

5<sup>th</sup> International Conference and Exhibition on

# MECHANICAL & AEROSPACE ENGINEERING

October 02-04, 2017 Las Vegas, USA



## *Shaaban Abdallah*

*University of Cincinnati, USA*

### **Review of the state of the art and new mathematical formulations for solutions of the incompressible Navier-Stokes equations in velocity and pressure derivatives**

Numerical simulations of the incompressible Navier-Stokes equations in primitive variables employ the velocity and pressure as dependent variables. The lack of an equation for the pressure causes numerical difficulties in calculating the pressure. Two formulations have been developed for solving the pressure problem. The Artificial Compressibility method modifies the continuity equation by adding a time dependent pressure term, and the pressure Poisson method enforces the continuity equation through the divergence of the momentum equation. The pressure equations are solved iteratively and upon convergence of the numerical solution, the continuity equation is satisfied. Mostly these techniques are developed and tested using explicit schemes. An implicit approach for solving the incompressible Navier Stokes equations known as the Fractional Step method derives an approximate pressure Poisson equation. It is important to state here that the velocity field of incompressible flows does not depend on the pressure but it depends on the pressure derivatives. Recently, we developed two methods that employ the velocity and pressure derivatives as dependent variables. The pressure derivatives increase the dependent variables to four in two-dimensions and six in three dimensions. Additional governing equations are obtained from the identity that the Curl Gradient of the pressure is zero. In the second method, the continuity equation is replaced by its spatial derivatives and enforces the continuity equation at the boundary. These techniques are developed and tested using explicit time-marching schemes. They do not require boundary conditions for the pressure and they satisfy the continuity equation to machine zero. In this study, we re-derived the above mentioned methods using lower and upper (LU) factorization of the discretization matrix of the governing equations. In addition, we developed the implicit technique of the velocity and pressure derivatives formulation. Exact LU factorization of the matrix representing the discrete Navier-Stokes equations is obtained. The main advantage of this approach, in addition to the above mentioned advantages of the velocity and pressure derivatives explicit methods, is that no matrix inversion of the implicit operator is required.

#### **Biography**

Shaaban Abdallah is working as a Professor of Aerospace Engineering and has been at the University of Cincinnati since 1989. He obtained his PhD in Aerospace Engineering at the University of Cincinnati in 1980. He joined Penn State University from 1981 to 1988. His research interests include computational fluid dynamics, nano fluids, turbo-machines, unmanned aerial vehicles and medical devices. He has two US patents on centrifugal compressors and three disclosures with university of Cincinnati on medical devices.

Shaaban.abdallah@uc.edu

#### **Notes:**