SOME PROBLEMS ON KAHLERIAN SPACE
WITH SEMI-SYMMETRIC METRIC F-CONNECTIONS

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ABSTRACT:
This paper delineates the study of Kaehlerianspace with semi-symmetric metric F-connections. We have obtained few important theorems.

KEY WORDS:
Riemannian space, Kaehlerian space, semi-symmetric, F-connection.

1. INTRODUCTION:
An 2m-dimensional Kaehlerian space is a Riemannian space if it admits a structure tensor F_j^i satisfying [2,6]:

(1.1) F_j^i F_k^j = \delta_k^i
(1.2) F_j^i = - F_j^i
(1.3) \nabla_k F_j^i = 0,
(1.4) F_l^j - F_j^l
(1.5) F_j^i = 0,
(1.6) g^{ij} F_k^i = F_k^j.

It is easy to verify that in a totally real subspace M_n of Kaehlerian space M_{2m}, the following equations are satisfied [2,6,7]:

(1.7) F_s^a B_x^b = F_s^a B_x^b = 0,
(1.8) F_s^a C_i^x - F_s^a C_i^x = 0
Consequently yields
(1.9) F_s^a B_x^b = F_s^a B_x^b = 0,
(1.10) F_s^a B_x^b = F_s^a B_x^b = 0
By virtue of equations (1.2), (1.7), (1.8) and (1.13), we obtain
(1.14) f_a^i = f_a^i

Applying the complex structure tensor to equations (1.7), (1.8) and using equations
(1.9), (1.10), (1.11) and (1.14), we obtain
(1.16) f_a^i = f_a^i
(1.17) f_a^i = f_a^i
(1.18) f_a^i = f_a^i

Let *R_b^a c^d be the curvature tensor of the connection *F_b^a, then we have [1,7]:
(1.20) *R_b^a c^d = M_b^a c^d - \delta_b^a p_c d + \delta_b^a q_b d + p_a g_c d + p_a g_c d - F_b^a q_c d + F_a^a q_b d - q_a^a F_b d + q_a^a F_b d

Wherein M_b^a is the Riemannian curvature tensor of a Kaehlerian space M_{2m} and
(1.21) p_a = \nabla_a F_b - \nabla_a F_b + (1/2)p_a p_c q_b d
(1.22) q_a = \nabla_a q_b - \nabla_a q_b - (1/2)p_a p_c F_b d
(1.23) p_a = p_a g_c d
(1.24) q_a = q_a g_c d

In this regard, we have
(1.25) p_a = q_a F_b d
(1.26) q_a = - p_a F_b d
(1.27) p_a - p_b a = 0.

2. KAHLERIAN SPACE WITH SEMI-SYMMETRIC METRIC F-CONNECTIONS:
In n-dimensional totally real subspace M_n of a Kaehlerian space M_{2m} admits special semi-symmetric metric F-connection, then we observe that the equations (1.16), (1.17), (1.18) and (1.19), gives the following relations [3,5,6]:

Mathematica Aeterna, Vol. 5, 2015, no. 3, 417 - 419
(2.1) \( f_i^j = 0 \)

(2.2) \( f_j^i f_j^x = \delta_i^x \) and \( (2.3) \) \( M_{xyij} f_i^j f_w^j = M_{xyw} \)

Wherein

(2.4) \( M_{xyzw} = M_{xyzw}^e g_{yw} \), \( (2.5) \) \( M_{xyij} = M_{xyij}^k g_{kj} \)

and \( M_{xyij}^k \) denotes the curvature tensor of the connection induced in the normal bundle.

Ricci equation is given by [4]:

(2.6) \( M_{xyij} = M_{abcd} B_{abc}^x C_{cd}^y T_{xyij} \)

Wherein

(2.7) \( M_{abcd} = M_{abcd}^e g_{ed} \)

and \( (2.8) \) \( T_{xyij} = H_{x,i} H_{y,j} - H_{x,i}^y H_{x,j} \)

Contracting the covariant form of equation (1.20) with \( B_{ab x} C_{cd} \) and making use of equations \((1.2), (1.4), (1.5), (1.7), (1.8), (1.9), (1.10), (1.11), (1.12), (1.21), (1.22), (1.23), (1.24), (1.25), (1.26), (1.27), (2.1) \) and \( (2.6) \), we obtain

(2.9) \* \( R_{abcd} B_{xy} C_{cd}^i j = M_{xyi} + T_{xyij} - f_{xj} p_{bc} B_{bc}^{xy} f_{ij} + f_{xj} p_{bc} B_{bc}^{xy} f_{ij} - f_{yj} p_{bc} B_{bc}^{xy} f_{ij} \)

\( + f_{yj} p_{bc} B_{bc}^{xy} f_{ij} \)

**Definition 2.1:**

A Riemannian space \( M_n \) is called to be a M-Einstein [3] if

(2.10) \( M_{xy} = \frac{1}{n} M g_{xy} \)

Now, we assume that

(2.11) \* \( R_{abcd} = \mu_{bc} F_{da} \)

and \( (2.12) \) \* \( R_{a b c d} = \mu_{bc} F_{da} \)

for some tensor \( \mu_{bc} \) in Kaehlerian space \( M_{2m} \).

In this regard, we have the following theorems:

**Theorem 2.1:**

Let \( M_n \) be a totally real subspace of a Kaehlerian space \( M_{2m} \) with special semi-symmetric F-connection whose curvature tensor assumes the form \( (2.12) \). If the second fundamental tensor of \( M_n \) commute and \( n = 1 \), then \( M_n \) is conformally flat.

**Proof:**

Let \( *R_{abcd} \) assume the form \( (2.12) \) and the second fundamental tensors of \( M_n \) commute i.e. \( T_{xyij} \) vanishes, then the equation \( (2.9) \) in the view of equations \( (1.11) \) and \( (2.1) \) reduces to the form

(2.13) \( M_{xyij} = f_{xj} p_{bc} B_{bc}^{xy} f_{ij} + f_{yj} p_{bc} B_{bc}^{xy} f_{ij} - f_{xj} p_{bc} B_{bc}^{xy} f_{ij} \)

Contracting equation \( (2.13) \) with \( f_i^j f_w^j \) and making use of equations \( (1.14), (1.16), (2.1), (2.2), (2.3) \) and \( (2.6) \), we obtain

(2.14) \( M_{xyzw} = g_{s w} p_{bc} B_{bc}^{xy} g_{xy} p_{bc} B_{bc}^{xy} - g_{s w} p_{bc} B_{bc}^{xy} - f_{xj} p_{bc} B_{bc}^{xy} g_{xy} \)

Contracting the equation \( (2.14) \) with \( g_{xy} g_{zw} \) and using the equation \( B_{bc}^{xy} = B_{bc}^{xy} g_{xy} \), we get

(2.15) \( M_{xy} = (n - 1) g_{s w} p_{bc} B_{bc}^{xy} \)

If \( n = 1 \) then we obtain

(2.16) \( M_{xy} = 0 \).

This establishes the validity of the theorem.

**Theorem 2.2:**

Let \( M_n \) be a totally real subspace of a Kaehlerian space \( M_{2m} \) with special semi-symmetric F-connection whose curvature tensor assumes the form \( (2.12) \). If the second fundamental tensor of \( M_n \) commute, then \( M_n \) is M-Einstein.

**Proof:**

Contracting the equation \( (2.15) \) with \( g_{xy} \), we get

(2.17) \( M = m (n - 1) p_{bc} B_{bc}^{xy} \)

From equations \( (2.15) \) and \( (2.17) \), we get

(2.18) \( M_{xy} = (1/n) M g_{xy} \).
This establishes the validity of the theorem.

**Theorem 3.3:**

For $M_4$ be a totally real subspace of a Kaehlerian space $M_{2m}$ with special semi-symmetric $F$-connection whose curvature tensor assumes the form (2.12) and satisfying the condition $F_{bc}M_{xyzw} = 0$.

**Proof:**

Multiplying equation (2.14) by $F_{bc}$ and using equation (1.9) then we obtain

(2.19) $F_{bc}M_{xyzw} = 0$.

**REFERENCES:**


Received: May, 2015