

The weighted mean and its improved dispersion

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Statistics is involved in every physical measurement. The weighted mean appears when a physical quantity is measured by different methods in different laboratories, producing different results. The formula is: The formula is:

$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i} = \sum p_i x_i \quad (1)$$

with w_i - absolute weights and p_i - relative weights.

A relation exists:

$$w_i = \frac{1}{\sigma_i^2} \quad (2)$$

with σ_i - the individual standard deviations.

To w_i , two different dispersions are associated, D_1 (internal) and D_2 (external).

The ratio D_2/D_1 is:

$$\frac{D_2}{D_1} = \sum w_i (x_i - \bar{x}_w)^2 = \sum \frac{(x_i - \bar{x}_w)^2}{\sigma_i^2} = \sum p_i^2 \quad (3)$$

In practice $(D_2/D_1) > 1$. So D_2 is the confident one.

In principle p_i should produce as great deviations $(x_i - \bar{x}_w)$ as the associated σ_i are great.

For equal treatment of these deviations, relative deviations may be considered:

$$d_i = \frac{x_i - \bar{x}_w}{\sigma_i} \quad (4)$$

which express the deviations in units.

The n values from (4) must have same near (equivalent) values. Their arithmetical mean tends to zero. Thus, as in the case of a unique x , we may reach the minimum of the expression:

$$\sum_{i=1}^n \frac{(x_i - \bar{x}_w)^2}{\sigma_i^2} \quad (5)$$

The annulation of the derivative for \bar{x}_w , gives:

$$\sum_{i=1}^n \frac{(x_i - \bar{x}_w)}{\sigma_i^2} = \sum_{i=1}^n \frac{x_i}{\sigma_i^2} - \bar{x}_w \sum_{i=1}^n \frac{1}{\sigma_i^2} = 0 \quad (6)$$

and finally formulas (1) and (2) are obtained.

This calculus accepts great σ_i even with systematic uncertainties. In practice the theoretical unknown values x_i , are replaced in formulae (1), (2) by the experimental value x_i , which fluctuate.

A dispersion D_3 is obtained, by error propagation, and added to D_1 , D_2 .

First are calculates:

$$\frac{\partial \bar{x}_w}{\partial w_i} = \frac{x_i \sum w_j - \sum w_j x_j}{(\sum w_j)^2} = \frac{x_i - \bar{x}_w}{\sum w_j} \quad (7)$$