Analysis of Automotive Passive Suspension System with Matlab Program Generation

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Abstract

Automotive suspension systems are designed with the objective of isolating the vibrations produced because of road disturbances from being transferred to driver. In this paper a quarter car model with 2DOF is designed. The effect of speed bump as step function is analyzed for overshoot and settling time of sprung and unsprung mass. Matlab program was developed for the analysis using state space model. Program developed here can be used for and vehicles quarter car model with 2DOF for analysis, which will be beneficial in saving money required for test rigs and circuits.

Keywords **:** state space model, passive suspension, overshoot, settling time, DOF

1. Introduction:

Shock absorption in automobile is an important area of concern for the design engineers. Suspension system prevents shocks in automobiles which may be because of irregular road profile, drag forces, drivetrain or engine vibrations, wheel/tire non uniformity. Speed bumps and pot holes are the main cause of vehicle body vibration through wheel/tire assembly and suspension system. Comfort of passengers gets affected by overshoot and settling time of vehicle under vibration.

Matlab Program has been developed to analyze overshoot and settling time of a 2DOF quarter car model. A passive suspension system state space model has been used for analysis. Dynamic quarter car model can be used as a base for the analysis of full car model response to speed bump.

2. Mathematical Modeling

Modeling of automotive suspension is of great interest for automotive and vibration engineers. Vehicles ride quality is prime concern for the engineers when a vehicle passes over the speed bump. For our analysis 2 DOF quarter car model (Fig. 1) has been developed with following assumptions [6]:

- Vehicle is rigid body with the suspension
- Suspension consists suspension spring, absorber, sprung, un-sprung mass of the body
- Tire stiffness and tire absorptivity is considered separately

Parameters used for mathematical modeling are as follows:
M = Sprung Mass
m = Un-sprung Mass
K_s = Suspension spring stiffness
K_t = Tire stiffness
C_s = Damping coefficient of absorber
C_t = Damping coefficient of tire
w = Road input (height of speed bump)
X_1 = Sprung mass vertical movement
X_2 = Un-sprung mass vertical movement
Δ = Suspension Travel
The equation of motion for sprung and unsprung mass of the model considering it moving over a speed bump will become[2][5]:

$$M\ddot{X}_1 + K_S(X_1 - X_2) + C_S(\dot{X}_1 - \dot{X}_2) = 0 \quad (1)$$

$$m\ddot{X}_2 + K_t(X_2 - w) + C_t(\dot{X}_2 - \dot{w}) - K_S(X_1 - X_2) - C_S(\dot{X}_1 - \dot{X}_2) = 0 \quad (2)$$

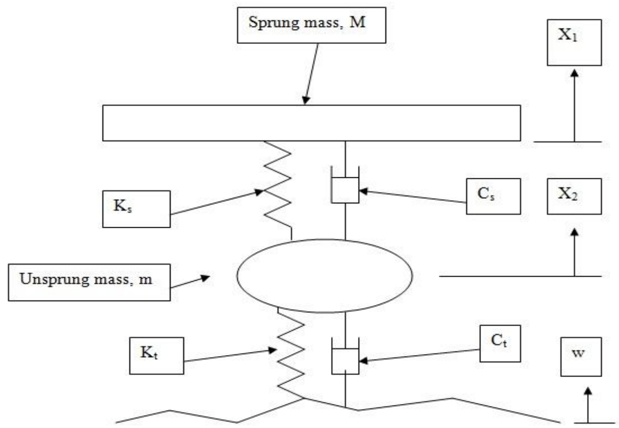

Fig. 1: 1/4 Car Model , Front Suspension

State Space Model

Equations (1) and (2) can be written in state space model as [3][4]:

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

Where,

A = State space matrix , B = Input matrix, C = Output matrix, D = Direct transmission matrix, U = Input of system

Let , $\dot{X}_1 = V_1$, $\dot{X}_2 = V_2$, $(X_1 - X_2) = \Delta$, $\dot{\Delta} = (V_1 - V_2)$

Therefore eq. (1) and (2) can be written as:

$$\dot{V}_1 = -[K_S / M]\Delta - [C_S / M](V_1 - V_2) \quad (3)$$

$$\dot{V}_2 - [C_t / \mathrm{m}]\dot{w} = [K_S / \mathrm{m}]\Delta + [K_t / \mathrm{m}]\mathrm{w} - [K_t / \mathrm{m}]\mathrm{X}_2 + [C_S / \mathrm{m}]V_1 - [(C_S + C_t) / \mathrm{m}]V_2 \quad (4)$$

Let , $\dot{T} = \dot{V}_2 - [\mathrm{C}_t / \mathrm{m}]\dot{w}$, $T = V_2 - [\mathrm{C}_t / m]w$, $V_2 = T + [C_t / m]w$ (5)

Putting (5) in (4) gives:

$$\mathrm{T}' = [\mathrm{K}_S / \mathrm{m}]\Delta - [\mathrm{K}_t / \mathrm{m}]\mathrm{X}_2 + [\mathrm{C}_S / \mathrm{m}]\mathrm{V}_1 + \{(C_S + C_t) / m\}\mathrm{T} + \left[-(\mathrm{C}_S\mathrm{C}_t) / \mathrm{m}^2 - (\mathrm{C}_t^2 / \mathrm{m}^2) + K_t / \mathrm{m}\right]\mathrm{w} \quad (6)$$

Now , $\dot{\Delta} = (V_1 - V_2) = V_1 - [T + (C_t / m)w]$ (7)

From eq. (3)

$$\dot{V}_1 = -[K_S / M]\Delta - [C_S / M](V_1 - V_2)$$

Putting V_2 from (5) in above eq.

$$\dot{V}_1 = -[K_S / M]\,\Delta - [C_S / M]V_1 + [C_S / M]T + [(C_S C_t) / (Mm)]w \quad (8)$$

From(6) state variable are Δ , X_2 , V_1, T .

State space vector matrix can be written as[7]:

$$\begin{vmatrix} \dot{X}_2 \\ \Delta \\ V_1 \\ T \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -K_S/M & -C_S/M & C_S/M \\ -K_t/m & K_S/m & C_S/m & -[(C_S+C_t)/m] \end{vmatrix} \begin{vmatrix} X_2 \\ \Delta \\ V_1 \\ T \end{vmatrix} + \begin{vmatrix} (C_t/m) \\ -(C_t/m) \\ [(C_S C_t)/(Mm)] \\ \left[\dfrac{-(C_S C_t)}{m^2} - \dfrac{C_t^2}{m^2} + \dfrac{K_t}{m}\right] \end{vmatrix} |w|$$

Output matrix becomes:

$$\begin{vmatrix} X_1 \\ V_1 \\ \dot{V}_1 \\ X_2 \\ V_2 \\ \Delta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\dfrac{K_S}{M} & -\dfrac{C_S}{M} & \dfrac{C_S}{M} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} X_2 \\ \Delta \\ V_1 \\ T \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ \dfrac{(C_S C_t)}{(Mm)} \\ 0 \\ \dfrac{C_t}{m} \\ 0 \end{vmatrix} |w|$$

3. Matlab Program Development

Ordinary differential equations of linear and nonlinear type can be solved using MATLAB. For our 2DOF quarter car model Matlab programs are developed to analyze sprung and un-sprung mass displacement and velocity and for suspension travel responses of suspension system when it passes over a speed bump [1]. Input parameters for analysis are shown in Table 1. Different response programs are as follows:

Table 1. Input Parameters

S.No	Parameter	Value
1.	Sprung Mass , M	275 Kg
2.	Unsprung Mass, m	27 Kg
3.	Suspension Spring Stiffness, K_s	150000 N/m
4.	Suspension Damping Co-efficient, C_s	1120 N-s/m
5.	Tire Stiffness, K_t	310000 N/m
6.	Tire Damping Co-efficient, C_t	3100 N-s/m
7.	Speed Bump Height, w	10 Cm

Matlab Program

```
m=27; M=275; Cs=1120; Ct=3100; Ks=150000; Kt=310000;

A=[0 0 0 1;0 0 1 -1;0 -(Ks/M) -(Cs/M) (Cs/M);-(Kt/m) (Ks/m) (Cs/m) (-(Cs+Ct)/m)]
B=[(Ct/m);-(Ct/m);((Ct*Cs)/(m*M));[(-(Cs*Ct)/(m*m))-((Ct*Ct)/(m*m))+(Kt/m)]]
C=[1 1 0 0]          % for sprung mass displacement %
  =[0 0 1 0]         % for sprung mass velocity,V1 %
  =[1 0 0 0 ]        % for unsprung mass displacement, X2 %
  =[0 0 0 1 ]        % for unsprung mass velocity, V2 %
  =[0 1 0 0 ]        % for suspension travel, X1-X2 %
D=[0]
t=0:0.01:10;
u=0.1*ones(size(t));
[Y,X]=lsim(A,B,C,D,u,t)
plot(t,Y)
```

4. Results and Discussion

Analysis results of suspension system for ¼ car model for speed bump of 0.1 m (step input) shows that vehicle sprung mass displacement (Fig.2) has overshoot of 70% and acceleration amplitude (Fig.4) of 1.75 m/s^2. These values are very high and are undesirable from comfort point of view. High overshoot is not desirable for better working of suspension and for its long life. The result of unsprung mass displacement (Fig.3) also has overshoot of 30% and acceleration (Fig.5) drops suddenly from 4 m/s^2 to 0.7 m/s^2 which are also undesirable and uncomfortable from drivers point of view and ride quality [9][10]. Even though the settling time is quite satisfactory. These problems with passive suspension system can be overcome by design an active suspension system with a controller input which will vary according to road input and will keep vehicle body movement within the range.

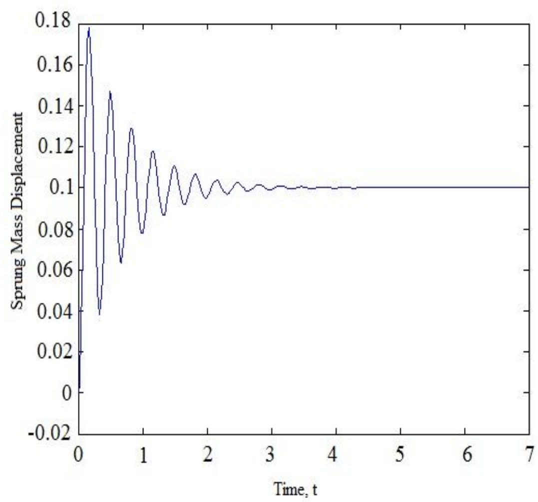

Fig. 2 Sprung Mass Displacement

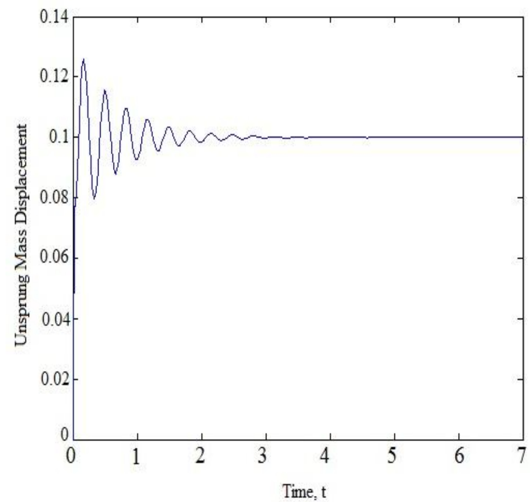

Fig. 3 Unsprung Mass Displacement

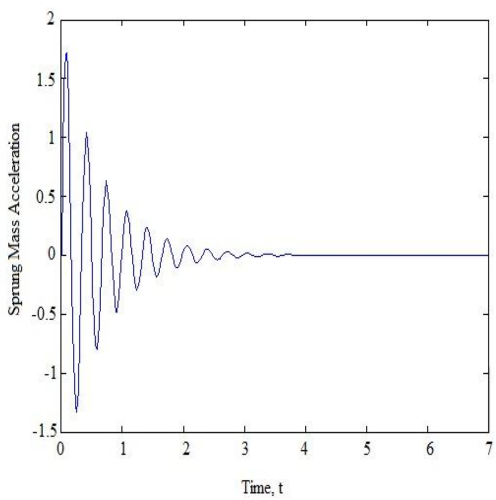

Fig. 4 Sprung Mass Acceleration

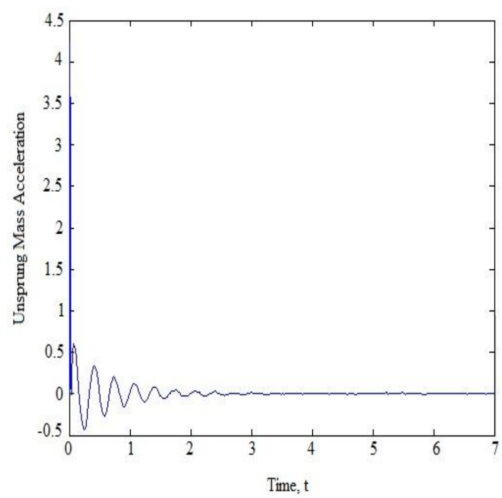

Fig. 5 Unsprung Mass Acceleration

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