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Hom-pre-Lie algebras of three dimension

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Abstract

In this paper, we give some examples of Hom-pre-Lie algebras of three dimension. They can be obtained from pre-Lie algebras by twisting along any algebra endomorphism.

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1 Preliminaries

Pre-Lie algebras were introduced in the studies of different geometry and Lie group, which have a close relationship with the Lie algebras. They have already been introduced by Cayley in 1896 as a kind of rooted tree algebras [5]. They affine manifolds and affine structures on Lie group [9,10]. They also play very important roles in the studies of certain in tegrable systems, classical and quantum Yang-Baxter equations [6,8] , and so on. The Pre-Lie algebra is also called a left-symmetric algebra or Vinberg algebra. Hom-pre-Lie algebras were first introduced by Makblouf and Silvestor as a special case of G-Hom-associative algebras [4]. Pre-Lie algebra appears in many fields in mathematics and mathematical physics. So far, there have been many results in the study of Hom- pre-Lie algebra, but there are still some problems that have not been solved. In this paper, we shall give some examples of the 3-dimensional Hom-pre-Lie algebras.

Definition 1.1 [1] *Let A be a vector space over a field $\boldsymbol{F}$ equipped with a bilinear product $\mu : A^{\otimes 2} \to A$. A is called a pre-Lie-algebra if for any $x, y, z \in A$,*

$$(xy)\,z - x\,(yz) = (yx)\,z - y\,(xz)\,, \tag{1}$$

Definition 1.2 [2] *Let A be a vector space over a field **F** equipped with a bilinear product* $\mu : A^{\otimes 2} \to A$. *A is called a pre-Lie-algebra if for any* $x, y, z \in A$,

$$\alpha(xy) = \alpha(x)\alpha(y), \tag{2}$$

$$(xy)\alpha(z) - \alpha(x)(yz) = (yx)\alpha(z) - \alpha(y)(xz), \tag{3}$$

Definition 1.3 [7] *Let A be a vector space over a fieldF,* $\mu : A^{\otimes 2} \to A$ *is bilinear map, and* $\alpha : A \to A$ *is a linear map.* (A, μ, α) *is called Hom-Novikov algebra if the linear map satisfies equation* (2) *and* (3), *and the following condition for* $x, y, z \in A$:

$$(xy)\alpha(z) = (xz)\alpha(y), \tag{4}$$

Definition 1.4 [3] *Let A be a vector space over a field **F**,* $\mu : A^{\otimes 2} \to A$ *be bilinear map, and* $\alpha : A \to A$ *be a linear map.* (A, μ, α) *is called G-Hom-associative algebra if any* $x_i \in A (i = 1, 2, 3)$:

$$\sum_{\sigma \in G} (-1)^{\varepsilon(\sigma)} \left\{ (x_{\sigma(1)} x_{\sigma(2)}) \alpha(x_{\sigma(3)}) - \alpha(x_{\sigma(1)})(x_{\sigma(2)} x_{\sigma(3)}) \right\}, \tag{5}$$

where $\varepsilon(\sigma)$ is the signature of σ.

(1) A Hom-associtive algebra is a G-Hom-associative algebra in which G is the trivial subgroup $\{e\}$. The G-Hom-associativity now takes the form

$$(xy)\alpha(z) = \alpha(x)(yz), \tag{6}$$

which we call Hom-associativity.

(2) A Hom-pre-Lie algebra is a G-Hom-associative algebra in which $G = \{e, (12)\}$. The $\{e, (1,2)\}$ Hom-associativity axiom (5) is equivalent to

$$(xy)\alpha(x) - \alpha(x)(yz) = (yx)\alpha(z) - \alpha(y)(xz). \tag{7}$$

Theorem 1.5 [3] *Let* $A = (A, \mu)$ *be a G-associative algebra and* $\alpha : A \to A$ *be a linear map such that* $\alpha \circ \mu = \mu \circ \alpha^{\otimes 2}$. *Then* $(A, \mu_\alpha = \alpha \circ \mu, \alpha)$ *is a G-Hom-associative algebra. Moreover,* α *is multiplicative with respect to* μ_α, *i.e.,* $\alpha \circ \mu = \mu \circ \alpha^{\otimes 2}$.

Theorem 1.6 [3] *Let* $A = (A, \mu)$ *be a not necessarily associative algebra and* $\alpha : A \to A$ *be a algebra morphism. Write* A_α *for the triple* $(A, \mu_\alpha = \alpha \circ \mu, \alpha)$.

(1) If A is an associative algebra, then A_α is a Hom-associative algebra;
(2) If A is a Lie algebra, then A_α is a Hom-Lie algebra;
(3) If A is a pre-Lie algebra, then A_α is a Hom-pre-Lie algebra;
(4) If A is a Lie-admissible algebra, then A_α is a Hom-pre-Lie algebra.

2 Main results

Let A be a 3-dimensional Lie algebra with a fixed linear basis $\{e_1, e_2, e_3\}$, and the characteristic matrix of A be

$$M(\mu) = \begin{pmatrix} e_1e_1 & e_1e_2 & e_1e_3 \\ e_2e_1 & e_2e_2 & e_2e_3 \\ e_3e_1 & e_3e_2 & e_3e_3 \end{pmatrix}, \tag{8}$$

where $e_ie_j = \sum_{k=1}^{3} d_k^{ij} e_k$. If $\alpha : A \to A$ is an algebra morphism and the $\mu_\alpha = \alpha \circ \mu$ is the associated Hom-pre-Lie algebra product, then its characteristic matrix is defined similarly as

$$M(\mu_\alpha) = \begin{pmatrix} \alpha(e_1e_1) & \alpha(e_1e_2) & \alpha(e_1e_3) \\ \alpha(e_2e_1) & \alpha(e_2e_2) & \alpha(e_2e_3) \\ \alpha(e_3e_1) & \alpha(e_3e_2) & \alpha(e_3e_3) \end{pmatrix}, \tag{9}$$

We denote a linear map $\alpha : A \to A$ by its 3×3-matrix with respect to the basis $\{e_1, e_2, e_3\}$ and its matrix is

$$M(\alpha) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}, \end{pmatrix} \tag{10}$$

where $\alpha(e_i) = \sum_{k=1}^{3} a_{ki}e_k, 1 \le i \le 3$.

The classification of pre-Lie algebras of dimension three given in [3] is divided five classes, A, H, N, D and E. Together with H and E associated Hom-pre-Lie products (Theorem 2.2), the classifications of algebra morphisms on 3-dimensional pre-Lie algebras are listed in the table 1.

$$\begin{aligned} &\alpha(e_1e_2) \\ &= \alpha(e_1)\alpha(e_2) \\ &= (a_{11}e_1 + a_{21}e_2 + a_{31}e_3)(a_{11}e_1 + a_{21}e_2 + a_{31}e_3) \\ &= a_{11}e_1 + a_{11}a_{21}(e_2 + e_3) + a_{11}a_{31}e_3 + a_{21}a_{11}e_2 + a_{31}a_{11}e_3 \\ &= a_{11}^2e_1 + (a_{11}a_{21} + a_{21}a_{11})e_2 + (a_{11}a_{21} + 2a_{31}a_{11})e_3. \end{aligned} \tag{11}$$

And because $\alpha(e_1e_1) = \alpha(e_1)$, then $a_{11}^2e_1 + (a_{11}a_{21} + a_{21}a_{11})e_2 + (a_{11}a_{21} + 2a_{31}a_{11})e_3 = a_{11}e_1 + a_{21}e_2 + a_{31}e_3$. We can get the equations

$$a_{11}^2 = a_{11}, 2a_{11}a_{21} = a_{21}, a_{31} = a_{11}a_{21} + 2a_{11}a_{31}.$$

According to the $\alpha(e_1e_2) = \alpha(e_2) + \alpha(e_3)$, we get the equations

$$a_{11}a_{12} = a_{12} + a_{13}, a_{11}a_{22} + a_{21}a_{12} = a_{22} + a_{23}, a_{12}a_{22} + a_{11}a_{32} + a_{31}a_{12} = a_{32} + a_{33}.$$

Table 1: The classifications of algebra morphisms on 3-dimensional pre-Lie algebras

Pre-Lie algebra $M(\mu)$	Algebra morphism $M(\mu)$	Hom-pre-Lie algebra $M(\mu)$
$(H-1)=\begin{pmatrix} e_1 & e_2+e_3 & e_3 \\ e_2 & 0 & 0 \\ e_3 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & b_3 & 0 \end{pmatrix}$	$\begin{pmatrix} e_1 & b_2e_2+b_3e_3 & 0 \\ b_2e_2+b_3e_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-2)=\begin{pmatrix} e_1 & e_2+e_3 & e_3 \\ e_2 & e_3 & 0 \\ e_3 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_3 & 0 \end{pmatrix}$	$\begin{pmatrix} e_1 & b_3e_3 & 0 \\ b_3e_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-3)=\begin{pmatrix} e_1 & e_3 & 0 \\ 0 & e_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & b_3 & 1 \end{pmatrix}$	$\begin{pmatrix} e_1 & e_3 & 0 \\ 0 & e_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_3 & 0 \end{pmatrix}$	$\begin{pmatrix} e_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-4)=\begin{pmatrix} e_1 & e_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & b_3 & b_2 \end{pmatrix}$	$\begin{pmatrix} e_1 & b_2e_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & b_3 & b_2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-5)=\begin{pmatrix} 0 & 0 & 0 \\ -e_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & a_1b_2 \end{pmatrix}$ $(b_2b_1=0, a_2a_1=0, a_2b_1=0)$	$\begin{pmatrix} 0 & 0 & 0 \\ -a_1b_2e_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-6)=\begin{pmatrix} 0 & 0 & 0 \\ -e_3 & e_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} b_2^2 & b_1 & 0 \\ 0 & b_2 & 0 \\ -b_1b_2 & b_3 & b_2^3 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ -b_2^3e_3 & b_2^2e_1--b_1b_2e_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-7)_\lambda=\begin{pmatrix} e_3 & e_3 & 0 \\ 0 & \lambda e_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}, (\lambda\neq 0)$	$\begin{pmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 2a_1b_2 \end{pmatrix}$ $(\lambda a_1^2+\lambda a_1a_2+\lambda^2a_2^2$ $=b_1^2+b_1b_2+\lambda b_2)$	$\begin{pmatrix} 2a_1b_2e_3 & 2a_1b_2e_3 & 0 \\ 0 & 2\lambda a_1b_2e_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(a_1b_2+a_2b_1=0)$
$(H-8)=\begin{pmatrix} 0 & \frac{1}{2}e_3 & 0 \\ -\frac{1}{2}e_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-9)=\begin{pmatrix} 0 & e_3 & 0 \\ 0 & e_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} b_2^2 & b_1 & 0 \\ 0 & b_2 & 0 \\ b_1b_2 & b_3 & b_2^3 \end{pmatrix}$	$\begin{pmatrix} 0 & b_2^3e_3 & 0 \\ 0 & b_2^2e_1+b_1b_2e_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(H-10)=\begin{pmatrix} 0 & \frac{\lambda}{\lambda-1}e_3 & 0 \\ \frac{1}{\lambda-1}e_3 & \lambda e_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} (\lambda\neq 0,1)$	$\begin{pmatrix} b_2^2 & b_1 & 0 \\ 0 & b_2 & 0 \\ \frac{\lambda+1}{\lambda(\lambda-1)}b_1b_2 & b_3 & b_2^3 \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{\lambda}{\lambda-1}b_2^3e_3 & 0 \\ \frac{1}{\lambda-1}b_2^3e_3 & \lambda b_2^2e_1+\frac{\lambda+1}{\lambda-1}b_1b_2e_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(E-1)_\lambda=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_1 & e_1+e_2 & \lambda e_3 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$(E-2)=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_1 & e_1+e_2 & e_2+e_3 \end{pmatrix}$	$\begin{pmatrix} 1 & b_1 & c_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_1 & e_1+e_2 & (b_1+c_1)e_1+(1+b_1)e_2+e_3 \end{pmatrix}$
$(E-3)=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -e_1 \\ e_1 & e_2 & 0 \end{pmatrix}$	$\begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & c_2 \\ 0 & 0 & c_3 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_1e_1 & o \end{pmatrix}$
	$\begin{pmatrix} a_1 & b_1 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -a_1e_1 \\ a_1e_1 & b_1e_1+a_1e_2 & o \end{pmatrix}$
$(E-4)_\lambda=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda e_1 \\ e_1 & e_2+(\lambda+1)e_1 & 0 \end{pmatrix}$ $(\lambda\neq 0,-1)$	$\begin{pmatrix} 0 & b_1 & c_1 \\ 0 & 0 & c_2 \\ 0 & 0 & c_3 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_1e_1 & 0 \end{pmatrix}$
	$\begin{pmatrix} a_1 & b_1 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda a_1e_1 \\ a_1e_1 & (\lambda+1)a_1e_1+b_1e_1+a_1e_2 & 0 \end{pmatrix}$
$(E-5)=\begin{pmatrix} 0 & 0 & 0 \\ 0 & e_3 & 0 \\ e_1 & e_2+e_1 & 2e_3 \end{pmatrix}$	$\begin{pmatrix} a_1 & b_1 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & e_3 & 0 \\ a_1e_1 & (a_1+b_1)e_1\pm e_2 & 2e_3 \end{pmatrix}$
$(E-6)=\begin{pmatrix} 0 & 0 & 0 \\ 0 & e_1 & -e_1-e_2 \\ e_1 & 0 & -e_3-e_2 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & e_1 & -e_3-e_2 \\ e_1 & 0 & \frac{1}{2}e_1-e_3-2e_2 \end{pmatrix}$
$(E-8)=\begin{pmatrix} 0 & 0 & -e_1 \\ 0 & 0 & -e_2 \\ 0 & e_1 & e_3-e_2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -c_1e_1-e_3 \end{pmatrix}$
$(E-9)=\begin{pmatrix} 0 & 0 & e_1 \\ 0 & e_1 & 0 \\ 2e_1 & e_1+e_2 & e_3-e_2 \end{pmatrix}$	$\begin{pmatrix} 1 & -c_2 & -\frac{1}{2}c_2 \\ 0 & 1 & c_2 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & e_1 \\ 0 & e_1 & 0 \\ 2e_1 & (1-c_2)e_1+e_2 & -\frac{1}{2}c_2e_1+e_3+e_2 \end{pmatrix}$

According to the $\alpha(e_1e_3)=\alpha(e_3)$, we get the equations

$$a_{11}a_{13}=a_{13}, a_{11}a_{23}+a_{21}a_{13}=a_{23}, a_{11}a_{23}+a_{11}a_{33}+a_{31}a_{13}=a_{33}$$

According to the $\alpha(e_2e_1)=\alpha(e_2)$, we get the equations

$$a_{12}a_{11}=a_{12}, a_{12}a_{21}+a_{22}a_{11}=a_{22}, a_{12}a_{21}+a_{12}a_{31}+a_{32}a_{11}=a_{32}.$$

According to the $\alpha(e_1e_2)=0$, we get the equations

$$a_{12}^2=0, 2a_{12}a_{22}=0, a_{12}a_{22}+2a_{12}a_{32}=0$$

. According to the $\alpha(e_2e_3)=0$, we get the equations

$$a_{12}a_{13}=0, a_{12}a_{23}+a_{22}a_{13}=0, a_{12}a_{23}+a_{12}a_{33}+a_{32}a_{13}=0.$$

According to the $\alpha(e_3e_1)=\alpha(e_3)$, we get the equations

$$a_{13}a_{11}=a_{13}, a_{13}a_{21}+a_{23}a_{11}=a_{23}, a_{33}=a_{13}a_{21}+a_{13}a_{31}+a_{33}a_{11}.$$

According to the $\alpha(e_3e_2)=0$, we get the equations

$$a_{13}a_{12}=0, a_{13}a_{22}+a_{23}a_{12}=0, a_{13}a_{22}+a_{13}a_{32}+a_{33}a_{12}=0.$$

According to the $\alpha(e_3e_3)=0$, we get the equations

$$a_{13}^2=0, 2a_{13}a_{23}=0, a_{13}a_{23}+2a_{13}a_{33}=0.$$

So we obtain the matrixes of α is

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{or} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & b_2 & 0 \end{pmatrix}.$$

As if $e_ie_j=\sum_{k=1}^{3} d_k^{ij}e_k$, and the corresponding characteristic matrixes are

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} e_1 & b_2e_2+b_3e_3 & 0 \\ b_2e_2+b_3e_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

References

[1] Bai C M. Bijective 1-cocycles and classification of 3-dimensional left-symmetric algebras. Communications in Algebra. 2009, 37: 1016-1057.

[2] Yau D. Hom-Novikov algebras. Journal of physics A: Mathematical and Thoretical. 2011, 44(8).

[3] Yau D. Hom-algebras and Homology. Journal of Lie Theory. 2009, 19: 409-421.

[4] Makhiouf A, Silvestrov S D. Hom-algebra structures. Journal of Generalized Lie Theory and Application. 2008, 2: 51-64.

[5] Cayley, A. . On the theoryof analytic forms called trees. In: Cayley, A. ed. Collected Mathematical Papers of Arthur Cayley, Vol. 3, oxford: Cambridge University Press,1980. 242-246.

[6] Diatta, A.Medina, A.Classical Yang-Baxter equation and left invariant affine geometry on Lie groups.Manuscripta Math. 2004, .144(4):477-486.

[7] Bai C M, Meng D J. The classification of Novikov algebras in low dimensions. Journal of Physics A: Mathematical and General. 2001, 34(8): 1581-1594.

[8] Koszul, J.L. Domaines bornes homogenes et orbites de transformations affines. Bull.Soc.Math.France. 1961, 89:515-533.

[9] Matsushima,Y. Affine structures on complex mainfolds. Osaka J.Math.1968, 5: 215-222.

[10] Liu L G, Ni X, Bai C M. L-quadri-algebras. Scientia Sinica: Mathematica. 2011,44: 105-124.

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