

## A New Method to Predict Ratio of Width to Height Rock Pillar in Twin Circular Tunnels

Asghar Siahmansouri<sup>1\*</sup>, Javad Gholamnejad<sup>2</sup> and Mohammad Fatehi Marji<sup>3</sup>

<sup>1</sup>M.Sc. Student, Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran

<sup>2</sup>Associate Professor, Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran

<sup>3</sup>Associate Professor, Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran

### Abstract

Twin tunnels can be used for many applications. Interaction between two tunnels is an important problem in tunnel engineering that should be studied specially. Numerical investigations are well adapted to field data and numerical methods can be used in design of rock pillar of twin circular tunnels. So far no relationship has been provided to estimate the minimum stable rock pillar. In this paper the interaction between twin circular tunnels has been studied using 2D finite element analysis. To do this, a great number of twin tunnels were modeled in Phase2 software with different conditions of rock mass (RMR value) and depth of tunnel. Models were analyzed and minimum stable rock pillar was determined. This process was repeated for three different ratios of K (ratio of horizontal stress to vertical stress, 0.5, 1 and 1.5). Finally, according to the linear and nonlinear regression methods, the best merit function was fitted to result of numerical analysis. Then, a new approximate formula was proposed to predict ratio of width to height rock pillar according to RMR value and depth of twin circular tunnels with different K values. The formulae are very accurate (coefficient of correlation equals to 0.96) that can be used for predicting ratio of width to height rock pillar in twin circular tunnels.

**Keywords:** Twin circular tunnels; Rock pillar; Numerical modeling; Non linear regression; Phase2

### Introduction

Nowadays, multiple excavations can be used for many applications such as: roads, railways, subways, hydraulic tunnels, etc. Of these, the twin tunnels are used more than other spaces. Interaction between these tunnels is an important problem in tunnel engineering that should be studied specially.

Multiple excavations have been studied in models and results are presented by Obert and Duvall [1,2]. Under elastic conditions, two tunnels interact with each other if separated by a thickness of rock less than two times the sum of their dimensions in the direction parallel to the separation. As tunnels approach each other, the average stress in the rock pillar between them increases and approaches the maximum tangential stress (Figure 1).

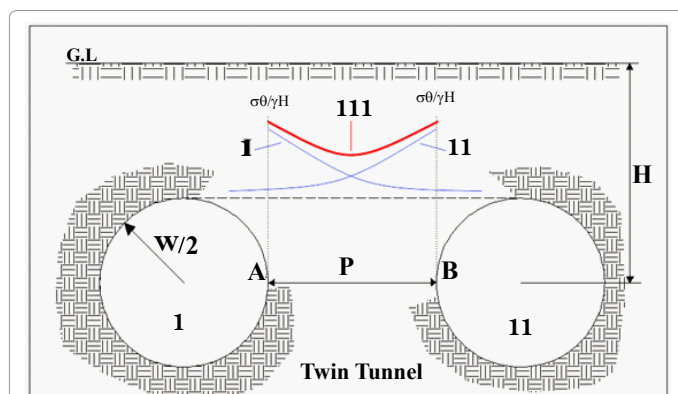
As a greater number of discontinuities are contained within a sample of rock, its strength must decrease. Accordingly, when the span

of a tunnel is many times greater than the average spacing between excavations, the tunnel cannot be expected to stand without artificial support [3].

Stress concentrations will generally be the least troublesome if smooth shapes are used, without corners and reentrants, and if the major axis is aligned to the major principal stress, with the ratio of width to height proportional to K (horizontal stress to vertical stress ratio) [4]. Openings of other shapes have been solved mathematically and solutions can be found in Muskhelishvili [5]. Elliptical and other idealized shapes are discussed by Jaeger and Cook and Obert and Duvall [1,6].

For twin tunnels, by decrease in tunnels spacing, the total project cost would decrease; using geotechnical studies, preparation and cross cut can be reduced. On the other hand, when the spacing between tunnels is very little, plastic zones of the tunnels would join each other; therefore rock pillar placed in plastic zone and tunnels may become unstable. So, the minimum stable rock pillar between twin tunnels should be determined by technical and economic studies. To do this, the spacing between two tunnels should be far enough to prevent plastic zones from meeting.

Stresses around multi-excavations are determined using modeling and numerical methods [7]. A numerical investigation of rock pillar failure mechanism in underground openings was done by Mortazavi et al. [8]. They found that numerical investigations are well adapted to the field data and numerical methods can be used in the design



**Figure 1:** Geometrical parameters of twin circular tunnels in massive rock  
I: Ratio of secondary stress to initial stress around circular tunnel I ( $\sigma_{\theta}/\gamma H$ )  
II: Ratio of secondary stress to initial stress around circular tunnel II ( $\sigma_{\theta}/\gamma H$ )  
III: Interaction of I and II secondary stress on rock pillar between twin circular tunnels (I+II).

\*Corresponding author: Asghar Siahmansouri, M.Sc. student, Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran, E-mail: [a.s.mansouri@stu.yazduni.ac.ir](mailto:a.s.mansouri@stu.yazduni.ac.ir)

Received February 10, 2012; Accepted April 30, 2012; Published May 02, 2012

Citation: Siahmansouri A, Gholamnejad J, Marji MF (2012) A New Method to Predict Ratio of Width to Height Rock Pillar in Twin Circular Tunnels. J Geol Geosci 1: 103. doi: 10.4172/2329-6755.1000103

Copyright: © 2012 Siahmansouri A, et al. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

of rock pillars in underground structures [8]. In this paper, the interaction between two tunnels has been studied using 2D finite element analysis. To do this, at first the stress concentrations of twin tunnels was investigated. Then, a great number of twin tunnels were modeled in Phase2 software with different conditions of rock mass and depth of tunnels for three different ratios of K (0.5, 1 and 1.5). Models were analyzed and minimum stable rock pillar was determined. Finally, according to the linear and nonlinear regression methods, the best merit function was fitted to these data. Then, a new approximate formula for predicting ratio of width to height rock pillar in twin circular tunnels was proposed. The formula is very accurate with high correlation coefficient (0.96) that can be used for predicting ratio of width to height rock pillar in twin circular tunnels.

### Proposed Method

In rock stressed below its elastic limit, that is, below about one-half of the compressive strength, and in which joints are widely spaced and tightly pre-compressed or healed, it is often acceptable to consider an opening as a long hole of constant cross section in an infinite volume. This is the plane strain equivalent of a hole in a plate, and we can use the solution to the problem of a circular hole in a biaxially loaded plate of homogeneous, isotropic, continuous, linearly elastic material-the Kirsch solution [3].

### Geometrical modeling

In this paper a twin circular tunnels was considered in massive rock and depth of H. Geometrical parameters of twin circular tunnels are shown in Figure 1.

The Phase2 software is used for modeling. The Phase2 software in addition to simplicity is sufficiently accurate at modeling complex underground spaces. This software is based on finite element, and modeling is doing as two-dimensional assuming the plane strain [9]. The rock mass is a continuum and the software is a high speed for modeling of many of tunnel; so the software was chosen for stability analysis of the twin circular tunnels [9]. The diameter of each (W) is 10 meters. The boundaries of the model expanded as five times the diameter of the tunnel. Three angular elements are used for mesh generation in the model (Figure 2).

### Mechanical modeling

Bieniawski developed his scheme using data obtained mainly from civil engineering excavations in sedimentary rocks in South Africa [10-13]. Bieniawski's geomechanical classification system provides a general Rock Mass Rating (RMR) increasing with rock quality from 0 to 100. It is based upon five universal parameters: strength of the rock, drill core quality, groundwater conditions, joint and fracture spacing, and joint characteristics. A sixth parameter, orientation of joints, is entered differently for specific application in tunneling, mining, and foundations. Increments of rock mass rating corresponding to each parameter are summed to determine RMR [3,11-13]. RMR can show the quality of rock mass, as well. According to the RMR value, rocks are divided into 13 groups (mat 1 to mat 13). Geomechanical prosperities of rock groups are given in table 1.

Hoek and Brown criterion was used for stability analysis investigation. Hoek and Brown proposed the following criterion for rock masses [14-17].

$$\sigma_1 = \sigma_3 + \sigma_C \left[ m_b \frac{\sigma_3}{\sigma_C} + S \right]^a \quad (1)$$

Studies showed that above criterion is valid for massive rock, if RMR>25 [16]. Geological Strength Index (GSI) is calculated by [16]:

$$GSI = RMR_{1989} - 5 \quad (RMR_{1989} > 23)$$

Constant coefficient  $m_b$ , S and a are determined by:

$$\frac{m_b}{m_i} = \exp \left[ \frac{GSI - 100}{25} \right] \quad (3)$$

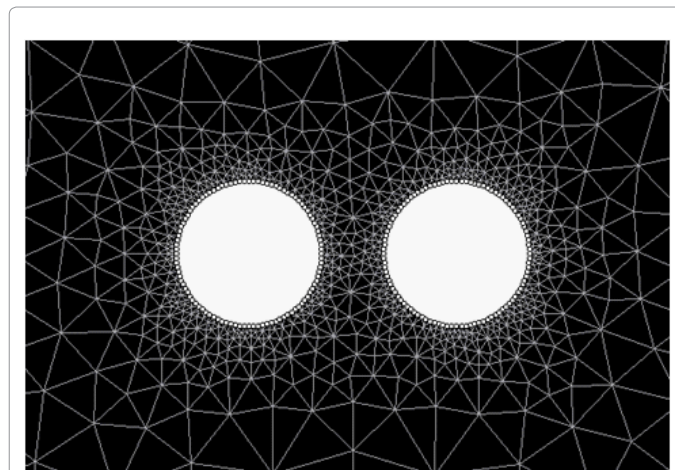


Figure 2: Modeling of twin circular tunnels in Phase2.

Mat.	RMR	UCS (MPa)	$E_m$ (GPa)	$\nu$	$\gamma$ (MN/m <sup>3</sup> )
mat 1	90	250	100	0.2	0.027
mat 2	85	225	75	0.209	0.027
mat 3	80	200	56	0.218	0.027
mat 4	75	175	42	0.227	0.027
mat 5	70	150	32	0.236	0.027
mat 6	65	125	24	0.245	0.027
mat 7	60	100	18	0.254	0.027
mat 8	55	75	13	0.263	0.027
mat 9	50	50	10	0.27	0.027
mat 10	45	30	7.5	0.277	0.027
mat 11	40	20	6	0.285	0.027
mat 12	35	10	4.2	0.293	0.027
mat 13	30	5	3.1	0.3	0.027

Table 1: Geomechanical prosperities of rock groups.

Mat.	GSI	$m_i$	$m_b$	S
mat 1	85	34	18.7	0.2
mat 2	80	31.5	14.1	0.11
mat 3	75	29	10.7	0.06
mat 4	70	26.5	8	0.035
mat 5	65	24	6	0.02
mat 6	60	21.5	4.4	0.012
mat 7	55	19	3.1	0.007
mat 8	50	16.5	2.3	0.004
mat 9	45	14	1.5	0.0022
mat 10	40	11.5	1	0.0013
mat 11	35	9	0.7	0.0007
mat 12	30	6.5	0.4	0.0004
mat 13	25	4	0.2	0.00025

Table 2: Hoek and Brown prosperities of rock groups.

$$S = \exp \left[ \frac{GSI - 100}{9} \right] \quad (4)$$

$$a = 0.5 \quad (5)$$

There  $m_i$  is material constant.

According to the above relations, Hoek and Brown parameters are calculated. These parameters are given in table 2.

### Stability Analysis

All rock groups were modeled at 5 difference depths (50, 150, 300, 500 and 750 meters). The models were analyzed for different K values (ratio of horizontal stress to vertical stress,  $K=1$ ,  $k=0.5$  and  $k=1.5$ ). For each model, the minimum stable rock pillar was determined using try and error. For this purpose, the plastic zones of the tunnels should be separated. Rock is placed in plastic zone if safety factor value is less than 1.5. For instance, safety factor contour value of rock group 4 is shown in Figure 3.

#### Stability analysis of K=1

The results of stability analysis are given in table 3, for K ratio equal to 1.

#### Stability analysis of K=0.5

The results of stability analysis are given in table 4, for K ratio equal to 0.5.

### Stability analysis of K=1.5

The results of stability analysis are given in table 5, for K ratio equal to 1.5.

In condition of non-hydrostatic ( $K=0.5$  and  $K=1.5$ ), weak rock mass ( $RMR < 50$ ) do not tolerate difference of vertical and horizontal stress; in this condition, the depth is more and rock is weaker, K is limited to 1. In non-hydrostatic conditions, number of data is less than hydrostatic condition.

### Results and Discussions

Regression analysis gives us the ability to summarize a collection of sampled data and fitting to a model that will accurately describe the data. Regression analysis can turn the sampled data points into a smooth continuous function. This measurement of agreement is called the merit function. There are two widely used and accepted methods for performing regression analysis. The first, and easiest to implement, is linear regression. The second more general method is called nonlinear regression. In this paper both linear and nonlinear approach has been used for the analysis [18,19].

According to the linear and nonlinear regression methods, a lot of functions are fitted to all data, and then the best merit function can be chosen. These functions must be comprehensive and have high

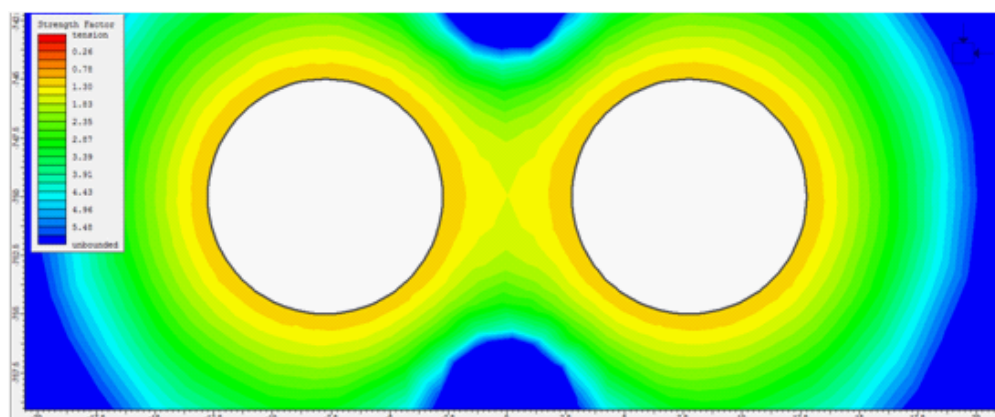


Figure 3: Safety factor contour value of rock group 4 at depth of 750 m (K=1) in Phase2.

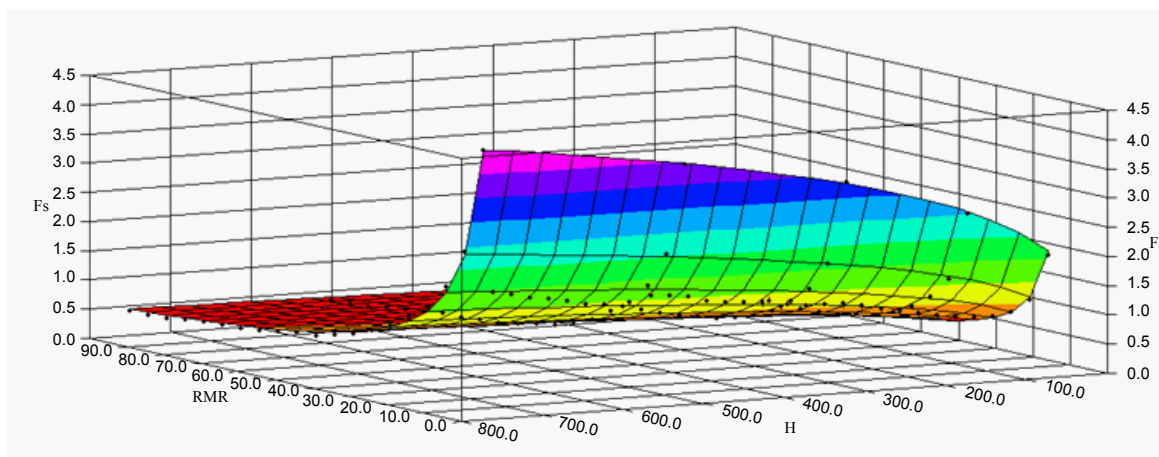


Figure 4: Regression method, curve fitting to all data (167#) for K=0.5, K=1, K=1.5, correlation coefficient,  $R^2=0.96$ .

Mat.	RMR	H (m)	P (m)	F	Mat.	RMR	H (m)	P (m)	F
mat 1	90	50	1	0.1	mat 7	70	500	6.4(10)	0.54
mat 1	90	150	2.1	0.21	mat 7	70	750	7.2(10)	0.55
mat 1	90	300	3.1	0.31	mat 8	65	50	4.2(10)	0.31
mat 1	90	500	3.8	0.38	mat 8	65	150	5.4(10)	0.44
mat 1	90	750	4.3	0.43	mat 8	65	300	6.4(10)	0.52
mat 2	85	50	1.2	0.12	mat 8	65	500	7.4(10)	0.55
mat 2	85	150	2.7	0.27	mat 8	65	750	8.4(10)	0.58
mat 2	85	300	3.8	0.38	mat 9	50	50	5(10)	0.39
mat 2	85	500	4.3	0.43	mat 9	50	150	6.4(10)	0.51
mat 2	85	750	4.7	0.47	mat 9	50	300	8(10)	0.54
mat 3	80	50	1.5	0.15	mat 9	50	500	10(10)	0.57
mat 3	80	150	3.2	0.32	mat 9	50	750	11(10)	0.62
mat 3	80	300	4.1	0.41	mat 10	45	50	5.8	0.43
mat 3	80	500	4.6	0.46	mat 10	45	150	8	0.52
mat 3	80	750	5.2	0.52	mat 10	45	300	10.2	0.56
mat 4	75	50	1.8	0.18	mat 10	45	500	12.2	0.6
mat 4	75	150	3.6	0.36	mat 10	45	750	13.6	0.68
mat 4	75	300	4.4	0.44	mat 11	40	50	7	0.44
mat 4	75	500	5.2	0.52	mat 11	40	150	10.2	0.55
mat 4	75	750	5.5	0.55	mat 11	40	300	12.8	0.59
mat 5	70	50	2.6	0.26	mat 11	40	500	15.2	0.65
mat 5	70	150	4.2	0.42	mat 11	40	750	17.8	0.7
mat 5	70	300	5.2	0.52	mat 12	35	50	9.8	0.48
mat 5	70	500	5.6	0.56	mat 12	35	150	14.6	0.57
mat 5	70	750	6	0.6	mat 12	35	300	18.4	0.64
mat 6	65	50	3.4	0.34	mat 12	35	500	22.2	0.71
mat 6	65	150	4.6	0.46	mat 12	35	750	25.2	0.78
mat 6	65	300	5.4	0.54	mat 13	30	50	15	0.565
mat 6	65	500	6	0.6	mat 13	30	150	22.4	0.4
mat 6	65	750	6.6	0.66	mat 13	30	300	27.2	0.327
mat 7	70	50	4	0.4	mat 13	30	500	32	0.278
mat 7	70	150	5.2	0.52	mat 13	30	750	36.6	0.241
mat 7	70	300	6	0.6					

\* F is ratio of minimum rock pillar width (P) to tunnel height (W). (Figure 1)

**Table 3:** Calculated coefficient of pillar for the different RMR and depth. (K=1).

Mat.	RMR	H (m)	P (m)	F	Mat.	RMR	H (m)	P (m)	F
mat 1	90	50	1	0.1	mat 6	65	50	2.8	0.28
mat 1	90	150	1.6	0.16	mat 6	65	150	4	0.4
mat 1	90	300	2.4	0.24	mat 6	65	300	5	0.5
mat 1	90	500	3	0.3	mat 6	65	500	5.8	0.58
mat 1	90	750	3.6	0.36	mat 6	65	750	6.6	0.66
mat 2	85	50	1.2	0.12	mat 7	70	50	3.6	0.36
mat 2	85	150	2	0.2	mat 7	70	150	5	0.5
mat 2	85	300	3	0.3	mat 7	70	300	5.8	0.58
mat 2	85	500	3.4	0.34	mat 7	70	500	3.6	0.36
mat 2	85	750	4	0.4	mat 7	70	750	5	0.5
mat 3	80	50	1.4	0.14	mat 8	65	50	3.6()	0.36
mat 3	80	150	2.6	0.26	mat 8	65	150	5()	0.5
mat 3	80	300	3.4	0.34	mat 8	65	300	6.4()	0.64
mat 3	80	500	4	0.4	mat 8	65	500	8()	0.8
mat 3	80	750	4.4	0.44	mat 8	65	750	10()	1
mat 4	75	50	2.2	0.22	mat 9	50	50	4.4()	0.44
mat 4	75	150	3.2	0.32	mat 9	50	150	6.4()	0.64
mat 4	75	300	3.8	0.38	mat 9	50	300	10()	1
mat 4	75	500	4.4	0.44	mat 9	50	500	12()	1.2
mat 4	75	750	5	0.5	mat 9	50	750	17()	1.7
mat 5	70	50	2.2	0.22	mat 10	45	50	5.8()	0.58
mat 5	70	150	3.6	0.36	mat 10	45	150	11()	1.1
mat 5	70	300	4.4	0.44	mat 11	40	50	7.8()	0.78
mat 5	70	500	5.2	0.52	mat 11	40	150	15.6()	1.56
mat 5	70	750	5.6	0.56					

**Table 4:** Calculated coefficient of pillar for the different RMR and depth. (K=0.5).



Mat.	RMR	H (m)	P (m)	F	Mat.	RMR	H (m)	P (m)	F
mat 1	90	50	1	0.1	mat 6	65	300	6	0.6
mat 1	90	150	2	0.2	mat 6	65	500	6.6	0.66
mat 1	90	300	3.1	0.31	mat 6	65	750	7	0.7
mat 1	90	500	4.2	0.42	mat 7	70	50	4.4	0.44
mat 1	90	750	4.6	0.46	mat 7	70	150	5.8	0.58
mat 2	85	50	1.2	0.12	mat 7	70	300	6.6	0.66
mat 2	85	150	2.7	0.27	mat 7	70	500	7	0.7
mat 2	85	300	4	0.4	mat 7	70	750	7.6	0.76
mat 2	85	500	4.8	0.48	mat 8	65	50	4.6()	0.46
mat 2	85	750	5.4	0.54	mat 8	65	150	5.8()	0.58
mat 3	80	50	1.5	0.15	mat 8	65	300	6.8()	0.68
mat 3	80	150	3.4	0.34	mat 8	65	500	7.4()	0.74
mat 3	80	300	4.8	0.48	mat 8	65	750	8()	0.8
mat 3	80	500	5.4	0.54	mat 9	50	50	5.4()	0.54
mat 3	80	750	6	0.6	mat 9	50	150	6.6()	0.66
mat 4	75	50	2	0.2	mat 9	50	300	7.8()	0.78
mat 4	75	150	4.2	0.42	mat 9	50	500	8.6()	0.86
mat 4	75	300	5.2	0.52	mat 9	50	750	9.6()	0.96
mat 4	75	500	5.8	0.58	mat 10	45	50	6()	0.6
mat 4	75	750	6.2	0.62	mat 10	45	150	8()	0.8
mat 5	70	50	2.8	0.28	mat 10	45	300	9.4()	0.94
mat 5	70	150	4.8	0.48	mat 10	45	500	11()	1.1
mat 5	70	300	5.6	0.56	mat 10	45	750	12.2()	1.22
mat 5	70	500	6.2	0.62	mat 11	40	50	7()	0.7
mat 5	70	750	6.4	0.64	mat 11	40	150	9.4()	0.94
mat 6	65	50	3.8	0.38	mat 11	40	300	11.4()	1.14
mat 6	65	150	5.4	0.54					

Table 5: Calculated coefficient of pillar for the different RMR and depth. (K=1.5).

Constant factors	
a	370
b	-2
c	0.33

Table 6: Constant factors of Eq. 6.

correlation coefficients. Also, merit function should be simple so that, geotechnical engineers can use it in the site easily. These models contain two independent variables, RMR and H, three constant parameters, a, b and c, and the dependent variable, F, estimated based on the independent variables and constants. Results of analysis and merit functions are investigated for difference K ratio.

Three-dimensional graph of merit function is shown in Figure 4.

The merit function for the graph shown in Figure 3 can be estimated as:

$$F_s = a(RMR)^b H^c \quad (6)$$

Where,  $F_s$  is coefficient of pillar, for the best fitted function constant factors are shown in Table 6.

Then coefficient of pillar,  $F_s$  can be written as:

$$F_s = 370(RMR)^{-2} H^{0.33} \quad (7)$$

This is a very accurate formula with correlation coefficient,  $R^2=0.96$ .

## Conclusion

In this paper the interaction between twin circular tunnels has been studied using 2D finite element analysis. For this purpose, a great number of twin tunnels were modeled in Phase2 software with different conditions of rock mass (RMR value), depth of tunnels and three different ratios of K (ratio of horizontal stress to vertical stress,

0.5, 1 and 1.5). Finally, according to the nonlinear regression methods, the best function was fitted to result of numerical analysis of twin tunnels. Then, a new approximate formula for estimating the minimum rock pillar according to RMR value and depth of twin circular tunnels was proposed for different K. The formula is very accurate with high correlation coefficient (0.96) that can be used for estimating the minimum rock pillar of twin circular tunnels.

## References

- Obert L, Duvall WI (1967) Rock mechanics and the design of structures in rocks. Wiley, New York.
- Obert L, Duvall WI, Merrill RH (1960) Design of underground opening in competent rock. USBM Bull.
- Goodman RE (1989) Introduction to rock mechanics. (2ndedn), John Wiley & Sons, Toronto.
- Duvall WI (1976) General principles of underground opening design in competent rock. The 17th US Symposium on Rock Mechanics (USRMS).
- Muskhelishvili NI (1953) Some basic problems of the mathematical theory of elasticity. (4thedn), translated by Radok JRM.
- Jaeger JC, Cook NGW (1979) Fundamentals of rock mechanics. (3rdedn), Chapman & Hall, London.
- Vutukuri VS, Katsuyama K (1994) Introduction to rock mechanics. Industrial Publishing & Consulting Inc., Tokyo.
- Mortazavi A, Hassani FP, Shabani M (2009) A numerical investigation of rock pillar failure mechanism in underground openings. Computers and Geotechnics 36: 691-697.
- Rocscience Incorporation (2005) Phase2 Manual.
- Brady BHG, Brown ET (1985) Rock mechanics for underground mining. George Allen & Unwin, London.
- Bieniawski ZT (1974) Geomechanics classification of rock masses and

- 
- its application in tunneling. Proc. 3rd Congress on Rock Mechanics, ISRM, Denver.
12. Bieniawski ZT (1976) Rock mass classifications in rock engineering. Proceedings of Symposium on Exploration for Rock Engineering, Balkema, Rotterdam 1: 97-106.
13. Bieniawski ZT (1989) Engineering rock mass classifications: a complete manual for engineers and geologists in mining, civil, and petroleum engineering. John Wiley & Sons, NewYork.
14. Hoek E, Brown ET (1980) Empirical strength criterion for rock masses. *Journal of Geotechnical and Geoenvironmental Engineering* 106: 1013-1035.
15. Hoek E, Brown ET (1988) The Hoek-Brown failure criterion – a 1988 update. Proceedings of the 15th Canadian Rock Mechanics Symposium. Symp, University of Toronto, Ontario, 31-38.
16. Hoek E, Wood D, Shah S (1992) A modified Hoek-Brown criterion for jointed rock masses. *Proc. Rock Characterization, Symp. Int. Sock. Rock*, 209-214.
17. Hoek E, Kaiser PK, Bawden WF (1995) Support of underground excavations in hard rock. Rotterdam, Balkema.
18. Draper NR, Smith H (1981) *Applied Regression Analysis*. Second edition, John Wiley & Sons, USA.
19. Ratkowsky DA (1990) *Handbook of nonlinear regression models*. Marcel Dekker, New York.