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Cardinal properties of Hattori spaces on the real lines and their superextensions

Farkhod G. Mukhamadiev

National University of Uzbekistan named after Mirzo Ulugbek
str.Universitet 4, 100174 Tashkent, Uzbekistan
farhod8717@mail.ru

Nodirbek K. Mamadaliev

Tashkent State Pedagogical University named after Nizami
str.Yusuf Khos Hojib 103, 100070 Tashkent, Uzbekistan
nodir_88@bk.ru

Abstract

In the work some cardinal and topological properties of Hattory space on the real lines and their superextensions are investigated.

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1 Introduction

In 1981 on the Prague topological symposium V.V.Fedorchuk [1] put forward the following common problems in the theory of covariant functors:

Let P be some geometrical property and F- some covariant functor. If X has a property P, then $F(X)$ has the same property P? Or on the contrary, i.e. for what functors, if $F(X)$ possesses a property P, it follows that X possesses the same property P?

In our work the property P is a cardinal number of topological spaces and functors $F = exp, \lambda, N, P, O$: an exponential functor, functor of superextension, the functor of complete linked systems, probability measures, weakly additive functionals, respectively.

2 Preliminary Notes

It is known that the family consisting of all sets $U \subset R$ with the property that for every $x \in U$ there exists an $\epsilon > 0$ such that $(x-\epsilon, x+\epsilon) \subset U$, generates the natural topology on the real line R.

In 1929 P.S.Alexandroff defined following space [2]: consider half-open interval $[0,1)$, the family of all half-open intervals $[\alpha,\beta)$, where $0 \leq \alpha < 1$, $\alpha < \beta \leq 1$, generates a base for some topology on $[0,1)$. Obtained topological space is called P.S.Alexandroff arrow.

In 1947 R.M.Sorgenfrey [3] defined a topology on the real line as following way:

Let $\Re$ be the family of all intervals $[x,r)$, where $x, r \in R, x < r$ and r is a rational number. It is easy to check that the family $\Re$ generates a base of a topology on R. Members of $\Re$ are clopen with respect to the topology generated by $\Re$. It is clear that $w(R) = c$ [4]. The space R with the topology defined above, is called Sorgenfrey line.

In 2010 Y.Hattory [5] defined a topology on R as following way:

Let R be the real line and $A \subseteq R$. The topology $\tau(A)$ on R is defined as follows:

(1) for each $x \in A$, $\{(x-\epsilon, x+\epsilon) : \epsilon > 0\}$ is the neighborhood base at X,

(2) for each $x \in R \backslash A$, $\{[x, x+\epsilon) : \epsilon > 0\}$ is the neighborhood base at x.

The space $(R, \tau(A))$ is called Hattory space.

Let τ_E be the Euclidean topology on R. Note that for any $A, B \subseteq R$ we have $A \supseteq B$ iff $\tau(A) \subseteq \tau(B)$, in particular $\tau(R) = \tau_E \subseteq \tau(A)$, $\tau(B) \subseteq \tau(\emptyset) = \tau_S$. Put $P_{top}(R) = \{\tau(A) : A \subseteq R\}$ and define a partial order $\leq$ on $P_{top}(R)$ by inclusion: $\tau(A) \leq \tau(B)$ iff $\tau(A) \subseteq \tau(B)$.

It is clear that Hattory topology generalizes Sorgenfrey topology and the topology of Alexandroff and the natural topology.

We give some definitions of cardinal functions of topological spaces.

A set $A \subset X$ is called dense in X if $[A] = X$. The density of a space X is defined as the smallest cardinal number of the form $|A|$, where A is a dense subset of X; this cardinal number is denoted by $d(X)$.

The character of a point x of a space X is the smallest cardinal number in the form $|\beta(x)|$, where $\beta(x)$ is a base of X at x; this cardinal number is denoted by $\chi(x, X)$.

The character of a topological space X is the supremum of all numbers $\chi(x, X)$ for $x \in X$; this cardinal number is denoted by $\chi(X)$.

A family $\beta(x)$ of neighborhoods of a point x of a space X is called a π-base of X at a point x if for any neighborhood V of x there exists an element $U \in \beta(x)$ such that $U \subset V$.

The π-character of a point x of space X is the smallest cardinal number in the form $|\beta(x)|$, where $\beta(x)$ is a π-base of X at x; this cardinal number is denoted by $\pi\chi(x, X)$.

The π-character of a topological space X is the supremum of all numbers $\pi\chi(x, X)$ for $x \in X$; this cardinal number is denoted by $\pi\chi(X)$.

The tightness of a point x in a topological space X is the smallest cardinal number $\tau \geq \aleph_0$ with the property that if $x \in \overline{C}$, then there exists a $C_0 \subset C$ such that $|C_0| \leq \tau$ and $x \in \overline{C_0}$; this cardinal number is denoted by $t(x, X)$.

The tightness of a topological space X is the supremum of all numbers $t(x, X)$ for $x \in X$; this cardinal number is denoted by $t(X)$ [6].

The smallest cardinal number $\tau \geq \aleph_0$ such that every family of pairwise disjoint nonempty open subsets of X has the cardinality $\leq \tau$, is called the Souslin number of a space X and is denoted by $c(X)$.

The spread $s(X)$ of the space X is the least infinite cardinal τ such that the cardinality of the discrete space X does not exceed τ, i.e. $s(X) = sup\{\tau : \tau = |Y|, Y \subset X, Y \text{ is discrete}\}$.

A cardinal $\tau \geq \aleph_0$ is said to be a caliber of the space X if for any family $\mu = \{U_\alpha : \alpha \in A\}$ of nonempty open in X sets such that $|A| = \tau$, there exists $B \subset A$, for which $|B| = \tau$, and $\bigcap\{U_\alpha : \alpha \in B\} \neq \emptyset$. Set $k(X) = \{\tau : \tau \text{ is a caliber of the space } X\}$.

A cardinal $\tau \geq \aleph_0$ is called to be a precaliber of the space X if for any family $\mu = \{U_\alpha : \alpha \in A\}$ of nonempty open in X sets such that $|A| = \tau$, there is $B \subset A$, for which $|B| = \tau$, and $\{U_\alpha : \alpha \in B\}$ is centered. Set $pk(X) = \{\tau : \tau \text{ is a precaliber of the space } X\}$.

The cardinal number $min\{\tau : \tau^+ \text{ is a caliber of } X\}$ is called the Shanin number of X and is denoted by $sh(X)$, where τ^+ is the least cardinal number from all cardinals strictly greater that τ.

The cardinal number $psh(X) = min\{\tau : \tau^+ \text{ is a precaliber of } X\}$ is called the predshanin number.

We say that the weakly density of a topological space X is equal to $\tau \geq \aleph_0$ if τ is the smallest cardinal number such that there exists a π-base in X coinciding with τ centered systems of open sets, i.e. there exists a π-base $B = \bigcup\{B_\alpha : \alpha \in A\}$, where B_α is centered system of open sets for every $\alpha \in A, |A| = \tau$ [7].

The weakly density of a topological space X is denoted by $wd(X)$.

The π-weight of a space X is the smallest cardinal number in the form $|\beta|$, where β is a π-base of X; this cardinal number is denoted by $\pi w(X)$.

Let φ be a cardinal invariant. Denote by $h\varphi$ the new cardinal defined by the following formula: $h\varphi(X) = sup\{\varphi(Y) : Y \subset X\}$. Invariants $hc(X)$, $hd(X)$, $h\pi w(X)$, $hsh(X)$, $hpsh(X)$, $hk(X)$, $hpk(X)$, $hwd(X)$, $hl(X)$, $he(X)$ denote the hereditary Souslin number, the hereditary density, the hereditary π-weight, the hereditary Shanin number, the hereditary pre-shanin number, the hereditary caliber, the hereditary pre-caliber, the hereditary weakly density, the hereditary Lindelof number, and the hereditary extent of the space X, respectively. The spread [6] $s(X)$ of the space X is the least infinite cardinal τ such that the cardinality of any discrete subspace of X does not exceed τ, i.e. $s(X) = sup\{\tau : \tau = |Y|, Y \subset X, Y\text{- discrete}\}$.

Let X be a T_1-space. The collection of all nonempty closed subsets of X we denote by $expX$. The family B of all sets in the form $O\langle U_1, U_2, ..., U_n\rangle = \{F : F \in expX, F \subset \bigcup U_i, F \cap U_i \neq \emptyset, i = 1, 2, ..., n\}$, where $U_1, U_2, ..., U_n$ is a sequence of open sets of X, generates the topology on the set $expX$. This topology is called the Vietoris topology. The $expX$ with the Vietoris topology is called the exponential space or the hyperspace of X [8].

Let X be a T_1-space. Denote by $exp_n X$ the set of all closed subsets of X cardinality of that is not greater than the cardinal number n, i.e. $exp_n X = \{F \in expX : |F| \leq n\}$.

A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called linked if every two elements of ξ have non-empty intersection. By Zorn lemma any linked system can be filled up to a maximal linked system (MLS), but such completion is not unique.

Proposition 2.1 *.[8]. A linked system ξ of a space X is MLS iff it has following density property:*
if a closed subset $A \subset X$ intersects all elements of ξ then $A \in \xi$.

The superextension λX of a topological space X is the set λX of all maximal linked systems of the topological space X generated by the Wallman topology, an open base of which consists of sets in the form $O(U_1, U_2, ..., U_n) = \{\xi \in \lambda X : \forall i = 1, 2, ..., n, \exists F_i \in \xi : F_i \subset U_i\}$, where $U_1, U_2, ..., U_n$ are open subsets of X.

A topological space X can be naturally embedded in λX identifying each point of X to the MLS $\xi_x = \{F \in expX : x \in F\}$, where $expX$ is the exponential space of X.

A.V.Ivanov [9] defined the space NX of complete linked systems (CLS) of a space X in a following way:

Definition 2.2 *.[9]. A linked system M of closed subsets of a compact X is called a complete linked system (CLS) if for any closed set of X, the condition*
"Any neighborhood OF of the set F consists of a set $\Phi \in M$" implies $F \in M$.

A set NX of all complete linked systems of a compact X is called the space NX of CLS of X. This space is equipped with the topology, the open basis of which is formed by sets in the form of $E = O(U_1, U_2, ..., U_n)\langle V_1, V_2, ..., V_s\rangle = \{M \in NX :$ for any $i = 1, 2, ..., n$ there exists $F_i \in M$ such that $F_i \subset U_i$, and for any $j = 1, 2, ..., s, F \cap V_j \neq \emptyset$ for any $F \in M$ $\}$, where $U_1, U_2, ..., U_n, V_1, V_2, ..., V_s$ are nonempty open in X sets.

Definition 2.3 *.[10]. A functional $v : C(X) \longrightarrow R$ is said to be*
1) weakly additive if $v(\varphi + c_X) = v(\varphi) + c$ for all $\varphi \in C(X)$ and $c \in R$;
2) order preserving if for all $\varphi, \psi \in C(X)$ from $\varphi \leq \psi$ it follows that $v(\varphi) \leq v(\psi)$;
3) normed if $v(1_X) = 1$.

For a compactum X by $O(X)$ we denote the set of all weakly additive, order preserving and normed functionals. For shortness, elements of the set $O(X)$ we shall call weakly additive functionals. Note [10] that any functional $v \in O(X)$ is a continuous mapping from $C(X)$ into R. Consequently, the set $O(X)$ is a subset of the space $C_p(C(X))$ of all continuous functions on $C(X)$ generated by the topology of pointwise convergence. With the topology of pointwise convergence the set $O(X)$ can be considered as a subspace of the space $C_p(C(X))$. Sets in the form $(v; \varphi_1, \varphi_2, ..., \varphi_k; \epsilon) = \{v' \in O(X) : |v(\varphi_i) - v'(\varphi_i)| < \epsilon, i = 1, 2, ..., k\}$, where $\varphi_i \in C(X), i = 1, 2, ..., k$ and $\epsilon > 0$, forms a base of neighborhoods of a weakly additive functional $v \in O(X)$.

We will need following propositions:

Theorem 2.4 *.[11]. Let X be a separable space. Then any uncountable cardinal number is a caliber of the space X.*

Proposition 2.5 *.[12]. A) $e(X) \leq s(X) \leq min\{hl(X), hd(X)\}$.*

Proposition 2.6 *.[13]. For any T_1- space X we have $c(X) \leq wd(X) \leq d(X)$.*

In [14] M.Talaat proved

Theorem 2.7 *.[14]. For any Xausdorff space X we have the following: The subspace $exp_3^0 X = \{F \in expX : |F| = 3\}$ of the hyperspace $expX$ is homeomorphic to the subspace $\lambda_3^0 X = \{\xi \in \lambda X : |supp\xi| = 3\}$ of the superextension λX where $supp\xi$ is the support of the MLS ξ.*

3 Main Results

It is possible to prove the following theorem easily.

Theorem 3.1 *. For any subset $A \subset R$ we have*
1) $d(R, \tau(A)) = \aleph_0$;
2) $wd(R, \tau(A)) = \aleph_0$;
3) $c(R, \tau(A)) = \aleph_0$;
4) $\pi w(R, \tau(A)) = \aleph_0$;
5) $\chi(x, (R, \tau(A))) = \aleph_0$;

6) $\pi\chi(x,(R,\tau(A))) = \aleph_0$;
7) $sh(R,\tau(A)) = \aleph_0$;
8) $psh(R,\tau(A)) = \aleph_0$;
9) $t(x,(R,\tau(A))) = \aleph_0$;
10) $l(R,\tau(A)) = \aleph_0$;
11) $e(R,\tau(A)) = \aleph_0$;
12) $k(R,\tau(A)) = c$;
13) $pk(R,\tau(A)) = c$;
14) $s(R,\tau(A)) = \aleph_0$.

Theorem 3.2 . *Let A be a subset of R such that $int(R\setminus A) \neq \emptyset$. Then for the Hattory space $(R,\tau(A))$ and the covariant functors F we have*
1) $s(R,\tau(A)) \neq s(F(R,\tau(A)))$;
2) $hd(R,\tau(A)) \neq hd(F(R,\tau(A)))$;
3) $h\pi w(R,\tau(A)) \neq h\pi w(F(R,\tau(A)))$;
4) $hsh(R,\tau(A)) \neq hsh(F(R,\tau(A)))$;
5) $hc(R,\tau(A)) \neq hc(F(R,\tau(A)))$;
6) $hk(R,\tau(A)) \neq hk(F(R,\tau(A)))$;
7) $hpk(R,\tau(A)) \neq hpk(F(R,\tau(A)))$;
8) $hpsh(R,\tau(A)) \neq hpsh(F(R,\tau(A)))$;
9) $hwd(R,\tau(A)) \neq hwd(F(R,\tau(A)))$;
10) $hl(R,\tau(A)) \neq hl(F(R,\tau(A)))$;
11) $he(R,\tau(A)) \neq he(F(R,\tau(A)))$
where, $F = \Pi^n, exp_n, exp, \lambda_n, \lambda, P, O, N$*- respectively, functor of degree, an exponential functor, functor of superextension, probability measures, weakly additive functionals, the functor of complete linked systems.*

Proof. At first we will prove for a functor *exp*. Let A be a subset of R such that $int(R\setminus A) \neq \emptyset$. Then there exist a point $a \in R\setminus A$ and neighborhood $[a,b)$ such that $[a,b) \subset R\setminus A$. In $exp_3^0 R$ we consider following set:

$$Y = \{F_t = \{a+t, \frac{a+b}{2}, b-t\} : 0 < t < \frac{b-a}{2}\}.$$

We show that Y is a discrete set of cardinality of c. Suppose $OF_t = \langle O_1^t, O_2^t, O_3^t\rangle$, where $O_1^t = [a+t, \frac{a+b}{2}), O_2^t = [\frac{a+b}{2}, b-t), O_3^t = [b-t, b)$. Let us show that $OF_t \cap Y = F_t$. In fact, suppose $F_t, a+t \in OF_t$, since $a+t^{'} < \frac{b-a}{2}$, we have $a+t^{'} \in O_1^t$ but from $a+t^{'} \in O_1^t$ hence $b-t^{'} \in O_3^t$, therefore $b-t^{'} > b-t$, we have $a+t^{'} < a+t$. Hence, Y is a discrete space of cardinality c. By definition of the spread we have $s(exp_3^0 R) = c$, hence $hd(exp_3^0 R) = c$.

We proved that the spread of the space is equal to $\aleph_0 = s(R,\tau(A)) \neq s(exp_3^0(R,\tau(A))) = c$. So the functor *exp* does not preserve the spread of the Hattory space on the real line $(R,\tau(A))$. Inequality 1) is proved.

2) In the first part we proved that the space $exp(R, \tau(A))$ contains the discrete space Y of cardinality c. So $exp(R, \tau(A))$ is not hereditarily separable space, i.e. $\aleph_0 = hd(R, \tau(A)) \neq hd(exp(R, \tau(A))) = c$. So that the functor exp does not preserve the hereditary density of the Hattory space on the real line $(R, \tau(A))$. 2) is proved.

3) In part 1) we proved that the space $exp(R, \tau(A))$ contains discrete subset of cardinality c. It is known that $\{[r_1, r_2) : r_1 < r_2, r_1, r_2 \in Q\}$ is a π-base of the Hattory space on the real line. So that we have $\aleph_0 = h\pi w(R, \tau(A)) \neq h\pi w(exp(R, \tau(A))) = c$. 3) is proved.

4) From theorem 2.4 [11] it follows that the Shanin number of the Hattory space on the real line is countable. Clear that the space $exp(R, \tau(A))$ contains a discrete set of cardinality c. Then we have $\aleph_0 = hsh(R, \tau(A)) \neq hsh(exp(R, \tau(A))) = c$. 4) is proved.

5) In fact, in the first part we proved the inequality $\aleph_0 = hc(R, \tau(A)) \neq hc(exp(R, \tau(A))) = c$. 5) is proved.

6) It is clear that the Hattory space on the real line $(R, \tau(A))$ is hereditarily separable. In that case by theorem 2.4 [11] any uncountable cardinal number is a caliber of $(R, \tau(A))$. So, the hereditary caliber of the Hattory space $(R, \tau(A))$ equals $hk(R, \tau(A)) = c$ continuum. On the other hand, the set Y is a discrete set of cardinality $hk(exp(R, \tau(A))) = c^+$. So, the hereditary caliber of space is $hk(exp(R, \tau(A))) = c^+$. From this it follows that the functor exp does not preserve the caliber of the Hattory space on the real line $(R, \tau(A))$. 6) is proved.

7) From theorem 2.4 [11] it follows that the caliber and the pre-caliber of the Hattory space on the real line $(R, \tau(A))$ are equal to c. Therefore, $hk(R, \tau(A)) = hpk(R, \tau(A)) = c$ is the cardinality of continuum, $hk(exp(R, \tau(A))) = hpk(exp(R, \tau(A))) = c^+$ is the next cardinal to c. So that the functor exp does not preserve the hereditary caliber and hereditary pre-caliber of the Hattory space on the real line $(R, \tau(A))$. 7) is proved.

8) The definition of the Shanin number and parts 6) and 7) implies the inequality $hpsh(R, \tau(A)) \neq hpsh(exp(R, \tau(A)))$. 8) is proved.

9) The Hattory space on the real line $(R, \tau(A))$ is hereditary separable. Then, by proposition 2.6 [13] the space $(R, \tau(A))$ is hereditary weakly separable. We showed that $exp(R, \tau(A))$ contains a discrete set Y of cardinality c. So, the space $exp(R, \tau(A))$ is not hereditary weakly separable, i.e. $\aleph_0 = hwd(R, \tau(A)) \neq hwd(exp(R, \tau(A))) = c$. Hence the functor exp does not preserve hereditary weakly density of the Hattory space $(R, \tau(A))$. 9) is proved.

10) It is known that the Hattory space $(R, \tau(A))$ is a hereditary Lindelof space, i.e. each subspace is finally compact. In 1) we shoved that the space $exp(R, \tau(A))$ contains a discrete subset Y of cardinality c. The set Y is not finally compact subspace of $exp(R, \tau(A))$. So that the space $exp(R, \tau(A))$ is

not hereditary Lindelof space. It implies that the functor exp does not preserve the Lindelof number of the space $exp(R,\tau(A))$. Inequality 10) is proved.

11) In part 1) we proved that the spread of the Hattory space is countable. From proposition 2.5 [12] we see that the extent $e(R,\tau(A)) = \aleph_0$ of the Hattory space is countable. It is clear that the subset Y is a discrete subset of cardinality c. A we know, any subset of a discrete space is also discrete and closed. So $\aleph_0 = he(R,\tau(A)) \neq he(exp(R,\tau(A))) = c$. We have proved 11).

From the theorem 2.7[14] follows that the theorem 3.2 it is right for functors λ_n and λ.

Remark 3.3 . *From van Mill's theorem [15] on coincidence of the tightness and the character of any normal space X it follows that for the Hattory space we have*

1) $\chi(\lambda(R,\tau(A))) \neq \chi(R,\tau(A))$;

2) $t(\lambda(R,\tau(A))) \neq t(R,\tau(A))$.

Obviously, any MLS ξ is a CLS, hence, $\lambda X \subset NX$. Therefore, the theorem 3.2 is right for a functor N.

T.Radul proved that the space of closed sets $expX$ and superextension λX are subsets of the space $O(X)$ of weakly additive functionals. In the work [10] proved that the functor of probability measures P is a functor subfunctor O. It is known that for any Tychonoff space X its square X^2 topologically is put century $P(X)$. Therefore the theorem 3.2 are true for functors of probability measures P and weakly additive functionals O. Theorem 3.2 is proved.

Let X be a compact. By $C(X)$ denote the space of all continuous functions $f : X \to R$ with usual (pointwise) operations and the sup-norm, i.e. with the norm $\|f\| = \sup\{|f(x)| : x \in X\}$. For each $c \in \mathrm{R}$ by c_X denote the constant function defined by the formula $c_X(x) = c$, $x \in X$. Suppose $\varphi, \psi \in C(X)$. An inequality $\varphi \le \psi$ means that $\varphi(x) \le \psi(x)$ for all $x \in X$.

A functional $\nu : C(X) \to \mathrm{R}$ is called:

1) weakly additive if for all $c \in \mathrm{R}$ and $\varphi \in C(X)$ the equality $\nu(\varphi + c_X) = \nu(\varphi) + c \cdot \nu(1_X)$ holds;

2) order-preserving, if for functions $\varphi, \psi \in C(X)$ from $\varphi \le \psi$ it follows $\nu(\varphi) \le \nu(\psi)$;

3) normed if $\nu(1_X) = 1$;

4) positively-uniform if $\nu(\lambda\varphi) = \lambda\nu(\varphi)$ for all $\varphi \in C(X)$, $\lambda \in \mathrm{R}_+$, where $\mathrm{R}_+ = [0, +\infty)$;

5) semiadditive if $\nu(f+g) \le \nu(f) + \nu(g)$ for all $f, g \in C(X)$ [16].

For a compact X by $OS(X)$ denote the set of all weakly additive , order-preserving, normed, positively-uniform functionals [10]. These sets are equipped with the pointwise topology. Sets in the form

$$\langle\mu;\varphi_1,...,\varphi_k;\varepsilon\rangle = \{\nu \in OS(X) : |\mu(\varphi_i) - \nu(\varphi_i)| < \varepsilon,\ i = 1,...,k\}$$

where $\varphi_i \in C(X)$, $i = 1, ..., k$, $k \in N$, $\varepsilon > 0$, generates a neighborhood base of a functional $\mu \in OS(X)$.

We say that the weakly density of a topological space X is equal to $\tau \geq \aleph_0$ if τ is the smallest cardinal number such that there exists a π-base in X coinciding with τ centered systems of open sets, i.e. there exists a -π-base $B = \bigcup B_\alpha : \alpha \in A$, where B_α is centered system of open sets for every $\alpha \in A$, $|A| = \tau$. The weakly density of a topological space is denoted by $wd(X)$.

The following diagram holds:

$$\begin{array}{ccccccc} P & \rightarrow & OS & \rightarrow & OH & \rightarrow & O \\ & & \uparrow & & \uparrow & & \\ & & exp & \rightarrow & \lambda & & \end{array},$$

where $F \rightarrow G$ means that a functor F is a subfunctor of G.

From works [17] and [18] we can get following theorem:

Theorem 3.4 . *For the normal functor OS and for any infinite Tychonoff space X, we have*

1. $d\left(OS^\beta(X)\right) \leq d(X)$,

2. $wd\left(OS^\beta(X)\right) \leq wd(X)$, *where OS^β is natural extension of the functor OS over the category $Tych$ of Tychonoff spaces*

3) $c(OS(X)) = \sup\{c(X^n) : n \in N\}$, *where c is the Souslin number of the space X.*

Corollary 3.5 . *Functors Π^n, exp_n, exp, OS do not preserve Hattory space on the real line, where $n \in N$.*

Corollary 3.6 . *The product of two Hattory spaces on the real line may not be the Hattory space.*

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