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Research Article

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A Validation Study of the Control-Value Theory within the Domain of Mathematics at High School: A Latent Profile Analysis

Sabry M Abd-El-Fattah*

Department of Educational Psychology, Faulty of Education, Minia University, Egypt

Abstract

The present study aimed at validating the control-value theory within the domain of mathematics at high school level. It adopts a person-centered approach to explore whether high school students can be grouped into different achievement emotions profiles within the domain of mathematics and whether these profiles differ in mathematics achievement and mathematics tracking. The sample of the study included 355 high school students (185 males and 170 females) in Egypt. Students responded to the Learning-Related Emotions Scales- Mathematics-Arabic version (LRES-M-AR) in addition to a question concerning their intention to pursue a mathematics track at high school. A latent profile analysis showed three different achievement emotions profiles within the domain of mathematics: positive emotions, negative emotions, and positive emotions with high boredom. Students belong to the positive emotions profile had higher mathematics achievement and reported more mathematics tracking than students belong to the negative emotions profile or the positive emotions with high boredom profile. Students belong to the negative emotions profile had higher mathematics achievement and reported more mathematics tracking than students belong to the positive emotions with high boredom profile.

Keywords: Control-value theory; Emotions; Mathematics; High school; Latent profile analysis

Introduction

Emotions are multicomponent processes that include specific affective, cognitive, physiological, motivational, and expressive elements [1-3]. For example, feeling anxious prior to taking a tough test can trigger nervous and tense feelings (affective), worries about getting a low grade or failing the test (cognitive), sweating and high heart rate (physiological), desire not to take the test (motivational), and uneasy body language and facial expressions (expressive). Baumeister and Bushman [4] defined emotions as a subjective state often accompanied a bodily reaction and an evaluative response to some situation, action, or event.

Within an academic context, emotions are omnipresent. Students can experience a wide range of positive and negative emotions in different academic settings including class attendance, doing homework, and test taking. Students may feel enjoyment and excitement for learning new material during classes or they may feel bored and tedious for spending several hours at school. When doing their homework, students may feel proud for completing homework successfully, discomfiture for not making a progress on homework, shame for getting a low grade on homework, or uninterested about completing homework at all [1]. In a test situation, students may feel hope for success, afraid of failure, anxious because they are not ready for the test, proud for passing the test, shame for failure, or indifferent about the test grade at all [5].

Although emotions seem ubiquitous within different academic settings, for several years, researchers within the field of education and psychology have extensively overlooked the role of emotions in students' learning and development with most studies focusing exclusively on the role of behavioral, cognitive, and motivational factors [6]. Two exceptions are studies on test anxiety [7] and research on causal attributions (success x failure) as predictors of emotions [8]. However, more recently, researchers have recognized that students' emotional experiences within achievement and academic contexts can have a critical influence on their learning processes and achievement outcomes and thus research on emotions has begun to thrive [5,9,10]. This emerging research path has explicated the important contribution of emotions towards students' learning and academic performance during class time [10], homework [11], small group work [12], and computerized learning [13]. As Schutz and Lanehart [14] highlighted in an introduction to Special Issue of Educational Psychologist on Emotion in Education "*Emotions are intimately involved in virtually every aspect of the teaching and learning process and, therefore, an understanding of the nature of emotions within the school context is essential*".

The control-value theory of achievement emotions

Pekrun and his colleagues have recently proposed the control-value theory of achievement emotions as a comprehensive framework of the emotional experiences within achievement and academic settings [1,10,15]. Achievement emotions refer to the feelings that students experience in educational and achievement contexts and are related to achievement activities (i.e., studying) and outcomes (i.e., success or failure) [1,3,10]. The control-value theory proposes that achievement emotions can be theorized as temporary or momentary manifestations within a specific situation at a particular point of time (State achievement emotions; e.g., state fear of failure before a test) or they can be theorized as habitual dispositional emotions that a person experienced with regard to achievement-related activities and outcomes (Trait achievement emotions; e.g., trait fear of failure). It is important to note that time-based (temporal) generalization rather than situational generalization represents the essential characteristic that differentiates between trait and state achievement emotions because trait achievement

***Corresponding author:** Sabry M Abd-El-Fattah, Department of Educational Psychology, Faulty of Education, Minia University, Minia City, Minia Governorate, Egypt, Tel: 08623342601; E-mail: sabryrahma@hotmail.com

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emotions can be tied to a specific situation as well (e.g., trait science emotions are related to representative emotional feelings in science-related situations) [2,3].

Furthermore, the control-value theory of achievement emotions proposes that the cognitive appraisal of control over and the subjective value of achievement activities and outcomes (i.e., predictors or antecedents) have the potential to stimulate achievement emotions (i.e., mediator) which in turn affect learning processes and outcomes (i.e., outcomes or precedent) [3]. Thus, the control-value theory posits two types of appraisals that are particularly pertinent for achievement emotions: (1) subjective control that refers to perceived control and influence over achievement-related activities and achievement outcomes (i.e., working hard and persistence can make success possible), and (2) subjective value that refers to perceived value of achievement activities and outcomes (i.e., perception that success is important) [1]. Within an achievement context, students can appraise achievement activities and outcomes according to their subjective appraisals of control and value. For example, if students feel control of their learning outcomes and they value their learning tasks, they are more likely to feel happiness and enjoyment, whereas students who do not feel such control and they do not value their learning tasks are more likely to feel anxiety, boredom, and unhappiness [1,16].

The three-dimensional taxonomy of achievement emotions

Pekrun and his colleagues [1,3] proposed the three-dimensional taxonomy of achievement emotions (Table 1). According to this taxonomy, emotions can differ according to object focus, valence, and activation [16]. The object focus refers to whether emotions are linked to an achievement activity (i.e., activity emotions) or an achievement outcome (outcome emotions). The activity emotions are emotions associated with continuous achievement-related activities including, for example, excitement and enjoyment for commencing a new academic project, boredom for spending several hours at school, and frustration when not making progress on an academic course. The outcome emotions are emotions tied to the outcomes of achievement-related activities (success or failure) [17,18]. The outcome emotions can be classified into prospective emotions that focus on future outcomes (e.g., hope for success and fear of failure) or retrospective emotions that arise when a person think back on the outcomes of an achievement task (e.g., feeling anger or shame after failure or feeling pride and relief after success) [1,16].

Valence refers to whether emotions are perceived as positive (i.e., enjoyment) or negative (i.e., anxiety). Activation refers to whether the emotions are physiologically activating (e.g., excitement) or deactivating (i.e., relaxation) [1,3,10,16,19]. The organization of the bipolar dimensions of valence (positive/negative) and activation (activation/deactivation) render four groups of emotions: positive activated (e.g., excitement), positive deactivated (e.g., relaxation), negative activated (e.g., angry), and negative deactivated (e.g., exhausted) emotions [3].

Domain specificity of achievement emotions

Within real academic settings, it is possible to examine students' achievement emotions from a domain-specific perspective because students are likely to experience different emotions for different subject matters [1]. For example, a student may feel anxious in a math class but relax in a language class. One advantages of examining achievement emotions from a domain specific perspective is that it helps determine the most appropriate interventions and assistance to be offered to students who struggle with a specific subject matter. Another advantage is that researchers and teachers in schools may need to know whether students are experiencing an achievement emotion in a specific subject matter or if this achievement emotion is a general emotion that students experience across all subject matters. An examination of possible communalities among subject matters may account for students' achievement emotions within these subject domains [3,17]. Furthermore, control-based and value-based variables have been shown to be planned in a domain-specific paths (e.g., students' self-concept and interests; Bong, [20]; self-efficacy, Bandura, [21]). By implication, the emotions determined by control and values should be domain specific too, in contrary to the traditional viewpoint concerning achievement emotions as general personality traits. Assumptions on domain specificity of students' achievement emotions were examined in a few recent studies [22].

For example, Pekrun and his colleagues utilized the domain-specific perspective of achievement emotions and conducted a series of exploratory studies to test the three-dimensional taxonomy of achievement emotions with samples of high school and university students using interview and questionnaire techniques. They found that anxiety was the most often reported achievement emotion; it was reported 15-27% of all emotional scenarios across different academic situations. Emotions of enjoyment, pride, happiness, anger, shame, boredom, and relief were reported frequently as well. However, emotions such as envy, gratitude, admiration, and hopelessness were reported less frequently [3]. Furthermore, several studies have examined achievement emotions within specific subject matters including foreign language [17], mathematics [22-24] and anatomy [25]. For example, Goetz et al. [19] investigated the within and the between domain relations of academic emotions (e.g., boredom, anger, anxiety, enjoyment, and pride) in different subject matters including English, physics, mathematics, and German using a sample of 8th and 11th graders. They reported that the domain specificity of emotions was stronger in the 11th grade compared to the 8th grade. Pride showed the strongest correlations of emotions between domains but enjoyment and anxiety showed the weakest correlations. Mathematics was found to be the strongest in terms of correlations within domain followed by physics, English, and German, respectively. Similarly, Goetz et al. [17], using a sample of 7th graders, examined the domain-specificity of achievement emotions for several domains including mathematics, Latin, German, and English. Achievement emotions included enjoyment, anxiety,

Object focus	Positive [a] Activating	Positive [a] Deactivating	Negative [b] Activating	Negative [b] Deactivating
Activity	Enjoyment	Relaxation	Anger Frustration	Boredom
Outcome/Prospective	Hope Joy	Relief [c]	Anxiety	Hopelessness
Outcome/Retrospective	Joy Pride Gratitude	Contentment Relief	Shame Anger	Sadness Disappointment

Table 1: A three-dimensional taxonomy of achievement emotions. **Note:** [a]Positive=pleasant emotion; [b]Negative=unpleasant emotion; [c]Antipatory joy/relief. Adapted from "Achievement emotions: A control-value approach" by Pekrun et al. [16]

and boredom. Domain specificity of emotions was evident by means of multi-trait and multi-method technique and confirmatory factor analysis. Enjoyment was found to be the most dominant specific type of achievement emotions.

Mathematics achievement and mathematics tracking in Egypt

Mathematics learning and understanding are important to occupational success and effective personal management in everyday life. A study by the National Research Council [26] in the United States showed that the basics of mathematics and geometry fields are necessary for specialization in 75% of all occupations. Furthermore, mathematics represents a principal discipline in education across primary, secondary, and higher education [27]. Achievement in mathematics constitutes a substantial factor in decisions concerning students' placement, selection, and admission across most educational systems worldwide [28].

In Egypt, at the end of the second year of high school students are expected to decide which track they would like to pursue for the final year of high school. There are three major tracks: literature, science, and mathematics. One of the important decisions that students have to make is whether or not they will continue with the mathematics track. The present study focuses on mathematics achievement and mathematics tracking at high school in Egypt for three primary reasons. Firstly, increasing mathematics expertise of Egyptian students is recognized as an area of special national need because international comparative assessments, such as those obtained in the third Trends in International Mathematics and Science Study (TIMSS), have indicated that Egyptian high school students lag behind those of other countries in both mathematics and science even for those of high ability [29]. Secondly, despite recent educational reforms, high school students in Egypt remain underrepresented within mathematics and science tracks which both depend on skills in mathematics. For example, the annual report published by the Ministry of Education in Egypt showed that the number of students who choose mathematics track at high school has decreased by up to 15% in the last 5 years, and that the proportion of boys who choose mathematics track had fallen to 87.22%; among girls it was only 82.66% in the last year (Ministry of Education, 2015). Therefore, understanding how to promote mathematics achievement and mathematics tracking at high school and thereby promoting a stronger educational and career pipeline for the advancement of Egyptian high school students is of considerable national significance to combat adolescents' loss of interest in mathematics. Thirdly, research within Egyptian context has shown that students who come to university with poor mathematics achievement or who choose to skip mathematics at high school have a difficult time progressing in the disciplines they have chosen to major in at university [30].

A person-centered perspective of achievement emotions

Contemporary research on the role of human emotions in educational settings have demonstrated that individuals can experience multiple and blended emotions simultaneously in an achievement context and this may have imperative impacts on their academic functioning and academic achievement [31-33]. In fact, Pekrun [1] recognized that multiple emotional experiences may co-occur in an achievement context, and that these mixed emotional experiences may reveal distinctive patterns of motivation and behavior. Thus, a more appropriate approach to examine the influences of a combination of emotions "emotion profile" is to use person-centered analysis techniques (e.g., cluster analysis and latent profile analysis). This type of analysis detects the set of emotional experiences of each individual (profile), and then groups together individuals with similar emotional experiences [31,34]. Furthermore, a person-centered analysis enables researchers to examine the associations among multiple emotion variables with no need to utilize higher-order interaction terms or apply linear relationships. The notion of linear relationship may be of particular importance to the investigation of emotions at achievement contexts because some research highlighted that relationship between some emotions (i.e., anxiety) and academic achievement may be curvilinear [35,36].

Research on achievement emotions using a person-centered perspective

Among the few recent studies that have adopted the person-centered approach is the study by Barker, et al. [37] who investigated the covariance between positive and negative effect within a sample of 187 first year university students over time and how different patterns of affective states and traits are linked to academic achievement. A multilevel linear modeling analysis showed that students who experienced a high trait-like positive effect coupled with bouts of negative effect showed best growth in academic achievement than students who experienced either positive or negative affect alone. Barker and colleagues concluded that the impact of trait-like negative affect may differ according to whether it has been experienced with positive trait-like affect or not and that different groups of affect may be linked distinctively to academic achievement.

A study by Jarrell et al. [32] investigated the relationship of the achievement emotions experienced by students during performing a task and their appraisals of task control, value, perceived performance, and actual performance on a diagnostic reasoning task with a Computer-based Learning Environment (CBLE), BioWorld. The sample of the study included 26 university medical students. A cluster analysis grouped participant students into one of three different achievement emotions profiles: positive emotion, negative emotion, and low emotion. Students with a positive emotion profile showed more task control and task value compared to the students with a negative emotion profile. Students with a positive emotion profile had the greatest perceived performance followed by the students with low emotion and students with negative emotion profiles. Students with a low emotion profile achieved higher than students with positive or negative emotion profiles who showed similar mean levels of performance.

Jarrell et al. [33], using a sample of 30 university medical students, examined the relationship between retrospective achievement emotions experienced by students after performance a task and their performance on a diagnostic reasoning task with a Computer-based Learning Environment (CBLE), BioWorld. A cluster analysis grouped participant students into one of three different achievement emotions profiles: positive emotion, negative emotion, and low intensity emotion profiles. Students with positive emotion profile had the greatest performance; those with the negative emotion profile had the poorest performance; and students with the low emotion profile had moderate performance.

Molk [38] examined the effect of achievement emotions on students' mathematics achievement at high school and their expectations to obtain a university degree in mathematics. The sample of the study included 216 college students in South Africa. A cluster analysis grouped participant students into one of three different achievement emotions clusters: positive emotions, negative emotions, and moderate-low emotions. Students within the positive emotions cluster reported higher mathematics achievement and greater expectations to obtain a university degree in mathematics more than students within the negative or moderate-low clusters who showed similar mean levels of mathematics achievement and expectations of obtaining a university degree in mathematics.

Ganotice et al. [24], in Study 1, used cluster analysis to group a sample of 1147 high school students into four distinctive achievement emotion profiles: adaptive shame, moderate group, maladaptive group, and adaptive group. Results showed that students with the adaptive shame and the adaptive profiles demonstrated the greatest levels of autonomous and controlled motivation, while students with the maladaptive profile had the lowest controlled motivation. In Study 2, a cluster analysis replicated the four achievement emotion profiles described in Study 1. Students with the adaptive cluster reported the highest level of positive emotions, while students with the maladaptive profile had the highest levels of negative emotions. Students with the adaptive with shame and adaptive profiles demonstrated the highest levels of engagement outcomes, and had the highest mathematics achievement.

Robinson et al. [25] examined the relationship between affective profiles and behavioral engagement, behavioral disengagement, and academic achievement in a college anatomy course. The sample of the study included 278 undergraduates. A cluster analysis grouped students into four different affective profiles: positive, deactivated, negative, and moderate-low. Students with a positive profile showed greater behavioral engagement than students with a negative profile. Students with positive and deactivated profiles showed significantly poorer disengagement than students with the negative and moderate-low profiles. Students with a deactivated profile showed higher levels of academic achievement than students with the negative and moderate-low profiles. Students with the positive profile showed significantly higher levels of academic achievement than students with the negative profile. A path analysis model showed that positive and deactivated profiles have positive indirect effects on academic achievement through engagement.

Within a non-academic context, Fernando et al. [31], conducted two studies using two samples of Australian (N=97) and Canadian citizens (N=190) and found that while a variable-centered analysis showed that sympathy was a significant predictor of orientation to help others, a person-oriented analysis showed that only those who experienced sympathy accompanied by other prosocial emotions were greatly inclined towards helping others. However, those who experienced high levels of sympathy and low levels of other prosocial emotions were less oriented to help others. This means that sympathy may promote a behavior to help others only when accompanied by other prosocial emotions.

Problem and Rationale of the Study

One important framework that has been frequently used to understand individuals' achievement emotions is the control-value theory [1,15]. This theory assumes that emotions can shape individuals' learning processes and thus can have a critical impact on achievement outcomes. The recent developments of the control-value theory have resulted in a multi-achievement emotions perspective which theorizes that an individual can ideally experience more than one emotion concurrently within an achievement context (i.e., achievement emotion profile) and this may have important implications for academic achievement and outcomes. Pekrun [1] argued that varied emotional experiences are common within academic and achievement contexts, and that these experiences may highlight distinctive patterns of motivation and behavior. The multi-achievement emotions perspective often uses person-centered analysis techniques (e.g., cluster analysis or latent profile analysis) wherein the differences in the process and the outcome variables are examined for various subgroups/clusters of individuals with similar achievement emotion profiles [31,34]. Although the idea of adopting multiple achievement emotions simultaneously has been recently investigated in a very limited number of studies [31], there is still debate regarding which combination of achievement emotions leads to the most adaptive outcomes, and how the effects of achievement emotions are best revealed in terms of educational outcomes.

The debate over the most adaptive achievement emotions profile becomes more stimulating and interesting when discussed from a cross-cultural perspective. To date, most of research on the multi-achievement emotions perspective has been conducted almost exclusively with subjects from Western countries [5,6] with most studies generally overlooking the utilization of this research approach with samples from Non-Western countries [3]. In fact, one should pay attention to possible differences in achievement emotions conceptualization and effect from culture to culture. This is true because emotions are not only biologically determined, but also influenced by cultural and social factors [1]. Culture can determine how individuals should feel in a given situation and how they should express their emotions in such situations according to predefined cultural norms [39]. Culture was also found to contribute towards differences in emotion expressions including facial expression and body language, recognition of emotions, affect valuation, and the use of terms and words to express emotions. Furthermore, research on cross-cultural differences in emotional arousal has shown that individuals from Western countries are characterized with high levels of emotional arousal compared to individuals from Non-Western countries. These cultural differences may be interpreted within the framework of the distinctive characteristics of individualist and collectivist cultures [1,3]. These cultural differences necessitate the validation of the control-value theory in Non-western countries because models and theories developed in the West might not work properly in non-Western settings due to cultural differences [10].

Furthermore, early research within the framework of the control-value theory has adopted a domain-general perspective to examine the effect of achievement emotions within academic settings. This approach has focused on investigating how students' achievement emotions may affect their overall academic achievement based on the assumption that students experience similar achievement emotions across various subject-matters [15]. A more recent approach, however, proposed that a full understanding of achievement emotions entails the recognition of their multiplicity and specificity. Every subject-matter may trigger a specific achievement emotion which may have a distinct influence on academic achievement of this subject-matter. For example, a student may feel anxious in a mathematics class but relax in a science class. Thus, the domain-specific perspective of achievement emotions supports the investigation of achievement emotions at the course level. While there is consensus that an understanding of the functionality of achievement emotions from a domain-specific perspective is advantageous [3, 17,22], there is a limited research at present that adopt this approach. More research is therefore needed to understand how domain-specific achievement emotions may influence academic achievement and streaming within a particular subject-matter such as mathematics.

To summarize, the present study adopts a multi-achievement emotions perspective and utilizes a person-centered analysis technique to examine the role of achievement emotions within an academic context. The general problem tackled in the present study concerns the validation of the control-value theory within a non-Western context (i.e., Egypt) using a domain-specific perspective (i.e. mathematics). One goal of the present study is to examine whether high school students can be grouped into different achievement emotions profiles within the domain of mathematics. A second goal of the present study is to investigate whether students with different achievement emotions profiles within the domain of mathematics will differ in mathematics achievement and mathematics tracking.

Questions of the Study

The present study seeks to answer the following research questions:

1. Do students show subgroups (i.e., classes) with different achievement emotions profiles within the domain of mathematics?

2. Do students with different achievement emotions profiles within the domain of mathematics differ significantly in their mathematics achievement and mathematics tracking?

Aims of the Study

The following aims guide the present study:

1. Validate the control-value theory within a non-Western context (i.e., Egypt) using a domain-specific perspective (i.e. mathematics) to identify different achievement emotions profiles shown by students.

2. Examine the practical utility of the control-value theory by identifying whether students with different achievement emotions profiles within the domain of mathematics will differ in mathematics achievement and mathematics tracking.

Significance of the Study

The significance of the present study is summarized as following:

1. The application of the control-value theory within a non-Western context (i.e., Egypt) provides evidence for the cross-cultural external validity of the theory. It is possible that careful examination and validation of the control-value theory in Non-western contexts bring about profitable use of the theory and its applications within these contexts because emotions are not only biologically determined, but also influenced by cultural and social factors.

2. The investigation of the control-value theory within the subject matter of mathematics provides evidence for the utility of the theory within domain-specific contexts.

3. The examination of students' achievement emotions profiles linked to the subject matter of mathematics is particularly important because mathematics achievement and mathematics tracking at high school level are important factors for students' academic and professional career.

Methods

Participants

Sample 1: This sample included 225 first year high school students (125 males and 100 females) enrolled in five public schools in the main cities of three provinces (i.e., Minia, Samalut, and Abu Qurqas) in Minia governorate in Egypt. All schools were from metropolitan areas and had single-gender population. The age mean and standard deviation were 15.66 and 0.42 for boys and 15.37 and 0.29 for girls respectively. The percentage of missing data was less than 1%. Only students with complete data were retained for the present study. The analysis of demographic data showed that 98% of the participants belong to the middle socioeconomic class. The data collected from this sample would be used to examine the psychometric properties (validity and reliability) of the measurements of the present study.

Sample 2: This sample included 355 first year high school students (185 males and 170 females) enrolled in seven public schools in the main cities of four provinces (i.e., Minia, Samalut, Maghagha, and Mallawi) in Minia governorate in Egypt. All schools were from metropolitan areas and had single-gender population. The age mean and the standard deviation were 15.20 and 0.55 for boys and 15.41 and 0.46 for girls respectively. The percentage of missing data was less than 2%. Only students with complete data were retained for the present study. The analysis of demographic data showed that 97% of the participants belong to the middle socioeconomic class. The data collected from this sample would be used to run the main statistical analyses to test the hypotheses of the present study.

Measurements

The Learning-related Emotions Scales-Mathematics-Arabic version (LRES-M-AR)

The LRES description: The Learning-Related Emotions Scales (LRES) is one of three sections that constitute the original Achievement Emotions Questionnaire (AEQ, [40]). The LRES is a multidimensional self-report measurement that assesses students' achievement emotions pertaining to studying at college and university. The LRES uses a situation-reaction questionnaire format providing a description of the studying situation the assessment refers to and then asking respondents to report how they feel in the situation.

The LRES includes 75 items distributed over eight different emotions scales: enjoyment (10 items; e.g., "I *enjoy acquiring new knowledge*"), hope (6 items; e.g., "*I have an optimistic view toward studying*"), pride (6 items; e.g., "*I think I can be proud of my accomplishments at studying*"), boredom (11 items; e.g., "*Studying is dull and monotonous*"), anger (9 items; e.g., "*I get annoyed about having to study*"), anxiety (11 items; e.g., "*I get tense and nervous while studying*"), hopelessness (11 items; e.g., "*I feel hopeless when I think about studying*"), and shame (11 items; e.g., "*I feel ashamed because I am not as adept as others in studying*"). These scales focus on both activity-related emotions and outcome-related emotions. Within the LRES, items are placed in three blocks assessing emotional experiences before, during, and after being in the achievement situations described by scales. Students responded to each item of the LRES by rating their emotional experiences on a 5-point Likert type scale that ranged from 1 (Strongly Disagree) to 5 (Strongly Agree). A high score on any of the eight scales of the LRES indicates a high level of the type of achievement emotion described by this scale pertaining to studying in college and university.

The adaption of the LRES to mathematics: According the AEQ manual [40], the original instructions of the AEQ intends to assess students' typical emotional experiences at college and university (i.e., trait achievement emotions), thus it assesses students' achievement emotions within a general academic setting. However, these instructions can be modified to assess students' emotions in a specific course (course-specific emotions) or in a given achievement situation on a specified points of time (i.e.., state achievement emotions). Consistent with these guidelines, the researcher in the present study adapted the LRES to the subject-matter of mathematics and thus developed the Learning-Related Emotions Scales- Mathematics (LRES-M).

The adaption process was based upon the guidelines suggested by Cohen et al. [41] and also Kaplan et al. [42]. Firstly, the researcher adapted the general instructions of the original LRES into a more-specific instructions pertaining to the subject-matter of mathematics at high school level. Thus, respondents were asked to describe how they feel about studying the subject-matter of mathematics rather than how they feel about studying at high school in general. Secondly, the items of the eight scales of the LRES were re-worded and modified slightly to reflect achievement emotions in the subject-matter of mathematics. For example, in the enjoyment scale one item was changed from "*I enjoy acquiring new knowledge*" into "*I enjoy acquiring new knowledge in mathematics*". Thirdly, the interpretation of the scores of the LRES was modified to reflect students' achievement emotions within the subject-matter of mathematics and thus a high score on any of the eight scales of the LRES-M indicates a high level of the type of achievement emotion described by this scale within the domain of mathematics.

Translation of the LRES-M into Arabic: Applying a blind back translation strategy to the LRES-M, the author translated the LRES-M from English into Arabic using the back-translation method. Two bilingual professors of educational psychology, working without referencing to the English version of the LRES-M, independently translated the Arabic version back to English. Finally, two certified translators independently compared the original English version of the LRES-M to the new English version that was translated back from Arabic, and rated the match between the two versions on a scale of 0 or 1. A score of zero represented no match, whereas a score of 1 represented perfect match. The average percentage of match was 97% which could be considered highly acceptable [43]. Furthermore, the interarter agreement was calculated using SPSS 23.0 program Crosstabs function. This produces a Kappa statistic for level of agreement [44]. Cohen [45] argued that Kappa values can range between -1 and +1. Kappa>0 refer to greater than chance agreement, Kappa<0 refers to less than chance agreement, and Kappa=0 refers to chance agreement. Kappa<0.41 is weak, 0.41 $\leq$ Kappa $\leq$ 0.60 is moderate, and Kappa>0.60 is large level of agreement [46]. The interrater agreement Kappa value for the LRES-M was 0.78 which indicated high levels of interrarter agreement. The Arabic version of the LRES-M (LRES-M-AR) was used in all further analyses of the present study and its available for free form the author upon request.

Using the LRES-M-AR at high school: Although the original AEQ [40] was typically developed to assess students' achievement emotions at college of university, several recent studies have shown that the AEQ is a valid and a reliable measurement when used with samples of high schools students [47-49]. Thus, the present study intends to use the LRES-M-AR with a sample of high school students.

Validity of the LRES-M-AR

Consistent with the analysis strategy applied to the original AEQ [40], the present study tested a first-order correlated eight-factor model of the LRES-M-AR by means of confirmatory factor analysis to examine the construct validity of the measurement. Confirmatory factor analysis is used to validate and confirm the structure of a measurement tool [50]. Figure 1 shows the hypothesized model of the LRES-M-AR. Within this model, the items function as observed variables for their designated factors (scales) which in turn function as latent variables. The model includes the following correlated eight factors (subscales): enjoyment (10 items), hope (6 items), pride (6 items), boredom (11 items), anger (9 items), anxiety (11 items), hopelessness (11 items), and shame (11 items). The data showed both univariate (skewness and kurtosis values were between -1 and +1) and multivariate normality (Mardia's coefficient=39.16 and the Normalized estimate=21.33) [50] and thus the variance-covariance matrix was analyzed using the full information maximum likelihood estimation method [51,52]. The AMOS 20 program [53] was used to run the analysis. The fit indices used to evaluate the model fit to the data included: (1) Absolute fit

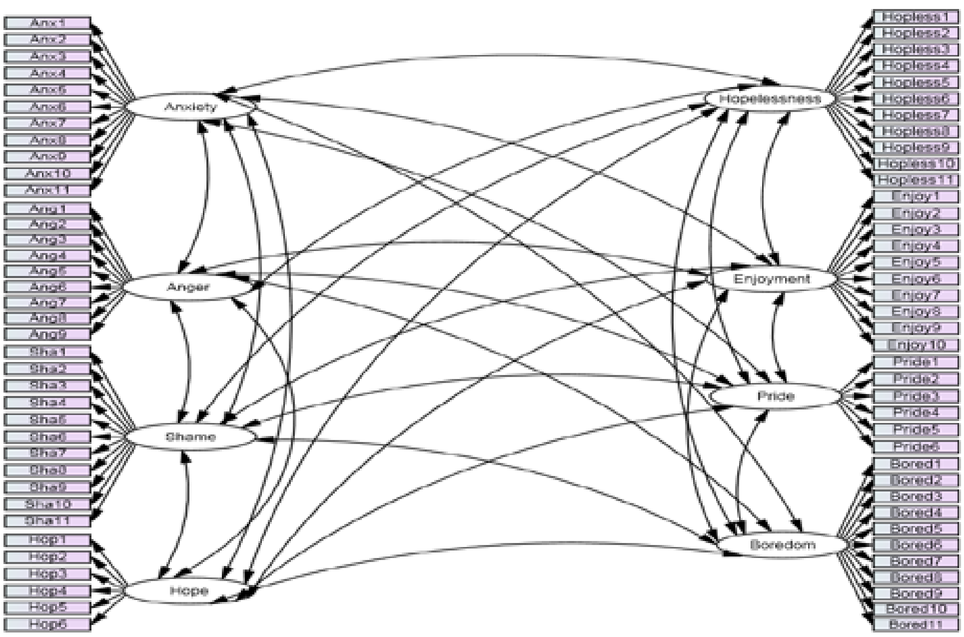

Figure 1: The hypothesized model of the LRES-M-AR.

indexes which included Chi-square (χ^2)/df<2.0, Standardized Root Mean-Square Residual (SRMR) $\leq$ 0.08, and Root-Mean-Square Error of Approximation (RMSEA) $\leq$ 0.06, and (2) Relative fit indexes which included Comparative Fit Index (CFI) $\geq$ 0.95 and Non-normed Fit Index (NNFI) $\geq$ 0.90) [54,55].

The final LRES-M-AR model had an acceptable fit to the data (χ^2=2950.22, df=2043, χ^2/df=1.44, RMSEA=0.04, SRMR=0.04, CFI=0.97, NNFI=0.97) after trimming 7 items from the model (2 from boredom, 2 from anxiety, 2 from hopelessness, and 1 from shame factors). The standardized regression weights of these 7 items on their designated factors were substantially low and ranged from 0.14 to 0.23 with critical ratio ranging from 0.73 to 1.63, suggesting that these standardized regression weights were not statistically significant (p>0.05). The critical ratio is the test statistic used to investigate the statistical significance of the factor loadings. The critical ratio values need to be > ± 1.96 for factor loading to be acceptable (i.e., statistically different from zero) [51,56]. The eight factors within the model correlated significantly with each other (r=0.31 to 0.52, p<0.01). The standardized regression weights (factor loadings) of the items on their designated factors ranged from 0.49 to 0.78. The standardized regression weights of the LRES-M-AR model were considered statistically significant because their CR values ranged from 3.55 to 12.56. Table 2 shows the item loadings of the LRES-M-AR model on their designated factors and the critical ratio values associated with these loadings.

Factor/subscale	Item	Item loading(1)	Critical ratio (2)
Anxiety	Anx1	0.54	4.56
	Anx2	0.62	5.33
	Anx3	0.57	4.66
	Anx4	0.60	7.88
	Anx5	0.63	4.96
	Anx6	0.67	5.55
	Anx7	0.60	8.55
	Anx8	0.64	10.33
	Anx9	0.69	7.60
Hopelessness	Hopl1	0.66	6.33
	Hopl2	0.55	5.10
	Hopl3	0.59	8.11
	Hopl4	0.63	4.33
	Hopl5	0.65	6.66
	Hopl6	0.60	7.77
	Hopl7	0.66	6.20
	Hopl8	0.71	8.40
	Hopl9	0.78	9.45
Shame	Sh1	0.64	11.22
	Sh2	0.66	10.67
	Sh3	0.61	8.44
	Sh4	0.72	10.15
	Sh5	0.63	4.55
	Sh6	0.69	6.22
	Sh7	0.67	7.45
	Sh8	0.65	8.33
	Sh9	0.62	9.25
	Sh10	0.59	7.63
Hope	Hop1	0.49	4.50
	Hop2	0.51	6.13
	Hop3	0.58	7.20
	Hop4	0.63	8.88
	Hop5	0.66	10.22
	Hop6	0.62	12.56
Enjoy	Enj1	0.65	7.60
	Enj2	0.60	4.33
	Enj3	0.56	5.22
	Enj4	0.59	4.37
	Enj5	0.63	7.88
	Enj6	0.70	4.52
	Enj7	0.66	3.55
	Enj8	0.63	5.52
	Enj9	0.65	6.60
	Enj10	0.67	7.20
Anger	Ang1	0.70	6.55
	Ang2	0.59	11.77
	Ang3	0.67	9.52
	Ang4	0.65	8.10
	Ang5	0.67	6.32
	Ang6	0.72	7.45
	Ang7	0.66	6.25
	Ang8	0.68	7.42
	Ang9	0.70	4.60
Pride	Prid1	0.73	5.62
	Prid2	0.70	7.33
	Prid3	0.69	8.11
	Prid4	0.73	9.64
	Prid5	0.77	11.30
	Prid6	0.74	8.90
Boredom	Bor1	0.69	4.95
	Bor2	0.65	4.69
	Bor3	0.58	5.30
	Bor4	0.62	7.22
	Bor5	0.66	6.50
	Bor6	0.60	4.33
	Bor7	0.73	6.11
	Bor8	0.70	7.32
	Bor9	0.68	6.54

Table 2: Item loadings on the eight factors of the LRES-M-AR and critical ratios[1]. **Note:** N = 355, (1) 7 items were deleted from the model, (2) Critical ration p < 0.01 for all instances.

Reliability of the LRES-M-AR

The reliability of the LRES-M-AR was tested by two methods:

Test-retest reliability: The LRES-M-AR was re-administered to a subsample of Sample 1 (n=115) after 4 weeks. Table 3 shows values of the test-retest reliability coefficient (stability coefficient) which ranged between 0.82 and 0.92. These values support the stability of students' performance scores on the LRES-M-AR subscales.

Internal-consistency reliability: The internal-consistency reliability of the LRES-M-AR was estimated using Sample 1 (N=225). Table 3 shows values of the internal consistency reliability coefficients (Cronbach's Alpha) for the subscales of the LRES-M-AR which ranged from 0.86 to 0.93. These values support the internal consistency reliability of the LRES-M-AR subscales.

Mathematics achievement

Students' mathematics achievement scores over the two semesters of the 2016/2017 school year were obtained from their school records. Students study three branches of mathematics in each semester:

LRES-M-AR subscales	Number of items	Test-retest coefficient(1)	Internal-consistency coefficient(2)
1. Enjoy	10	0.87	0.88
2. Anger	9	0.91	0.90
3. Pride	6	0.89	0.92
4. Boredom	9	0.84	0.87
5. Anxiety	9	0.92	0.87
6. Hopelessness	9	0.86	0.93
7. Shame	10	0.82	0.91
8. Hope	6	0.85	0.86

Table 3: Test-retest and Cronbach's alpha reliability coefficients for the LRES-M-AR subscales. **Note:** (1)n=115; (2)*N*=225.

Variables	1	2	3	4	5	6	7	8	9	10
1. Enjoy	-	-0.41	0.44	-0.53	-0.39	-0.44	-0.39	0.41	0.62	0.61
2. Anger		-	-0.49	0.41	0.37	0.33	0.40	-0.44	-0.56	-0.59
3. Pride			-	-0.40	-0.37	-0.49	-0.43	0.36	0.52	0.54
4. Boredom				-	0.41	0.37	0.47	-0.44	-0.58	-0.60
5. Anxiety					-	0.47	0.37	-0.41	-0.65	-0.64
6. Hopelessness						-	0.33	-0.36	-0.51	-0.50
7. Shame							-	-0.31	-0.53	-0.57
8. Hope								-	0.57	0.60
9. Mathematics achievement									-	0.46
10. Mathematics tracking (1)										-
M (2)	3.48	3.40	3.35	3.36	4.15	3.52	3.29	3.33	89.40	76.34
SD	0.41	0.33	0.29	0.30	0.37	0.44	0.61	0.54	1.94	1.14
Skewness	0.54	0.73	0.44	-0.52	-0.63	0.41	0.48	-0.56	0.49	0.63
Kurtosis	0.75	-0.41	0.65	-0.74	0.63	0.52	0.47	-0.83	0.81	0.41

Table 4: Descriptive statistics and interrelations amongst measured variables. **Note:** *N*=355. M=Mean, SD=Standard deviation. Pearson correlation coefficients were statistically significant for all instances ***p*<0.01. (1)Mathematics achievement and mathematics tracking are expressed as percentages. (2)Actual mean scores for the eight types of mathematics emotions were scaled out of 5 on a 5-point scale with the theoretical mean=3.

algebra, trigonometry, and analytical geometry. The scores for each semester represent the aggregated total scores of these three branches of mathematics (i.e., the sum of the mathematics branch assignments and examinations scores) and were expressed as percentages. High scores indicate a high level of mathematics achievement.

Mathematics tracking

According to Brahier [57], to measure students' intention to pursue a mathematics track at high school, students were asked to respond to a question that measures the probability that they will choose a mathematics track at high school. The question asked, what is the probability that you will choice a mathematic track at high school? The students respond by giving a probability out of 100%.

Procedure

An oral approval to conduct this study at the selected schools in Minia governorate was obtained from the authority at each school. Students were recruited for participation during their normal classes at their schools. Within each school, specific classes participated in data collection depending on students' classroom schedules. Students were instructed that participation was voluntary and that their responses would be kept confidential. The LRES-M-AR was administered to students by trained experimenters according to standardized instructions during the second week of the first semester of the 2016/2017 school year. Students completed the LRES-M-AR in 20 to 25 min. The cover sheet of the LRES-M-AR included questions about name, gender, age, province, and socio-economic status of students. Students were also asked to respond to a question that measures the probability that they will choose to pursue a mathematics track at high school.

Results

Descriptive statistics and correlation analyses

Table 4 shows descriptive statistics and Pearson's correlations among the eight types of mathematics achievement emotions, mathematics achievement, and mathematics tracking.

Table 4 shows that students had above average achievement emotions related to learning mathematics (M=3.29 to 4.15 on a five-point scale) in both positive and negative directions with anxiety being the most dominant emotion (M=4.15 on a five-point scale). The correlations among the eight mathematics achievement emotions were moderate to high and in both positive and negative directions ($-0.31 \leq r \leq -0.53$, $p<0.01$). The highest correlation coefficient was between enjoyment and boredom (r=- 0.53, $p<0.01$) and the lowest correlation coefficient was between shame and hope (r=-0.31, $p<0.01$). The eight mathematics achievement emotions correlated highly and significantly with mathematics achievement (r=-0.51 to -0.65, $p<0.01$) and mathematics tracking (r=-0.50 to -0.64, $p<0.01$). Anxiety had the highest correlations with mathematics achievement and mathematics tracking whereas hopelessness had the lowest correlations. Mathematics achievement and mathematics tracking correlated significantly at 0.46, $p<0.01$.

Latent profile analysis

Aim of the latent profile analysis: The eight types of mathematics achievement emotions were analyzed using a Latent Profile Analysis (LPA) technique. LPA is an extension of Latent Class Analysis (LCA) that can accommodate continuous, ordinal, and categorical indicators. LPA is based on the assumption that frequencies with which different item endorsement profiles occur can be explained by the existence of a few mutually exclusive respondent classes. LPA is a form of latent variable mixture modeling that aims at identifying different latent clusters, classes, or subgroups of individuals with similar response patterns based upon a set of categorical or continuous cluster indicators [58,59]. The main advantage of LPA over common cluster analytic techniques such as cluster analysis is that LPA is model-based, whereas hierarchical and most non-hierarchical techniques of cluster analysis are not [60]. In the present study, LPA was used to derive classes or subgroups of students who share similar profiles of mathematics achievement emotions.

Running the LPA: The analysis was run using Mplus 7.11 [58]. The analysis proceeded in two steps. Firstly, based on students' responses to the polytomous items of the LRES-M-AR, LPA was used to classify students who share a common pattern of mathematics achievement emotions into homogeneous latent classes. LPA uses all observations of the eight subscales scores of the LRES-M-AR to define these classes *via* maximum likelihood estimation method. LPA calculates the probability that a person is appropriately classified in order to categorize each person in the best-fitting latent class. Secondly, to determine the number of classes, the analysis allowed the comparison of several latent class models with different numbers of latent classes in terms of their relative model fit [59,61,62].

Model fit in LPA: Several models were estimated wherein latent classes were added iteratively to decide which model achieves the best fit to the current data. Each model was evaluated using the following fit indices: (1) the Lo-Mendell-Rubin Adjusted Likelihood Ratio Test (LMRT), (2) the Bootstrapped Likelihood Ratio Test (BLRT), (3) Akaike Information Criteria (AIC), and (4) Sample size-adjusted Bayesian Information Criteria (sBIC), (5) Logarithmic value of the likelihood (the log-likelihood or LL), and (6) The entropy statistics. The LMRT and BLRT fit indices compare the fit of the target model (e.g., 2 class model) to the specified model which specifies one less class (e.g., 1 class model). The p-value of LMRT and BLRT indicates whether the solution with more classes ($p<0.05$) or less classes ($p>0.05$) fits better. The smaller the values of the AIC and sBIC the better model fits the data [58,60,61].

The log-likelihood or LL of the final parameter estimates is a measure of model fit with higher values (e.g., closer to 0) indicating better fit than lower values. The entropy statistics is used to assess the classification utility of the model. It rang from 0 to 1 with higher values indicative of higher classification utility. A common cutoff value for the entropy statistic to highlight a model acceptable classification utility is 0.80. The entropy statistic is best used to compare the classification utility of different models fit to the same sample or of the same model fit to different samples [61]. In addition to these fit indices, the interpretability of each model was assessed to determine whether the latent classes actually represented different categories, instead of being a product of a non-normal distribution. Those classes that contain <5% of the sample are generally considered as spurious classes; a condition related to extracting many latent classes or profiles [63], therefore class size was also considered when determining the optimal number of classes [58,59].

LPA in the present study: Several latent profile models with 2 to 8 latent classes were compared using several model fit indices in order to determine the appropriate number of latent classes within a model. The fit indices for each model are summarized in Table 5. The LMRT ($p<0.007$) and BLRT ($p<0.008$) were statistically significant for the two-class solution. In addition, the model classification utility was within an acceptable level as determined by the entropy statistics (0.85). However, the three-class solution was superior to the two-class solution due to significant LMRT ($p<0.001$) and the BLRT ($p<0.002$) and substantially lower AIC and sBIC values. In addition, the model classification utility as determined by the entropy statistics was higher (0.94) compared to the two-class solution (0.85).

Furthermore, the fit indices indicated that the four- to eight- class solutions failed to achieve an acceptable fit to the current dataset. The non-significant values of the LMRT ($p<0.10$ to 0.37) and the BLRT ($p<0.12$ to 0.41) suggested that additional classes did not result in a significant improvement in fit over the three-class solution. The AIC and the sBIC values for the four- to eight-solution decreased slightly compared to the three-class solution. Although the model classification utility values as indicated by the entropy values were within acceptable range (0.83 to 0.84), they were notably lower than that of the three-class solutions (0.94). In terms of the interpretability, classes of all models contained more than < 5% of the sample and thus they were not considered spurious classes, except for the eight class solution that yielded a class size that was too small to be of substantive value (12 individuals for 3.3% of the sample). In conclusion, the 3-class solution achieved the best fit to the current dataset.

Three classes of mathematics achievement emotions: The three classes identified through the LPA include Class 1 (positive emotions profile) which constituted 40.3% (n=143) of the sample. Students with this profile had high levels of positive mathematics achievement emotions and low levels of negative mathematics achievement emotions. Students' mean scores on the positive mathematics achievement emotions ranged from 3.70 to 4.10 whereas their mean scores on the negative mathematics achievement emotions ranged from 1.16 to 1.74. Class 2 (negative emotions profile) constituted 33.5% (n=119) of the sample. Students with this profile had high levels of negative mathematics achievement emotions and low levels of positive mathematics achievement emotions. Students' mean scores on the negative mathematics achievement emotions ranged from 3.55 to 4.25 whereas their mean scores on the positive emotions ranged from 1.11

Solution/Class	LMRT (p)	BLRT (P)	AIC	sBIC	Log L	Entropy
2 class	133.17 ($p<0.007$)	138.28 ($p<0.008$)	2302.15	4227.33	-43658.22	0.85
3 class	115.11 ($p<0.001$)	119.66 ($p<0.002$)	1866.33	3880.22	-40365.17	0.94
4 class	74.23 ($p<0.10$)	79.33 ($p<0.12$)	1862.17	3875.96	-39966.35	0.84
5 class	66.55 ($p<0.14$)	71.17 ($p<0.16$)	1855.96	3868.12	-39949.22	0.84
6 class	55.13 ($p<0.21$)	60.23 ($p<0.23$)	1852.13	3864.33	-39933.10	0.83
7 class	44.36 ($p<0.29$)	51.22 ($p<0.27$)	1847.55	3858.12	-39902.14	0.83
8 class	33.11 ($p<0.37$)	37.14 ($p<0.41$)	1840.22	3851.88	-39892.36	0.84

Table 5: Fit indices for eight LPA models based on different numbers of two to eight latent classes. **Note:** N=355.

to 1.45. Class 3 (positive emotions high boredom profile) constituted 26.2% (n=93) of the sample. Students with this profile had high levels of positive mathematics achievement emotions and low levels of negative mathematics achievement emotions except for boredom which was remarkably higher than the other groups. Students' mean scores on the positive mathematics achievement emotions ranged from 3.89 to 4.35 and their score on boredom was 3.65. Table 6 show the mean and standard deviation for each achievement emotion within each of the three latent classes. Figure 2 displays the mathematics achievement emotions profiles of the three latent classes.

Multivariate analysis of variance

A Multivariate Analysis of Variance (MANOVA) was conducted to examine whether students' mathematics achievement and mathematics tracking differed as a function of their mathematics achievement emotions profiles. Students' mathematics achievement emotions profiles (3 latent classes) were set as an independent variable whereas students' mathematics achievement and mathematics tracking were set as dependent variables. Table 4 shows that the correlation coefficient between students' mathematics achievement and mathematics tracking was moderate and statistically significant (r=0.46, p<0.01). This finding supports the usage of the MANOVA [64].

The analysis showed that students' mathematics achievement and mathematics tracking differed significantly according to their mathematics achievement emotions profiles, Pillai's Trace=0.46, F (4, 704)=8.17, p<001,η^2=0.29 which meant that 29% of the variance in the canonical dependent variable of mathematics achievement and mathematics tracking was explained by students' mathematics achievement emotions profiles. Cohen's guidelines set 0.01 for small, 0.06 for medium, and 0.14 for large partial Eta Square effect size respectively [45]. Thus, the η^2=0.29 represents a large effect size. To examine the effect of the achievement emotions profiles on students' mathematics achievement and mathematics tracking separately and independently, a one-way analysis of variance (ANOVAs) was conducted for each dependent variable. Table 7 shows the means and the standard deviations of mathematics achievement and mathematics tracking for each of students' mathematics achievement emotions profiles.

In the first ANOVA, mathematics achievement emotions profiles were set as the independent variable and mathematics achievement was set as the dependent variable. The analysis showed significant differences in mathematics achievement among the three achievement emotions profiles, F (2, 352)=11.22, p<0.001, partial η^2=0.18. A partial Eta Square (η^2=0.18) indicated that 18% of the variance in mathematics achievement was accounted for by the differences in mathematics achievement emotions profiles. Post-hoc analyses using Scheffe's method showed that all mean comparisons were statistically significant (p<0.001). Specifically, students with positive emotions profile had the highest mathematics achievement (M=94.56, SD=1.12), followed by students with negative emotions profile (M=88.55, SD=0.72), and

Mathematics achievement emotions	Latent class 1 [1] (n = 143 (40.3%)) Positive emotions		Latent class 2 (n = 119 (33.5%)) Negative emotions		Latent class 3 (n = 93 (26.2%)) Positive emotions high boredom	
	M	SD	M	SD	M	SD
1. Enjoy	4.10	0.36	1.39	0.52	4.35	0.96
2. Pride	3.85	0.51	1.11	0.41	4.11	0.45
3. Hope	3.70	0.27	1.45	0.33	3.89	0.67
4. Boredom	1.66	0.41	4.25	0.77	3.65	0.65
5. Anxiety	1.42	0.50	4.12	0.54	1.33	0.73
6. Hopelessness	1.74	0.33	4.19	0.63	1.22	0.48
7. Shame	1.30	0.29	3.55	0.81	1.73	0.41
8. Anger	1.16	0.42	3.92	0.45	1.36	0.50

Table 6: Three latent class means and standard deviations by mathematics achievement emotions.
Note: N=355. [1]Means showed in bold are the highest means in the specified latent class.

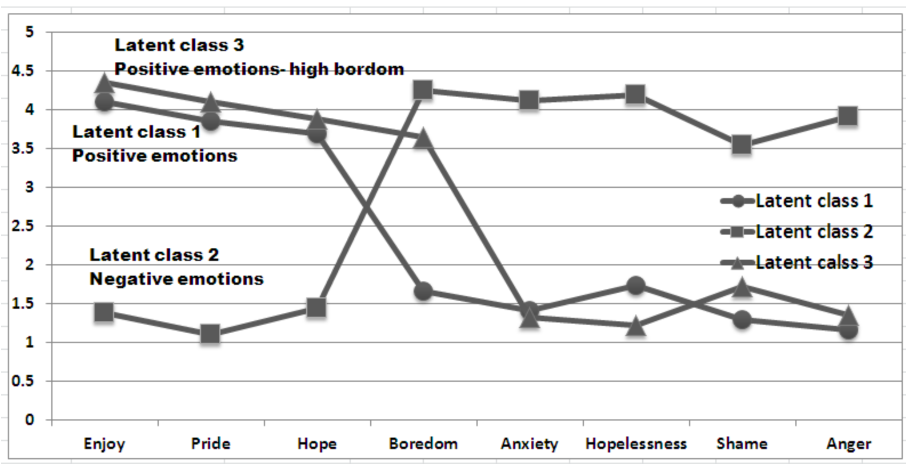

Figure 2: Mathematics achievement emotions profiles represented by three latent classes.

finally students with positive emotions high boredom profile (M=84.12, SD=0.45). In the second ANOVA, mathematics tracking was set as the dependent variable. The analysis showed significant differences in mathematics tracking among the three achievement emotions profiles, F (2, 352)=16.77, p<0.001, partial η^2=0.22. A partial Eta Square (η^2=0.22) indicated that 22% of the variance in mathematics tracking was accounted for by the differences in mathematics achievement emotions profiles. Post-hoc analyses using Scheffe's method showed that all mean comparisons were statistically significant (p<0.001). Specifically, students with positive emotions profile had the highest mathematics tracking (M=75.71, SD=1.19), followed by students with negative emotions profile (M=71.22, SD=0.72), and finally students with positive emotions high boredom profile (M=67.12, SD=0.47). Figure 3 shows means of mathematics achievement and mathematics tracking according to students' mathematics achievement emotions profiles.

Discussion

A primary objective of the present study was to validate the control-value theory within the domain of mathematics in a non-Western context. Specifically, the present study examined: (1) whether Egyptian high school students could be grouped into several achievement emotions profiles based upon their feelings relate to the subject matter of mathematics, and (2) whether identified groups of Egyptian high school students would differ in their mathematics achievement and mathematics tracking. Generally, the findings of the present study showed how multiple forms of achievement emotions can combine together to affect later mathematics achievement and mathematics tracking. Egyptian students were found to experience multiple types of achievement emotions concurrently within the domain of mathematics, distinct clusters of Egyptian students with diverse mathematics emotional experiences emerged in the classroom, and these multifaceted mathematics emotional experiences differentially impacted students' mathematics achievement and mathematics tracking. These findings broadly supported the basic notion of the control-value theory [1] that links achievement emotions and academic outcomes through emphasizing the adaptive impact of positive achievement emotions on academic outcomes and the maladaptation of negative achievement emotions.

The latent profile analysis identified three qualitatively different achievement emotions within the subject matter of mathematics that reflected the expected dimensions of positive and negative valence. Specifically, the analysis showed that Egyptian high school students could be organized into three distinct groups or clusters of achievement emotions associated with the subject matter of mathematics; positive-emotions, negative emotions, and positive-emotion with high boredom.

Dependent variables	Independent variable	*M*	*SD*	*F* (2, 352)	Partial Eta Square η^2
	Achievement emotions profiles				
Mathematics achievement	Positive emotions			11.22**	0.18
	Negative emotions	94.56	1.12		
	Positive emotions high boredom	88.55	0.72		
		84.12	0.45		
Mathematics tracking				16.77**	0.22
	Positive emotions	75.71	1.19		
	Negative emotions	71.22	0.72		
	Positive emotions high boredom	67.12	0.47		

Note: *N*=355. ** is significant at 0.01.

Table 7: Means, standard deviations, *F* values and partial eta square (η^2) for the effect of mathematics achievement emotions profiles on mathematics achievement and mathematics tracking.

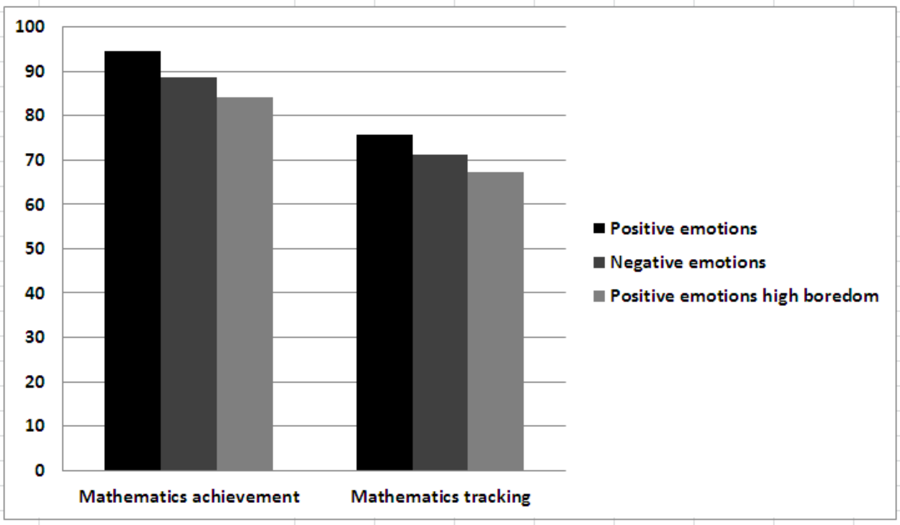

Figure 3: Means of mathematics achievement and mathematics tracking according to mathematics achievement emotions profiles.

This finding is consistent with the findings of other studies that reported similar achievement emotions profiles [24,38]. This finding provides support for research that clusters emotions into high-level classes based upon the dimensions of valence and arousal [3,32,65]. It also agrees with the premises of the control-value theory that the achievement emotions determined by control and values should be examined from a domain-specific perspective [19].

The emergence of three different achievement emotions profiles highlighted the complexity and the interrelatedness of the factors associated with the subject matter of mathematics that can have different effects on mathematic achievement emotions formation and integration. According to the control-value theory, certain discrete achievement emotions arise from various combinations of appraisal antecedents and show qualitative differences in their components. Thus, a full understanding of these emotions entails the recognition of their multiplicity. In fact, the formation of the three achievement emotions profiles supports the multi-emotion perspective that different groups of students can experience multiple simultaneous emotions within the same academic setting and that these set of emotions can have important implication for students' academic success [3]. Pekrun [1] agreed with this viewpoint and proposed that it is common for individuals to experience several emotions simultaneously within an academic context and that these emotional experiences may imply distinctive types of motivation and behavior. However, it should be emphasized that the notion of achievement emotions multiplicity stands against the variable-centered approach that links achievement emotions individually and distinctively to important achievement-related activities and outcomes and does not take into account the experience of multiple simultaneous emotions within an achievement context [10].

Furthermore, the emerged achievement emotions profiles and in particular the positive-emotion with high boredom profile as well as the conceptual differences among these profiles add to the theory and the previous research on the control-value theory that an assessment of achievement emotions in educational settings should include both valence (positive/negative) and activation (activation/deactivation) dimensions [1]. Both valence and activation stand out as factors that discriminate qualitatively different achievement emotions experiences from one another within the domain of mathematics. These findings, in combination with previous studies, offer further evidence that different groups of students can experience multiple concurrent achievement emotions within the same academic setting. An outstanding finding of the latent profile analysis in the present study concerns the merging of the positive emotions high boredom profile which combines and blends positive active emotions (e.g., enjoyment, hope, and pride) with a negative deactivated emotion (i.e., boredom). This is a unique pattern of achievement emotions because previous research used to identify profiles characterized by valence (positive versus negative emotions) and intensity (low, moderate, and strong emotions) with scant attention been given to indicators of negative deactivated achievement emotions [66].

Another finding of the present study concerns the differential effects of achievement emotions profiles on students' mathematics achievement and mathematics tracking. The analysis showed that students with positive emotions profile had higher mathematics achievement and reported more mathematics tracking than students with negative emotions profile and the student with positive emotion with high boredom profile. This finding highlighted that emotion is functionally important for students' learning and performance and is consistent with the general principles of the control-value theory that emotions can influence the underlying mechanisms of coding new information into memory and retrieving information from memory and thereby affects learning and performance processes [1,67]. This finding also agrees with a previous conclusion that students in the positive emotions group outperformed students in the negative emotions group in several educational outcomes [24,33,37,38]. However, this finding is in contrast with a recent reported finding by Jarrell et al. [32] that students in the positive emotion group had academic performance levels similar to students in the negative emotion group.

Pekrun and Stephens explained how positive and negative emotions affect learning process and performance by proposing that emotions are expected to affect three main underlying mechanisms [1]: (a) cognitive resources that help individuals concentrate on completion of achievement tasks; (b) interest and motivation to perform achievement tasks; and (c) cognitive and metacognitive strategies including the self-regulated learning strategies. As for learning, the control-value theory proposes that emotions can distinctly influence learning strategies and self-regulation of learning [1,68]. Initiating positive emotions is widely considered adaptive because they facilitate the use of effective cognitive and meta-cognitive learning strategies that adapt learning to individual goals. In contrast, negative emotions are generally viewed as maladaptive because they stimulate surface cognitive processing and dependence on external guidance [1]. For example, there are some research evidence indicating that feeling of enjoyment during learning has been linked with better academic outcomes including higher academic achievement because this positive achievement emotion can help maintain and sustain cognitive resources during achievement activities [1]. In contrast, feeling bored is related to poorer academic outcomes because this negative emotion undermines concentration on the learning task [16].

As for performance, emotions can also impact performance during an achievement situation. In the control-value theory, positive achievement emotions can facilitate retrieval of self-regulatory strategies, self-related and task-related information (e.g., elaboration and organization of task material), sustain cognitive resources to perform the achievement task, draw attention to the achievement task, enable the use of effective cognitive strategies, and enhance interest and intrinsic motivation. All these factors have positive impact on overall performance under many task conditions [1,16]. However, negative achievement emotions are expected to deter information retrieval and occupy attention with task irrelevant thoughts [3], thus reducing cognitive resources available for task performance in term of working memory capacity, and thereby negatively affecting performance [67]. It is therefore possible to suggest that when emotions and cognition interact together, they can result in better performance outcomes when positive activating emotions are triggered, such as enjoyment and hope [10], and they can result in poorer performance outcomes when negative emotions are stimulated such as anxiety [7].

Furthermore, the analysis showed that students with positive emotions high-boredom profile had the poorest mathematics achievement and reported the least mathematics tracking compared to all other profiles. This was unexpected finding because positive emotions have typically been regarded as a positive predictor of students' academic performance [1]. However, it seems that experiencing positive emotions with bouts of deactivating negative feelings such as boredom can have a damaging effect on students' mathematics achievement and mathematics tracking. Positive emotions apparently failed to counterbalance the detrimental effect of students' boredom emotions on their mathematics achievement and mathematics tracking. Put bluntly, feeling boredom was shown to be able to take a great toll on students' mathematics achievement and mathematics tracking regardless of their positive emotions.

One possible interpretation of this finding is that boredom as a negative deactivating emotion typically harms performance by reducing cognitive resources available to think about the achievement task, discouraging both intrinsic and extrinsic motivation, hindering metacognitive processes necessary for understanding and learning, and stimulating superficial information processing [1,69,70]. For example, researchers have reported boredom as being related negatively to measures of students' motivation, school belongingness, academic commitments, study behavior, and academic achievement [1]. In addition, experimental research on boredom has shown that boredom can reduce performance on some very simple, repetitive tasks, such as assembly-line, vigilance, or data entry tasks [71].

Another possible interpretation for this finding is available through the findings of previous research that highlighted positive emotions as maladaptive. For example, Meinhardt and Pekrun [67] found that positive emotions can reduce task-related cognitive resources. Similarly, Seibert and Ellis [72] found that positive emotions can stimulate task irrelevant thinking. Rieber and Noah [73] reported that strong positive emotions such as enjoyment can be negatively associated with explicit learning. For example, high levels of enjoyment might have been triggered through exploratory learning (e.g., reading irrelevant content) or from engaging in off-task behaviors such as experimenting features (e.g., seeing how many lab tests can be run at once). Aspinwall [74] explained how research has documented that positive emotions may not be always be useful for learning and performance: (1) positive emotions can lead to overestimation of the probability of success and an underestimation of the probability of failure due to emotion-congruent retrieval of information related to positive outcomes, (2) positive emotions can encourage relaxation and consequently hinder effort expenditure by signaling that everything is fine, and thus exerting effort seems unnecessary, and (3) positive emotions can encourage motivation to keep pleasant mood by escaping negative thoughts and ignoring warning prevention of future hardships. From this perspective, according to Aspinwall "*our primary goal is to feel good, and feeling good makes us lazy thinkers who are oblivious to potentially useful negative information and unresponsive to meaningful variations in information and situation*" [74].

The findings of the present study are constrained by three important limitations. Firstly, we used self-report measures of achievement emotions rather than behavioral indices. Secondly, data were cross-sectional and consequently decisive conclusions about achievement emotions within the domain of mathematics cannot be drawn. Thirdly, data were collected from high school students only in Egypt, and hence it may not possible to generalize the findings across younger or older populations.

In conclusion, the findings of the present study provided valuable evidence to support the theoretical relationship between achievement emotions and key educational outcomes through a person-centered approach. This relationship held when examined in a non-Western context and within a specific academic domain such as mathematics and thus provide a valuable evidence for the validity of the control-value theory. Generally our findings indicated that students who experienced more positive emotions within the domain of mathematics and little negative emotions were found to achieve the best academic outcomes in terms of mathematics achievement and mathematics tracking.

Implications for Teaching and Learning Practices at School

The findings of the present study underscore several important implications for teaching and learning practices of mathematics:

(1) Classroom teacher should use mathematics learning tasks of high cognitive quality that are well-designed and structured and moderately challenging. These tasks are likely to promote high levels of self-confidence and self-efficacy, curiosity, and surprise during mathematics problem solving and therefore elicit feeling of enjoyment, happiness, and pride and decrease feelings of anxiety and fear.

(2) Classroom teacher should use and focus on mathematics content that is meaningful and related to students' current interests at and out of school including their daily-life activities, and academic and future career. This content can promote students' interest and intrinsic task value and consequently trigger their positive feelings of enjoyment and hope.

(3) Classroom teacher should help students being autonomous learners who are able to regulate their learning of mathematics by defining specific learning goals, select appropriate learning tasks and strategies for learning, and continuously monitor and evaluate students' learning progress. Learners' autonomy is likely to develop their feelings self-confidence and personal reasonability for their learning and consequently trigger positive feeling of enjoyment and excitement.

(4) Classroom teacher should create and design a social learning environment to teach mathematics through peer learning and team work. This social environment can satisfy students' needs for social interaction during learning mathematics and consequently increase their positive feelings.

(5) Classroom teachers should show their students the positive feeling of enjoyment and enthusiasm they experience about school life and specifically teaching the subject matter of mathematics because emotions are contagious and they can pass from teachers to students.

References

1. Pekrun R (2006) The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. Educational Psychology Review 18: 315-341.
2. Pekrun R, Linnenbrink-Garcia L (2014) International handbook of emotions in education. Routledge, New York.
3. Pekrun R, Perry RP (2014) Control-value theory of achievement emotions. In: R. Pekrun & L. Linnenbrink-Garcia (Eds.), International handbook of emotions in education. Rutledge, New York.
4. Baumeister RF, Bushman BJ (2007) Angry emotions and aggressive behaviors. In: G. Steffgen & M. Gollwitzer (Eds.), Emotions and aggressive behavior. Hogrefe & Huber, Ashland, OH.
5. Pekrun R, Linnenbrink-Garcia L (2012) Academic emotions and student engagement. In: SL Christensen, AL Reschly & C Wylie (Eds.), Handbook of research on student engagement. Springer, New York.
6. Pekrun R, Lichtenfeld S, Marsh HW, Murayama K, Goetz T (2017) Achievement emotions and academic performance: Longitudinal models of reciprocal effects. Child Dev 88: 1653-1670.
7. Zeidner M (2007) Test anxiety in educational contexts: What I have learned so far. In: PA Schutz & R Pekrun (Eds.), Emotion in education. Academic Press, San Diego, CA.
8. Weiner B (1985) Attributional theory of achievement motivation and emotion. Psych Review 92: 548-573.

9. Pons F, Hancock D, Lafortune L, Doudin PA (2005) Emotions in learning. Aalborg University Press, Aalborg.

10. Pekrun R (2009) Global and local perspectives on human affect: Implications of the control value theory of achievement emotions. In: M Wosnitza, SA Karabenick, A Efklides, P Nenniger (Eds.), Contemporary motivation research: From global to local perspectives. Hogrefe, Cambridge, MA.

11. Dettmers S, Trautwein U, Ludtke O, Goetz T, Frenzel AC, et al. (2011) Students' emotions during homework in mathematics: Testing a theoretical model of antecedents and achievement outcomes. Contemp Educ Psychol 35: 25-35.

12. Linnenbrink-Garcia L, Rogat TK, Koskey KL (2011) Affect and engagement during small group instruction. Contemp Educ Psychol 36: 13-24.

13. Baker RSJ, D'Mello SK, Mercedes MT, Graesser AC (2010) Better to be frustrated than bored: The incidence, persistence, and impact of learners' cognitive-affective states during interactions with three different computer-based learning environments. Int J Hum Comput Stud 68: 223-241.

14. Schutz PA, Lanehart SL (2002) Emotions in education. Educational Psychologist 37: 1-2.

15. Pekrun R, Frenzel A, Goetz T, Perry RP (2007) The control-value theory of achievement emotions: An integrative approach to emotions in education. In: PA Schutz & R Pekrun (Eds.), Emotion in education. Academic Press, San Diego, CA.

16. Pekrun R, Stephens EJ (2010) Achievement emotions: A control-value approach. Soc Personal Psychol Compass 4: 238-255.

17. Goetz T, Pekrun R, Hall NC, Haag L (2006) Academic emotions from a social-cognitive perspective: Antecedents and domain specificity of students' affect in the context of Latin instruction. Br J Educ Psychol 76: 289-308.

18. Pekrun R, Elliot AJ, Maier MA (2006) Achievement goals and discrete achievement emotions: A theoretical model and prospective test. J Educ Psychol 98: 583-597.

19. Goetz T, Frenzel AC, Pekrun R, Hall NC, Lu¨dtke O (2007) Between- and within-domain relations of students' academic emotions. J Educ Psychol 99: 715-733.

20. Bong M (2001) Between- and within-domain relations of motivation among middle and high school students: Self-efficacy, task value, and achievement goals. J Educ Psychol 93: 23-34.

21. Bandura A (2012) On the functional properties of perceived self-efficacy revisited. Journal of Management 38: 9-44.

22. Goetz T, Cronjaeger H, Frenzel AC, Ludtke O, Hall NC (2010) Academic self-concept and emotion relations: Domain specificity and age effects. Contemp Educ Psychol 35: 44-58.

23. Frenzel AC, Thrash TM, Pekrun R, Goetz T (2007) Achievement emotions in Germany and China: A cross-cultural validation of the Academic Emotions Questionnaire-Mathematics (AEQ-M). J Cross Cult Psychol 38: 302-309.

24. Ganotice FA, Datu JAD, King RB (2016) Which emotional profiles exhibit the best learning outcomes? A person-centered analysis of students' academic emotions. School Psychology International 37: 498-518.

25. Robinson KA, Ranellucci J, Lee Yk, Wormington SV, Roseth CJ, et al. (2017) Affective profiles and academic success in a college science course. Contemp Educ Psychol 51: 209-221.

26. National Research Council (2015) Mathematics and future career: An annual report. NRC publishing, New York.

27. Baloglu M, Kocak R (2006) A multivariate investigation of the differences in mathematics anxiety. Pers Individ Dif 40: 1325-1335.

28. Nasser F, Birenbaum M (2005) Modeling mathematics achievement of Jewish and Arab eighth graders in Israel: The effects of learner-related variables. Educational Research and Evaluation 11: 277-302.

29. Ministry of Education (2015) Egyptian students' performance in TIMSS. A comprehensive analysis. Cabinet of Egypt, Cairo.

30. Khalid F, Young G (2014) Factors affecting Egyptian students' mathematics achievement at high school and university: A longitudinal study. Chines Journal of Psychology 4: 55-70.

31. Fernando JW, Kashima Y, Laham SM (2014) Multiple emotions: A person-centered approach to the relationship between intergroup emotion and action orientation. Emotion 14: 722-732.

32. Jarrell A, Harley JM, LaJoie SP (2016) The link between achievement emotions, appraisals, and task performance: Pedagogical considerations for emotions in CBLEs. J Comp Edu 3: 289-307.

33. Jarrell A, Harley JM, LaJoie SP (2017) Success, failure and emotions: Examining the relationship between performance feedback and emotions in diagnostic reasoning. Educ Technol Res Dev 65: 1263-1284.

34. Bergman LR, Magnusson D, El Khouri BM (2003) Studying individual development in an interindividual context: A person-oriented approach. Psychology Press, Chicago.

35. Bodas J, Ollendick TH (2005) Test anxiety: A cross-cultural perspective. Clin Child Fam Psychol Rev 8: 65-88.

36. Keeley J, Zayac R, Correia C (2008) Curvilinear relationships between statistics anxiety and performance among undergraduate students: Evidence for optimal anxiety. Statistics Education Research Journal 7: 4-15.

37. Barker ET, Howard AL, Galambos NL, Wrosch C (2016) Tracking affect and academic success across university: Happy students benefit from bouts of negative mood. Dev Psychol 52: 2022-2030.

38. Molk GL (2012) College students' emotions during learning mathematics in South Africa: A person-centered analysis. Journal of High School Research 3: 45-57.

39. Abd-El-Fattah SM (2013) A cross-cultural examination of the Aggression Questionnaire–Short Form among Egyptian and Omani Adolescents. Journal of Personality Assessment 5: 539-548.

40. Pekrun R, Goetz T, Perry RP (2005) Achievement emotions questionnaire (AEQ). User's manual. Department of Psychology, University of Munich, Munich, Germany.

41. Cohen J, Swerdlik M (2009) Psychological testing and assessment: An introduction to tests and measurement. McGraw-Hill, New York.

42. Kaplan RM, Saccuzzo DP (2012) Psychological testing: Principles, applications, and issues. Cengage Learning, Washington DC.

43. Maneesriwongul W, Dixon JK (2004) Instrument translation process: A methods review. J Adv Nurs 48: 175-186.

44. Cohen J (1960) A coefficient of agreement for nominal scales. Educ Psychol Meas 20: 37-46.

45. Cohen J (1988) Statistical power analysis for the behavioral sciences. Lawrence Erlbaum Associates, Hillsdale, NJ.

46. Landis JR, Koch GG (1977) The measurement of observer agreement for categorical data. Biometrics 33: 159-174.

47. King RB (2010) What do students feel in school and how do we measure them? Examining the psychometric properties of the S-AEQ-F. Philipp J Psychol 43: 161-176.

48. Goetz T, Frenzel AC, Hall NC, Pekrun R (2008) Antecedents of academic emotions: Testing the internal/external frame of reference model for academic enjoyment. Contemp Educ Psychol 33: 9-33.

49. Peixoto F, Mata L, Monteiro V, Sanches C, Pekrun R (2015) The Achievement Emotions Questionnaire: Validation for pre-adolescent students. European Journal of Developmental Psychology 12: 472-481.

50. Kline RB (2011) Principles and practice of structural equation modeling. Guilford Press, New York.

51. Byrne BM (2010) Structural equation modeling with AMOS: Basic concepts, applications, and programming. Routledge Academic, Washington, DC.

52. Tabachnick BG, Fidell LS (2013) Using multivariate statistics. Pearson, Boston.

53. Arbuckle JL (2012) AMOS (Version 20). SPSS, Chicago.

54. Blunch NJ (2008) Introduction to structural equation modeling using SPSS and AMOS. Sage Publications Ltd, Thousand Oaks, CA.

55. Hoyle RH (2012) Handbook of structural equation modeling. The Guilford Press, London.

56. Hox JJ, Bechger T (2014) An introduction to structural equation modeling. Family Science Review 11: 354-373.

57. Brahier D (2001) Assessment in middle and high school mathematics: A teacher's guide. Routledge, London.

58. Muthen LK, Muthen BO (2012) Mplus user's guide. Muthen & Muthen, Los Angeles, CA.

59. Nylund KL, Asparouhov A, Muthén B (2007) Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo simulation study. Structural Equation Modeling 14: 535-569.

60. Kongsted A, Nielsen AM (2017) Latent class analysis in health research. J Physiother 63: 55-58.

61. Collins LM, Lanza ST (2010) Latent class and latent transition analysis with applications in the social behavioral and health sciences. Wiley, Washington, DC.

62. Marsh HW, Ludtke O, Trautwein U, Morin AJS (2009) Classical latent profile analysis of academic self-concept dimensions: Synergy of person- and variable-centered approaches to theoretical models of self-concept. Structural Equation Modeling 16: 191-225.

63. Hipp JR, Bauer DJ (2006) Local solutions in the estimation of growth mixture models. Psychol Methods 11: 36-53.

64. Meyers LS, Gamst G, Guarino AJ (2006) Applied multivariate research: Design and interpretation. Sage Publications, Thousand Oaks.

65. Harley JM, Bouchet F, Hussain S, Azevedo R, Calvo R (2015) A multi-componential analysis of emotions during complex learning with an intelligent multi-agent system. Comput Human Behav 48: 615-625.

66. Linnenbrink-Garcia L, Patall EA, Pekrun R (2016) Adaptive motivation and emotion in education research and principles for instructional design. Policy Insights Behav Brain Sci 3: 228-236.

67. Meinhardt J, Pekrun R (2003) Attentional resource allocation to emotional events: An ERP study. Cogn Emot 17: 477-500.

68. Zimmerman BJ, Labuhn SS (2012) Self-Regulation of learning: Process approaches to personal development. In: KR Harris, S Graham, T Urdan (Eds.), The educational psychology handbook: Theories, constructs, and critical issues. American Psychological Association, Washington DC.

69. Pekrun R, Goetz T, Daniels LM, Stupnisky RH, Perry RP (2010) Boredom in achievement settings: Exploring control-value antecedents and performance outcomes of a neglected emotion. J Educ Psychol 102: 531-549.

70. Pekrun R, Hall NC, Goetz T, Perry RP (2014) Boredom and academic achievement: Testing a model of reciprocal causation. J Educ Psychol 106: 696-710.

71. Kass SJ, Vodanovich SJ, Stanny C, Taylor T (2001) Watching the clock: Boredom and vigilance performance. Percept Mot Skills 92: 969-976.

72. Seibert PS, Ellis HC (1991) Irrelevant thoughts, emotional mood states, and cognitive task performance. Mem Cognit 19: 507-513.

73. Rieber LP, Noah D (2008) Games, simulations, and visual metaphors in education: Antagonism between enjoyment and learning. Educational Media International 45: 77-92.

74. Aspinwall LG (1998) Rethinking the role of positive affect in self-regulation. Motiv Emot 22: 1-32.