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Application of Multi-Objective Optimization by S and R* Optimal Combination Criteria to Determine the Freeze Drying Mode of *Penaeus monodon*

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Abstract

The establishment of the freeze drying mode of *Penaeus monodon* was based on the solution to multi-objective optimization problem. Experiments was carried out to set up the objective functions describing the influence of technological factors (temperature and pressure of freeze drying chamber, and time of freeze drying) to the freeze drying process. The restricted area method with R*(Z) optimal combination criterion was applied to solve the multi-objective optimization problem, determining the optimal technological mode of freeze drying process (correspondingly 33.96°C, 0.008 mmHg and 13.21h) in order that the objective functions reached the minimum value in terms of the finished product, including the energy consumption of 68.77 kWh/kg, the residual water content of 4.76%, the anti-rehydration capacity of 7.82%, the volume contraction of 8.82% and the loss of vitamin C of 1.91%.

Keywords: Multi-objective optimization; Freeze drying; *Penaeus monodon*

Introduction

Freeze drying is a complicated technique including three consecutive main stages which are represented in the process diagram of Figure 17. The first stage is freezing the material; the second and the third stage are respectively the sublimation drying and vacuum drying. Both these final stages determine the quality of the product [6,7].

According to Figure 1, the determination of the freeze drying mode required the outputs to reach the minimal level, including the energy consumption per weight ($\mathbf{y_1}$,kWh/kg), the residual water content ($\mathbf{y_2}$,%), the anti-rehydration capacity ($\mathbf{y_3}$,%), the volume contraction ($\mathbf{y_4}$,%) and the loss of vitamin C ($\mathbf{y_5}$,%) of the freeze-dried product (finished product). It should be emphasized that these 5 outputs were affected by the 3 technological factors: temperature of freeze drying chamber ($\mathbf{Z_1}$°C), pressure of freeze drying chamber ($\mathbf{Z_2}$,mmHg) and time of freeze drying ($\mathbf{Z_3}$,h).

However, the simultaneous consideration of all these outputs above to reach the minimal level resulted in the standard solution to the multi-objective optimization problem [4,11]. This problem regularly appears in reality and in different fields. The answer to the multi-objective optimization problem was found in the case of the application of the R*(Z) optimal combination criterion (also known as the restricted area method) for the freeze drying process of *Penaeus monodon*. By solving the heat and mass transfer model of the freeze drying dehydration [2,3], the multi-objective optimization results were used to establish the freeze drying mode of *Penaeus monodon* which was the closest to the utopian point but the furthest from the restricted area C, [4,11,12].

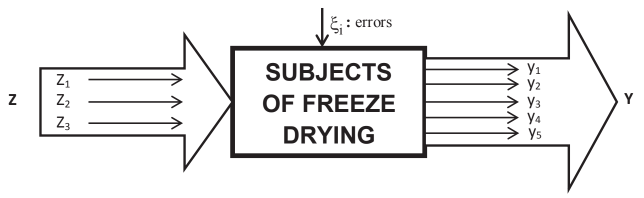

Figure 1: Diagram of subjects of freeze drying.

The fundamental of multi-objective optimization by S And R* optimal combination criteria

Basic concepts: The technological subjects including m objective functions $f_1(Z)$, $f_2(Z)$, ..., $f_m(Z)$ form the vector of these functions f(Z) = $\{f_j(Z)\} = \{f_1(Z), f_2(Z), ..., f_m(Z)\}$, where j = 1 ÷ m. Every objective function $f_j(Z)$ will be affected by n variables Z_1, Z_2, ..., Z_n which form the Z variable vector $Z = \{Z_i\} = (Z_1, Z_2, ..., Z_n)$, where i = 1 ÷ n. These variables vary in the identified domain Ω_Z and the function values form the domain of the objective function Ω_f (in the two-objective optimization problem, the domain can be performed geometrically in the closed curve **A – f(ZS) – f(ZR) – B – N – M**, Figure 2), [4,11,12].

Every objective function $f_j(Z)$ with Z variable vector $Z = \{Z_i\} = (Z_1, Z_2, ..., Z_n)$, where i = 1 ÷ n, is considered as the one-objective optimization problem. Hence, the m-objective optimization problem can be simply transformed into the problem to find the minimum value for the set of m one-objective optimization problems, [4,11,12]:

$$f_{jmin} = f_j(Z_1^{j\,opt}, Z_2^{j\,opt}, ..., Z_n^{j\,opt}) = \mathrm{Min}\, f_j(Z_1, Z_2, ..., Z_n) \qquad (1)$$

$$Z = \{Z_i\} = (Z_1, Z_2, ..., Z_n) \in \Omega_Z \qquad (2)$$

$$j = 1 \div m;\ i = 1 \div n \qquad (3)$$

The utopian plan and the utopian effect: If the variable vector $Z^{UT} = \{Z_i^{UT}\} = (Z_1^{UT}, Z_2^{UT}, ..., Z_n^{UT}) \in \Omega_Z$ is the test for all one-objective

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optimization problems (1) + (2) + (3), it means that $Z_i^{UT} = Z_i^{jopt}$ with i = 1 ÷ n. Thus, Z_i^{UT} is called the utopian plan or the utopian test of the m-objective optimization problem, [4,11,12].

In reality, Z_i^{UT} does not exist. However, every one-objective optimization problem (1) + (2) + (3) has its own f_{jmin} (with j = 1 ÷ m) respectively, so $f^{UT} = (f_{1min}, f_{2min}, ..., f_{mmin})$ does exist. Then, $f^{UT} = (f_{1min}, f_{2min}, ..., f_{mmin})$ is called the utopian effect or the utopian point. According to Figure 2, the utopian point f^{UT} of the two-objetive optimization problem exists but lies outside the identified domain Ω_f, i.e. the utopian test does not exist.

The dominant plan and the dominated plan: It is assumed that there are two variable vectors ZQ = {ZQ_i} and ZV = {ZV_i} with i = 1 ÷ n. Then, there exist respectivly two function vectors f(ZQ) = {f_j(ZQ)} and f(ZV) = {f_j(ZV)} with j = 1 ÷ m.

If with all j: $f_j(ZQ) < f_j(ZV)$, ZQ is called the dominant plan (or the dominant test) over ZV, symbolizing: ZQ '>' ZV; and ZV is called the dominated plan (or the dominated test), symbolizing: ZV '<' ZQ, [4,11,12].

The optimal paréto plan

The ZP plan is called the optimal Paréto plan in condition that ZP cannot be dominated by any other plans dependable on the identified domain Ω_Z. Then, f(ZP) would be called an optimal Paréto effect in the set of the optimal Paréto effects ΩfP. Figure 2 performs the set of the optimal Paréto effects ΩfP as the curve **A – f(ZS) – f(ZR) – B,** [4,11,12].

Theorem 1: (Theorem Paréto): If the multi-objective optimization problem has the test which is the so-called optimal one according to some definition, this test received has to be the optimal Paréto plan without the dependence on the chosen definition, [4,11,12].

Proof: If optimal Z test of the multi-objective optimization problem is not the optimal Paréto plan, it is certainly able to find at least one plan dominating Z. This proves that Z can not be recognized as the optimal test and leads to the conflict with the assumption that Z is the optimal test. Thus, Z must be a plan over which cannot be dominant, i.e. the Paréto plan is optimal.

Therefore, one test of the multi-objective optimization problem (1) + (2) + (3) found by any method, to be recognized as the optimal by the method chosen, must in advance be certified as the optimal Paréto plan.

Multi-objective optimization by the utopian point method with S(Z) optimal combination criterion

Considering the m-objective optimization problem (1) + (2) + (3): The optimal values $f_{1min}, f_{2min}, ..., f_{mmin}$ can be determined after solving each problems, and the fact that the utopian test (the test for the whole system) does not exist still identifies the utopian point $f^{UT} = (f_{1min}, f_{2min}, ..., f_{mmin})$. A S(Z) optimal combination criterion is defined by the following expression [4,12]:

$$S(Z) = \sqrt{\left[\sum_{j=1}^{m} s_j^2(Z)\right]} = \sqrt{\left[\sum_{j=1}^{m}\left(f_j(Z) - f_{j\,min}\right)^2\right]} \quad (4)$$

It is obvious that S(Z) is the distance from f(Z) to f^{UT}. Choosing S(Z) optimal combination criterion as an objective function, the m-objective optimization problem are restated as:

Find ZS = ($Z_1S, Z_2S, ..., Z_nS$) ∈ Ω_Z in order that the objective function S(Z) reaches the minimum value:

$$S_{min} = S(ZS) = \min\{S(Z)\} = \min\left\{\sqrt{\left[\sum_{j=1}^{m}\left(f_j(Z) - f_{j\,min}\right)^2\right]}\right\} \quad (5)$$

$\forall Z = (Z_1, ..., Z_n)\ \Omega_Z$

Theorem 2: (Theorem Paréto): If ZS of the optimization problem (5) does exist, ZS is the optimal Paréto test of the m-objective optimization problem (1) + (2) + (3), [4,11].

Proof: It is assumed that ZS is not optimal Paréto test. Then, it will be found that ZS^* is dominant over ZS. By definition, ZS^* must have at least an effect $f_k(ZS^*)$, where $1 \le k \le m$ in order that $f_k(ZS^*) < f_k(ZS)$. As a result, $S(ZS^*) < S(ZS)$. This contradicts the assumption that ZS is the optimal test (5). Hence, there does not exist any other dominant tests over ZS. Therefore, ZS must be an optimal Paréto test [4,11].

Symbol: f(ZS) = fPS = ($f_1PS, f_2PS, ..., f_mPS$). With the utopian point method (i.e. the m-objective optimization problem convert into the S optimal combination criterion), the optimal Paréto test ZS will be found to have the optimal Paréto effect f(ZS) = fPS closest to the utopian point $f^{UT} = (f_{1min}, f_{2min}, ..., f_{mmin})$. The case m = 2 (two objectives) is illustrated in Figure 2.

Optimizing the multi-objective functions by the restricted area method with R*(Z) optimal combination criterion

In fact, every objective function $f_j(Z)$ is restricted by the conditions set up by technology. Such as:

a) Case 1: The obligatory conditions $f_j(Z) < C_j$, ∀j = 1 ÷ m, $\forall Z \in \Omega_Z$ (6)

From (6), the restricted area would be made: C = {$f_j(Z) \ge C_j$}, with $f_j(Z)$ (7)

The restricted area method suggests the solution to the m-objective optimization problem (1) + (2) + (3) by R*(Z) optimal combination criterion, defined as, [4,12]:

$$R^*(Z) = \sqrt[m]{r_1(Z).\ r_2(Z)...\ r_m(Z)} = \sqrt[m]{\prod_{j=1}^{m} r_j(Z)} \quad (8)$$

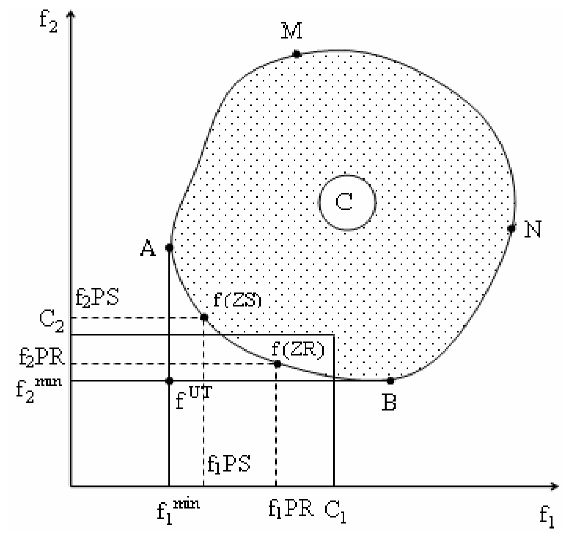

Figure 2: Dimension of objective functions of the two-objective optimization problem.

With $r_j(Z)=\left(\dfrac{C_j-f_j(Z)}{C_j-f_{jmin}}\right)$ when $f_j(Z)<C_j$ (9)

$r_j(Z)=0$ when $f_j(Z)\geq C_j$ (10)

According to (9), if $f_j(Z) \to f_{j\,min}$ and $\forall f_j(Z) < C_j$, $r_j(Z) \to r_{jmax} = 1$.

By choosing R*(Z) as the objective function, the m-objective optimization problem is restated as:

Find ZR = (Z_1R, Z_2R... Z_nR) $\in \Omega_Z$ in order that R*(Z) reaches the maximum value.

$$R^*_{max}=R^*(ZR)=\max\left\{R^*(Z)\right\}=\max\left\{\sqrt[m]{\left[\prod_{j=1}^{m} r_j(Z)\right]}\right\} \quad (11)$$

$\forall Z = (Z_1... Z_n) \in \Omega_Z$

From (9), it can be seen: $0 \leq R^*(ZR) \leq 1$. If R*(ZR) = 1, ZR = Z^{UT} – the utopian test. If R*(ZR) = 0, one of the values of $f_j(Z)$ violates (6), which means that $f_j(Z)$ belongs to the restricted area C (7).

Theorem 3: If the multi-objective optimization problem (11) has it own test ZR, this test ZR is also the optimal Paréto test of the m-objective optimization problem (1) + (2) + (3), [4,11].

Symbol: f(ZR) = fPR = (f_1PR, f_2PR, ..., f_mPR). With the optimal ZR, the optimal Paréto effect fPR = (f_1PR, f_2PR... f_mPR) would be the closest to the utopian point and the furthest from the restricted area C.

b) Case 2: The obligatory conditions $a_j < f_j(Z) < b_j$, $\forall j = 1 \div m$, $\forall Z \in \Omega_Z$ (12)

Then, setting a new objective function:

$$I_j(Z)=\left[f_j(Z)-\frac{a_j+b_j}{2}\right]^2 \quad (13)$$

From (13), it can be seen: if $I_j(Z) \to I_{j\,min} = 0$, $f_j(Z) \to f_{jmin} = (a_j + b_j)/2$, $\forall Z\ \Omega_Z$.

When $\dfrac{a_j+b_j}{2} < f_j(Z) < b_j$, then

$$0 < \left[f_j(Z)-\frac{a_j+b_j}{2}\right] < \frac{b_j-a_j}{2} \quad (14)$$

When $a_j < f_j(Z) \leq \dfrac{a_j+b_j}{2}$, then

$$-\frac{b_j-a_j}{2} < \left[f_j(Z)-\frac{a_j+b_j}{2}\right] \leq 0 \quad (15)$$

From (14) and (15), we have: $0 \leq \left|f_j(Z)-\dfrac{a_j+b_j}{2}\right| < \dfrac{b_j-a_j}{2}$ (16)

Combining (13) and (16), we have:

$$I_j(Z)=\left[f_j(Z)-\frac{a_j+b_j}{2}\right]^2 < \left(\frac{b_j-a_j}{2}\right)^2 = C_j \quad (17)$$

From (17), the restricted area of the new objective function is defined as:

C = {$I_j(Z) \geq C_j$}, with $C_j=\left(\dfrac{b_j-a_j}{2}\right)^2$ (18)

As a result of the fact that the restricted area (18) is the same as (6), the multi-objective optimization problem is solved similarly as case 1.

Materials and Methods

Materials

The material used for the freeze drying experiment was *Penaeus monodon* which had the approximate weight of (41÷50) prawns/pound, with the size coefficient K = 11, [1]. Blanched at 70°C during (15 ÷ 30) seconds, the material was processed minimally by peeling off the shells and cutting off the heads.

Apparatus

- The freeze drying machine DS-3 was controlled automatically by computer (Figure 3)

- The tools determining these factors such as energy consumption, residual water content, rehydration capacity, volume contraction, and loss of vitamin C could be referred to by [1,5].

Methods

Determining the energy consumption (**y_1, kWh/kg** product) for 1 kg finished product by Watt meter, [1,5].

$$y_1=\frac{U.I.\tau.\cos\varphi}{G} \quad (19)$$

Where: G [kg] – mass finished product;

U [V] – number of Voltmeter;

I [A] – munber of Amperemeter;

τ [s] – times

Determining the residual water content of the finished product (**y_2, %**) by mass sensor through computer, [1,5].

$$y_2=100-\frac{G_i}{G_e}(100-W_i) \quad (20)$$

Determining the anti-rehydration capacity of the finished product

Figure 3: The freeze drying system DS-3 with the auto-freezing (-50 ÷ - 45)°C.

(**y_3, %**) indirectly by IR [%], which is the rehydration capacity of the finished product: $y_3 = 100 - IR$, [1,5].

$$IR = \frac{G_1 - G_e}{G_i - G_e}.100\% \tag{21}$$

$$y_3 = 100 - IR = \frac{G_i - G_1}{G_i - G_e}100\% \tag{22}$$

where: G_i [kg] – weight of the initial material used for freeze drying; G_e [kg] - weight of the finished product, G_1 [kg] – weight of the finished product which was soaked into the water at 25^{0}C until the constant mass (the saturation of the water content), W_i [%] – initial water content of the material.

The ideal rehyration capacity of the product means that the in-water content is equal to the out-water content of the product, i.e. $G_1 = G_i$ and $IR_{max} = 1 = 100\%$, $y_{3min} = 0$. In fact, $y_3 > 0$.

Determining the volume contraction (**y_4, %**) by the volume of the initial material (V_1) and of the finished product after freeze drying (V_2), [1,5]:

$$y_4 = \frac{V_1 - V_2}{V_1}100\% = \frac{\Delta V}{V_1}100\% \tag{23}$$

The fact that the surface of the product is not rough and not contracted means $y_{4min} = 0$. In fact, $y_4 > 0$.

Determining the loss of vitamin C of the finished product (**y_5,%**) according to the method TCVN 4715 – 89, [1,5].

$$y_5 = \frac{m_1 - m_2}{m_1}100\% = \frac{\Delta m}{m_1}100\% \tag{24}$$

where: m_1 and m_2 [mg%] – the vitamin C content of the material before and after freeze drying respectively. The fact that the product achieves the best quality means $y_{5min} = 0$. In fact, $y_5>0$.

1. Determining the temperature and pressure by sensors in the machine.
2. Quadratic orthogonal experimental planning method, [4,10].
3. Establishing and solving 5-objective optimization problem by the restricted area method.

Results and Discussion

Establishing the constituent objective functions of the multi-objective problem

The constituent objective functions of the optimal freeze drying (**y_1, y_2, y_3, y_4, y_5**) depended on the parameters, including: temperature of freeze drying chamber (**Z_1, °C**), pressure of freeze drying chamber (**Z_2, mmHg**), time of freeze drying (**Z_3, h**), and were determined by the experimental planning method with the quadratic orthogonal experimental matrix ($k = 3$, $n_0 = 4$, carrying out 18 experiments). These variables x_1, x_2, x_3 were coded variables of Z_1, Z_2, Z_3. The value of the star point $\alpha = 1.414$.

The experimental parameters investigated by establishing and solving the heat and mass transfer model in the freeze drying [2,3] to reach the suitable residual water content of the finished product were summarized in Table 1.

The values of the objective functions y_1, y_2, y_3, y_4, y_5 were shown in Table 2 by carrying out the experiments with these parameters in Table 1.

The regression equations below were obtained after processing the experimental data, calculating the coefficients, testing the significance of the coefficients by the Student test, and testing the regression equations for the fitness of the experimental results by Fisher test.

$$y_1 = f_1(x_1,x_2,x_3) = 75.758 + 1.883x_1 + 10.167x_3 - 2.491x_2^2 + 2.487x_3^2 \tag{25}$$

$$y_2 = f_2(x_1,x_2,x_3) = 4.17 - 0.39x_1 - 0.614x_3{}^9$$
$$- 0.226x_1x_3 + 0.27.x_1^2 + 0.245x_2^2 + 0.142x_3^2 \tag{26}$$

$$y_3 = f_3(x_1,x_2,x_3) = 7.578 + 2.069x_1 + 0.575x_2 + 1.187x_3 + 0.851x_1^2 + 1.205x_3^2 \tag{27}$$

$$y_4 = f_4(x_1,x_2,x_3) = 8.307 + 1.45x_1 + 0.789x_2 + 0.76x_3$$
$$- 0.484x_2x_3 + 0.429x_1^2 + 0.607x_2^2 \tag{28}$$

$$y_5 = f_5(x_1,x_2,x_3) = 2.363 + 0.393x_1 + 0.375x_2 + 0.205x_3 - 0.178x_3^2 \tag{29}$$

Solving the one-objective optimization problems

These one-objective optimization problems were found to achieve: $y_{1min} = \min f_1(x_1, x_2, x_3)$; $y_{2min} = \min f_2(x_1, x_2, x_3)$; $y_{3min} = \min f_3(x_1, x_2, x_3)$; $y_{4min} = \min f_4(x_1, x_2, x_3)$; $y_{5min} = \min f_5(x_1, x_2, x_3)$, with the identified domain $\Omega_x = \{-1.414 \le x_1, x_2, x_3 \le 1.414\}$. By using the Excel – Solver software, the results of the optimal parameters of every objective function (25), (26), (27), (28) and (29) limited in the experimental domain were summarized in Table 3, [4,11,12]:

According to the Table 3, the utopian points were indentified: $f^{UT} = (f_{1min}, f_{2min}, f_{3min}, f_{4min}, f_{5min}) = (58.71, 3.119, 5.215, 5.163, 0.7497)$. However, the utopian plan did not exist, because of $x^{jopt} = (x_1^{jopt}, x_2^{jopt}, x_3^{jopt}) \ne x^{kopt} = (x_1^{kopt}, x_2^{kopt}, x_3^{kopt})$ with $j, k = 1 \div 5$, $j \ne k$.

Solving the multi-objective optimization problem by the restricted area method

The purpose of the experiment was to reach the targets of the freeze drying process which were expressed by 5 regression equations (25), (26), (27), (28) and (29), but the tests satisfying all function values (y_{1min}, y_{2min}, y_{3min}, y_{4min}, y_{5min}) could not be found. Hence, the idea of the multi-objective optimization problem was to find the optimal Paréto test for $yPR = (y_1PS, y_2PR, y_3PR, y_4PR, y_5PR)$ closest to the utopian point and the furthest from the restricted area, but $y_j = y_j(x) = f_j(x_1, x_2, x_3)$ must satisfy technological conditions with the initial requirements, [4,11,12]:

$$y_1 < C_1 = 86.21;\ 2 = a < y_2 < b = 6;\ y_3 < C_3 = 11.56;$$
$$y_4 < C_4 = 10.91;\ y_5 < C_5 = 2.98 \tag{30}$$

Setting the new objective functions as the followings:

$$I_1(x) = y_1(x) \text{ with } I_1(x) < C_1 = 86.21 \tag{31a}$$

$$I_2(x) = [y_2(x) - (a + b)/2]^2 = [y_2(x) - 4]^2 \text{ with } I_2(x) < C_2 = [(b - a)/2]^2 = 4 \tag{31b}$$

$$I_3(x) = y_3(x) \text{ with } I_3(x) < C_3 = 11.56 \tag{31c}$$

$$I_4(x) = y_4(x) \text{ with } I_4(x) < C_4 = 10.91 \tag{31d}$$

$$I_5(x) = y_5(x) \text{ with } I_5(x) < C_5 = 2.98 \tag{31e}$$

Parameters	Levels					Deviation ΔZ_i
	$-\alpha$ (-1.414)	Low (-1)	Central (0)	High (+1)	$+\alpha$ (1.414)	
Z_1 [^{0}C]	20.102	23	30	37	39.898	7
Z_2 [mmHg]	0.008	0.094	0.3	0.507	0.592	0.2065
Z_3 [h]	11.172	12	14	16	16.828	2

Table 1: Parameter level design.

N		X_0	X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	X_1^2-2/3	X_2^2-2/3	X_3^2-2/3	y_1	Y_2	Y_3	Y_4	Y_5
2^k	1	1	1	1	1	1	1	1	0.333	0.333	0.333	80.42	3.72	12.45	10.91	3.253
	2	1	-1	1	1	-1	-1	1	0.333	0.333	0.333	83.51	4.81	10.01	8.978	2.554
	3	1	1	-1	1	-1	1	-1	0.333	0.333	0.333	89.71	3.51	11.71	10.92	2.161
	4	1	-1	-1	1	1	-1	-1	0.333	0.333	0.333	84.34	5.04	7.73	8.525	1.192
	5	1	1	1	-1	1	-1	-1	0.333	0.333	0.333	66.52	5.36	10.94	10.31	2.581
	6	1	-1	1	-1	-1	1	-1	0.333	0.333	0.333	65.78	5.69	7.78	8.671	2.247
	7	1	1	-1	-1	-1	-1	1	0.333	0.333	0.333	65.89	5.39	10.09	8.478	1.628
	8	1	-1	-1	-1	1	1	1	0.333	0.333	0.333	64.82	5.87	5.97	6.19	1.287
2k	9	1	1.414	0	0	0	0	0	1.333	-0.667	-0.667	81.24	4.08	13.32	12.85	3.388
	10	1	-1.41	0	0	0	0	0	1.333	-0.667	-0.667	68.15	4.96	5.45	6.379	1.707
	11	1	0	1.414	0	0	0	0	-0.667	1.333	-0.667	70.62	4.93	8.61	11.63	2.748
	12	1	0	-1.41	0	0	0	0	-0.667	1.333	-0.667	73.73	4.01	7.75	8.307	2.653
	13	1	0	0	1.41	0	0	0	-0.667	-0.667	1.333	98.71	3.51	12.61	10.13	2.521
	14	1	0	0	-1.41	0	0	0	-0.667	-0.667	1.333	65.45	5.02	7.57	7.697	1.787
n_0	15	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	77.31	4.41	7.08	7.858	2.281
	16	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	75.52	4.21	7.12	8.018	2.263
	17	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	78.24	4.29	7.16	8.449	2.127
	18	1	0	0	0	0	0	0	-0.667	-0.667	-0.667	73.63	4.13	7.74	7.659	2.011

Table 2: The orthogonal experimental matrix level 2, k = 3, n_0 = 4.

Drying material	j	y_{jmin}	$x_1^{j\,opt}$	$x_2^{j\,opt}$	$x_3^{j\,opt}$
Penaeus monodon	1	58.71	-1.414	1.414	-1.414
	2	3.119	1.314	-6.11E-08	1.414
	3	5.215	-1.216	-1.414	-0.4925
	4	5.163	-1.394	-1.2137	-1.405
	5	0.7497	-1.241	-1.414	-1.341

Table 3: Minimum tests of each one-objective optimization problem.

$$\forall x = (x_1, x_2, x_3) \in \Omega_x \tag{31f}$$

From the system of equations (31a), (31b), (31c), (31d), (31e), (31f) and table 3, it can be easily found:

$I_{1min} = y_{1min} = 58.71$;

$I_{2min} = 0.00$;

$I_{3min} = y_{3min} = 5.215$;

$I_{4min} = y_{4min} = 5.163$;

$I_{5min} = y_{5min} = 0.7497$;

Establishing the R*-objective combination function $R^*(I_1, I_2, I_3, I_4, I_5) = R^*(y_1, y_2, y_3, y_4, y_5) = R^*(x_1, x_2, x_3) = R^*(x)$ as the followings [4,11, 12]:

$$\begin{cases} R^*(x) = R^*(x_1, x_2, x_3) = \sqrt[5]{\prod_{j=1}^{5} r_j(x_1, x_2, x_3)} = \sqrt[5]{\prod_{j=1}^{5} r_j(x)} \\ \Omega_x = \{-1{,}414 \le x_1, x_2, x_3 \le 1{,}414\};\ x = (x_1, x_2, x_3) \end{cases} \tag{32}$$

With: $$r_j(x) = \left(\frac{C_j - I_j(x)}{C_j - I_{jmin}}\right) \text{ when } I_j(x) < C_j \tag{33}$$

$$r_j(x) = 0 \text{ when } I_j(x) \ge C_j \tag{34}$$

The five-objective optimization problem needed to indentify xR = $(x_1R, x_2R, x_3R) \in \Omega_x$ in order that $R^*(x_1R, x_2R, x_3R) = \mathrm{Max}\{R^*(x_1, x_2, x_3)\}$. The maximum value of (32) was determined by using the Excel – Solver sofware:

$R^*(x)_{max} = \mathrm{Max}\{R^*(x_1, x_2, x_3)\} = R^*(x_1R, x_2R, x_3R) = 0.876$

With: $x_1R = 0.5659$; $x_2R = -1.414$; $x_3R = -0.3936$; Then, transforming into real variables:

$Z_1^{opt} = 33.96^\circ$C; $Z_2^{opt} = 0.008$mmHg; $Z_3^{opt} = 13.21$h

Substituting x_1R, x_2R, x_3R into these equations (31a), (31b), (31c), (31d), (31e) and (31f), the results were obtained as:

$I_1PR = 68.226$;

$I_2PR = 0, 7056$;

$I_3PR = 7.928$;

$I_4PR = 8.795$;

$I_5PR = 1.9$

Substituting x_1R, x_2R, x_3R into these equations (25), (26), (27), (28) and (29), the results were obtained as:

$y_1PR = 68.226$;

$y_2PR = 4.84$;

$y_3PR = 7.928$;

$y_4PR = 8.795$;

$y_5PR = 1.947$

The rehydration capacity of the product was determined as:

$IR = 100 - y_3PR = 92.072$

where: $xR = (x_1R, x_2R, x_3R)$ called optimal Paréto test; $yPR = (y_1PR, y_2PR, y_3PR, y_4PR, y_5PR)$ called the optimal Paréto effect.

As a result, through the calculation from the experimental models (25), (26), (27), (28) and (29), the parameters of the freeze drying process which satisfied the maximum R*-Optimal combination criterion were determined as: temperature of freeze drying chamber was Z_1^{opt} = 33.96⁰C, pressure of freeze drying chamber was Z_2^{opt} = 0.008mmHg, time of freeze drying was Z_3^{opt} =13.21h. The total energy consumption per weight of the product was y_1PR = 68.22 kWh/kg; the residual water content of the product was y_2PR = 4.84% (acceptable with the initial requirements of 2 ÷ 6 %); the rehydration capacity of the product was IR = 100 - y_3PR = 92.07 %; the volume contraction of the product was y_4 PR = 8.79 % and the loss of vitamin C of the product was y_5PR = 1.94%. Compared with the experimental results from the table 2, these results above were suitable and satisfying with the objectives of the problem.

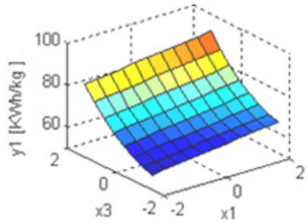

Figure 4: Energy consumption, x2 = -1.414.

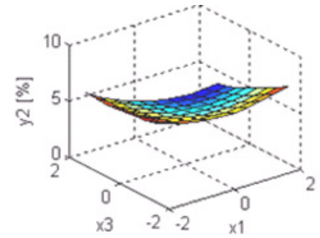

Figure 5: Water content of product, x2 = -1.414.

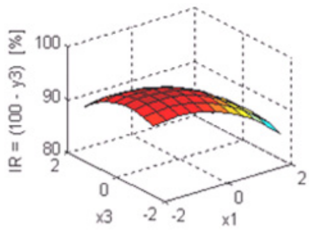

Figure 6: Rehudration capacity, x2 = -1.414.

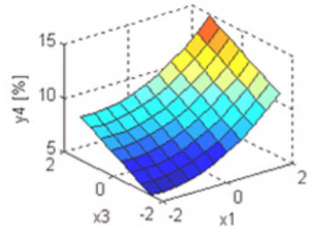

Figure 7: Volume contraction, x2 = -1.414.

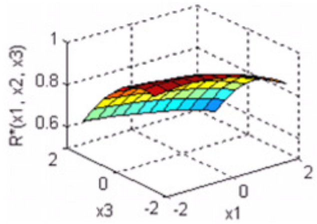

Figure 8: Loss of vitamine C, x2 = -1.414.

Experiment to test the results of multi-objective optimization problem

Carrying out the freeze drying process of *Penaeus monodon* at the optimal Paréto test: temperature of freeze drying chamber of Z_1^{opt} = 33.96⁰C, pressure of freeze drying chamber of Z_2^{opt} = 0.008mmHg, and time of freeze drying Z_3^{opt} = 13.21 hours, the experimental results were determined as: the energy consumption per product weight of y_1 = 68.77 kWh/kg, the residual water content of y_2 = 4.76%, the rehydration capacity of IR = 100 - y_3 = 92.17% (the anti-rehydration capacity of y_3 = 7.82%), the volume contraction of y_4 = 8.82% and the loss of vitamin C of y_5 = 1.91%.

Consequently, it was very noticeable that the results from the optimization problems of the freeze drying process had the fitness for the experimental results.

When the pressure of freeze drying chamber was fixed: x_2 = -1.414, respectively Z_2 = 0.008 mmHg, the relationship between y_1, y_2, y_3, y_4, y_5 and S combination function with 2 variables x_1, x_3 was performed geometrically in 3D (Figures 4, 5, 6, 7, 8, 9). When x_3 was fixed with constant values, the variation of x_1 was shown in Figures 10, 11, 12, 13, 14, 15.

Establishing the freeze drying mode

By the optimal Paréto test: the temperature, pressure of freeze drying chamber and the total drying time of 2 stages (2 and 3) respectively $Z_1^{opt} = T_f$ = 33.96⁰C, $Z_2^{opt} = P_m$ = 0.008mmHg and Z_3^{opt} = 13.21h, and by the temperature of the freezing chamber T_e = -45⁰C, the crystallization temperature of the water in *Penaeus monodon* T_{kt} = -1.21˚C [1], and the thermo-physical parameters of *Penaeus monodon* [1,13,14], the heat transfer model of the freezing process [2] and the heat and mass transfer model of the freeze drying process [3,13,14] were solved to determine the optimal freezing temperature, the time of stage 1 (**τ_1, h**), the time of sublimation drying at stage 2 (**τ_2, h**), and the time of vacuum drying at stage 3 (**τ_3, h**). The results were shown in Figure 17.

Stage 1: The frozen material was carried out at the optimal technological mode [2,3,13] (water completely frozen in the material) with the temperature of the freezing chamber of T_e = -45°C, the surface material temperature of T_s = -34°C, the central material temerature of T_c = -11.78°C and the average material temperature of T_m = -25.11°C, the frozen time of τ_1 = 2.5h.

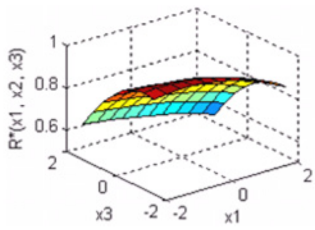

Figure 9: Conbination function R¹, x2 = -1.414.

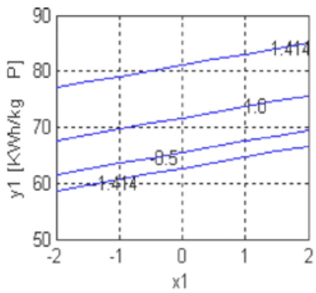

Figure 10: Energy consumption, x2 = -1.414.

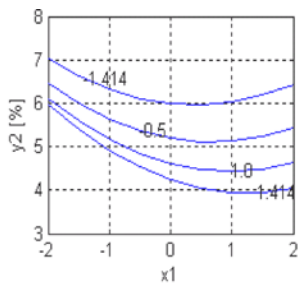

Figure 11: Water content of product, x2 = -1.414.

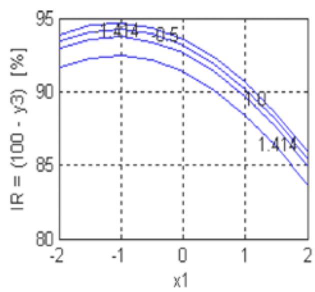

Figure 12: Rehudration capacity, x2 = -1.414.

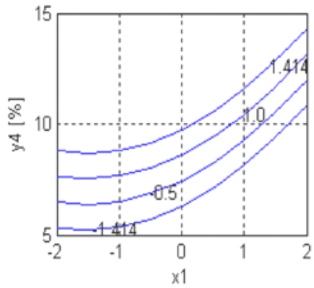

Figure 13: Volume contraction, x2 = -1.414.

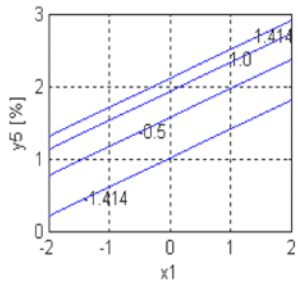

Figure 14: Loss of vitamine C, x2 = -1.414.

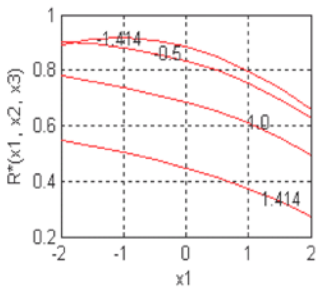

Figure 15: Conbination function R¹, x2 = -1.414.

Figure 16: The freeze-dried *Penaeus monodon*.

Stage 2: The sublimation drying was carried out with the temperature of freeze drying chamber of $Z_1^{opt} = T_f$ = 33.96°C, the pressure of freeze drying chamber of $Z_2^{opt} = P_m$ = 0.008mmHg, the sublimation temperature of the water frozen in the material of T_m = -25.11°C, the central material temperature of $T_c \leq$ -1.21°C, and the drying time of τ_2 =11.46h, [2,3,13].

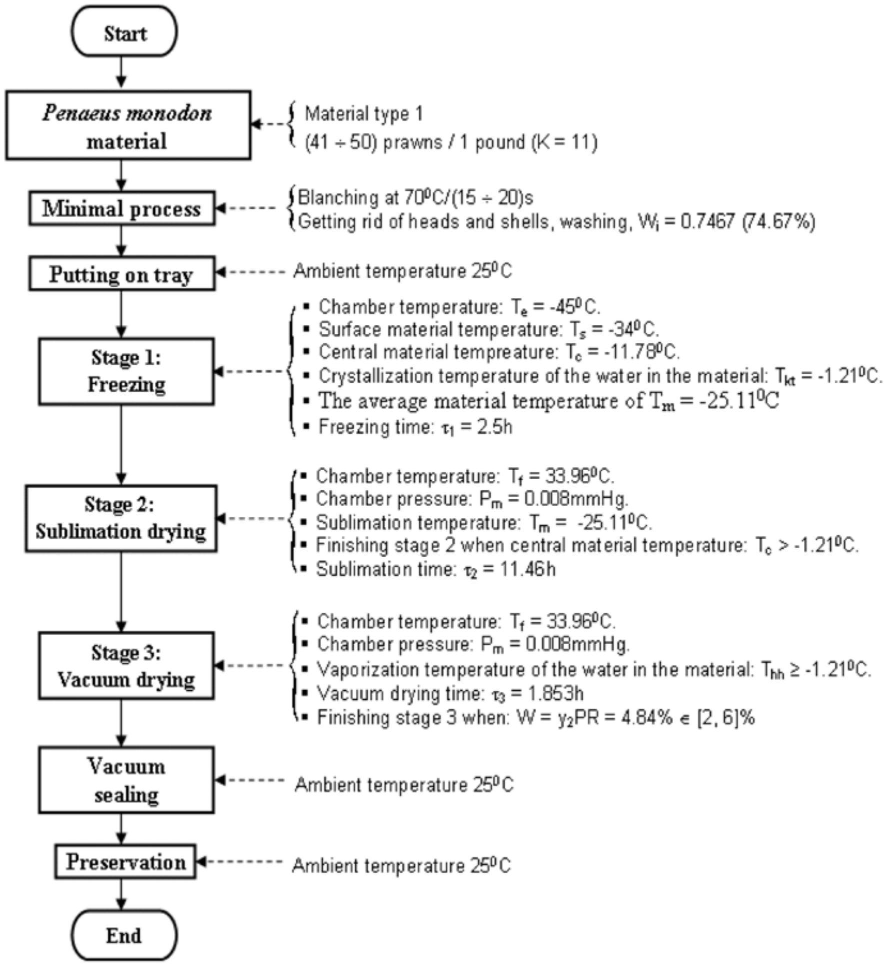

Figure 17: The freeze drying process diagram of *Penaeus monodon*.

Stage 3: The vacuum drying was carried out with the temperature of vacuum drying chamber of $Z_1^{opt} = T_f = 33.96^{\circ}C$, the pressure of vacuum drying chamber of $Z_2^{opt} = P_m = 0.008mmHg$, the central material temperature of $T_c > -1.21^{\circ}C$, and the drying time of $\tau_3 = 1.75h$. The residual water content of the finished product was $W = y_2PR = 4.84\%$, [2,3,13].

Thus, the total drying time of 2 stage was $Z_3^{opt} = \tau_2 + \tau_3 = 13.21h$, the total time of 3 stage was $\tau = \tau_1 + \tau_2 + \tau_3 = 2.5 + 11.46 + 1.75 = 15.71h$

The quality of the finished product was displayed in Figure 16 when all the parameters of the freeze drying process were applied as the above. The energy consumption per weight was the same as the determined in priority.

Conclusion

The results showed that the restricted area method with R*(Z) optimal combination criterion solving the five-objective optimization problem (y_1, y_2, y_3, y_4, y_5) determined three optimal technological parameters: Z_1^{opt} - temperature of freeze drying chamber, Z_2^{opt} - pressure of freeze drying chamber, Z_3^{opt} - time of freeze drying, which were entirely consistent with the experiments, [4,12].

The results also demonstrated that optimal Paréto test ZR = ($Z_1^{opt} = 33.96^0C$; $Z_2^{opt} = 0.008mmHg$; $Z_3^{opt} = 13.21h$) for optimal Paréto effect yPR = ($y_1PR = 68.226$; $y_2PR = 4.84$; $y_3PR = 7.928$; $y_4PR = 8.795$; $y_5PR = 1.947$) was the closest to the utopian point but the furthest from the restricted area.

The freeze drying process was researched systematically, combining mathematical method and experimental design.

The regression equations (25), (26), (27), (28) and (29) obtained from the experiments were the experimental statistical models which could well describe the impact of the freeze drying chamber temperature, the freeze drying chamber pressure and the freeze drying time on the energy consumption, the residual water content, the rehydration capacity, the volume contraction and the loss of vitamin C of the finished product.

The freeze drying mode was determined by the restricted area method in order to minimize the energy consumption per product weight and to maximize the quality of the finished product.

References

1. Dzung NT, Dzung TV, Ba TD (2008) Survery thermophysical properties of crustacean group (*Penaeus monodon, Penaeus Merguiensis, Penaeus Vannamei*) affected heat and mass transfer in processing freeze drying. Journal of Fisheries Science and Technology 3: 46-54.

2. Dzung NT, Dzung TV, Ba TD (2008) Researched the method to determine ice ratio in materials freeze and optimal temperature of freeze in stage 1 of processing freeze – drying. Journal of Science and Technology National University 11: 62-73.
3. Dzung NT, Dzung TV, Ba TD (2009) Research for the establishment and solution mathematical model of heat and mass transfer at the same time in processing freeze drying. Journal of Science and Technology National University11: 82-94.
4. Hai LX, Lan NT (2008) Multi-objective optimization with combination criteria applied to extraction of the high colour degree anthocyanin. Science and Technology development 11: 69-76.
5. Figura LO, Teixeira AA (2007) Food Physics: Physical properties Measurement and Application. Germany.
6. Pikal MJ, Shah ML (1984) Mass and Heat transfer in vial freeze drying of pharmaceuticals: role of the vial. J Pharm Sci 73: 1224-1237.
7. Liapis I, Bruttini R, Pikal MJ (1996) Research and development needs and opportunities in freeze drying. Journal of Food Engineering 14: 1265-1300.
8. Pikal MJ, Shah S, Roy ML, Putman R (1989) The secondary drying stage of Freeze drying: drying kinetics as a function of temperature and chamber pressure. Journal of Food Engineering 60: 203-207.
9. Khalloufi S, Robert JL, Ratti (2005) Solid foods Freeze drying simulation and experimental data. Journal of Food Engineering 28: 107-132.
10. Hai LX, Nguyen NV (2008) Multi-objective optimization for grouper (e.coioides) compounded feed formulation. Journal of Science and Technology 48: 123-136.
11. Dzung NT, Hai LX, Dzung TV, Ba TD (2010) Multi-objective optimization applied to determine regime technological freeze drying of penaeus vannamei. Journal of Fisheries Science and Technology 1: 82-93.
12. Dzung NT, Hai LX, Dzung TV (2010) Multi-objective optimization with optimal standard combination of R applied to determine regime technological freeze drying of *penaeus merguiensis.* Journal the Development of Science and Technology 13: 56-68.
13. Dzung NT, Dzung TV, Hai LX (2010) Building and solving a mathematical model of heat transfer and humidity seperation in conditions freeze drying by the finite element method. Journal of Science and Technology 48: 23-35.
14. Dzung NT, Dzung TV (2009) Building a mathematical model to determine the relationship between heat emission coefficient and pressure of the freeze drying environment of solid materials. Proceedings of the 11th Conference on Science and Technology, Viet Nam.