and NNGBM (1,1) for forecasting NTD against CNY.

The ARPE is defined as

$$\zeta(k) = \frac{1}{m-1} \sum_{k=2}^m |\xi(k)| \quad (10)$$

The forecasting model with smaller ARPE is regarded as a better one.

**Step 8:** Consider the following optimization problem.

$$\text{Min}_{\{n\}} [\xi(n | p = 0.5, X^{(0)}(1, m))]$$

where

$$n \in \mathbb{R} - \{1\},$$

$$X^{(0)}(1, m), \text{ the original series is exogenous.} \quad (11)$$

By computer software, we can find out the optimal value of p. It is the solution of NGBM.

**Step 9:** Consider the following optimization problem.

$$\text{Min}_{\{n, p\}} [\xi(n, p | X^{(0)}(1, m))]$$

where

$$p \in [0, 1],$$

$$n \in \mathbb{R} - \{1\},$$

$$X^{(0)}(1, m), \text{ the original series is exogenous.} \quad (11)$$

Thus, Nash solution can be defined as follows [16].

**Definition 1:**  $(p_n^*, n_n^*)$  is a Nash solution of formula (11), if

$$\xi(n_n^*, p_n^* | X^{(0)}(1, m)) \leq \xi(n, p_n^* | X^{(0)}(1, m)) \quad \forall n \in \mathbb{R} - \{1\} \quad (12)$$

$$\xi(n_n^*, p_n^* | X^{(0)}(1, m)) \leq \xi(n_n^*, p | X^{(0)}(1, m)) \quad \forall p \in [0, 1]$$

We can use the iterated technology to find a set of  $(p_n^*, n_n^*)$  that minimizes the ARPE.

## Forecasting Exchange Rate of Currency

Taiwan's top two trading partners are America and China. Taiwan is an island. Thus, export and import plays an essential role. Both the international traders and investors care about any fluctuation of foreign exchange rate. Thus, to forecast the currency exchange rate is very important for Taiwan.

The quarterly data are obtained from the website of the Ministry of Economic Affairs of Taiwan. NTD is New Taiwan Dollar, USD is US dollar and CNY is Chinese Yuan. The foreign exchange rate is direct quote in Taiwan.

The quarterly data period is from Q2 of 2014 to Q3 of 2015. To compare the performance of NGBM and NNGBM, this study lists the forecast values of NGBM and NNGBM. The forecast results discussed as follows.

In the case of America, ARPE of NGBM is 0.9955%. NNGBM's ARPE is 0.9663% as shown in Table 1. Both n and p are variables NNGBM. Thus, there is no doubt that NNGBM performs better. By using NGBM, the forecasted values are 31.4786, 31.3127, and 31.0877 NTD against USD for Q4 of 2015, Q1 of 2016 and Q2 of 2016, respectively. By using NNGBM, the forecasted values are 31.4123, 31.2074, and 30.9442 for Q4 of 2015, Q1 of 2016 and Q2 of 2016, respectively. The results show that NTD will appreciate.

In the case of China, ARPE of NGBM is 0.6974% as shown in Table 2. NNGBM's ARPE is 0.6033%. By using NGBM, the forecasted values are 4.9841, 4.9297, and 4.8664 for Q4 of 2015, Q1 of 2016 and Q2 of 2016, respectively. By using NNGBM, the forecasted values are 4.9704, 4.9099, and 4.8407 NTD against Chinese Yuan for Q4 of 2015, Q1 of 2016 and Q2 of 2016, respectively. The results show that NTD will also appreciate.

## Conclusions

Export and import are very important for Taiwan based on its business model. Foreign exchange rate affects export and import significantly. Thus, how to accurately forecast the trends of foreign exchange rate is an essential lesson for international traders and investors.

This study adopts grey forecasting models, NGBM and NNGBM, to forecast the foreign exchange rate of Taiwan's two major trading partners, America and China. The exchange rate is direct quote type, including NTD against USD and NTD against CNY. The results show that the trends of NTD exchange rate will slightly appreciate in the future which is totally opposite to the current public imagine after America announce to increase interest rate. The international traders and investors can take the conclusion as a reference in order to avoid currency exchange lost.

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