Mathematica Aeterna, Vol. 5, 2015, no. 3, 421 - 424

A NOTE ON FINSLER SPACE WITH HP-SCALAR CURVATURE

Dr. I.S. Chauhan[1], Dr. T.S. Chauhan[2], Dr. Mukesh Chandra[3], MohammadGauhar[4], Neelam[5]
1.Assistant Professor, Ganjdundwara College, Ganjdundwara, Kasganj(U.P.), 2.Associate Professor, Bareilly College, Bareilly(U.P.), 3.Associate Professor, IFTM University, Moradabad, (U.P.), 4.Research Scholar, IFTM University, Moradabad, (U.P.), 5.H.No.-373, Defence Colony, Airforce Gate, Bareilly, 243002

ABSTRACT:

Main aim of the present paper is to studythe Finsler space with HP-scalar curvature.In this paper, we have obtained some important theorems.

KEY WORDS:

HP-scalar curvature, h-curvature tensor, Finsler space, p-sectional curvature.

1.INTRODUCTION:

Definition 1.1:

Let F^nbe a Finsler space of n-dimension with the fundamental function L(x,x*) and g_{ab}(x,x*) be the fundamental tensor. The angular metric tensor$h_{ab}$is definedas[3]:

(1.1) $h_{ab} = g_{ab} - L^{-2}x^{*a}x^{*b}$.

Wherein

(1.2) $x^{*}_{a} = g_{ab}x^{*b}$,

(1.3) $h_{ab}h^{ab} = (n-1)$

and

(1.4) $\delta^a_b h_{ac} = h_{bc}$.

2. h-CURVATURE TENSOR OF BERWALD CONNECTION:

The relation between the h-curvature tensor R_{abcd} of the Cartan connectionand the h-curvature tensor $H_{abcd}$of the Berwald connection is asfollows[3]:

(2.1) $R_{abcd} = H_{abcd} + C_{abe}R^e_{cd} - P_{abc,d} + P_{abd,c} - Q_{abcd}$

Wherein

(2.2) $Q_{abcd} = P_{aec}P^e_{bd} - P_{aed}P^e_{bc}$

(2.3) $H_{ab} = H^e_{abe} = h^{cd}H_{acbd}$,

(2.4) $H^a_{bc} = H^a_{0bc} = R^a_{0bc}$,

(2.5) $h^{ab}R_{acbd} = R_{cd}$,

(2.6) $h^{ab}Q_{acbd} = Q_{cd}$,

(2.7) $h^{ab}Q_{ab} = (n-1)Q$.

Consequently, yields

(2.8) $H_{abcd} = H_{bacd} + 2(R_{abcd} + Q_{abcd})$

and

(2.9) $H_{abcd} = 2(P_{abc,d} - P_{abd,c}) - H_{bacd} - 2C^e_{ab}R_{ecd}$

Theorem 2.1:

If the h-curvature tensor R_{abcd} of a Finsler spaceF^n(n>2) of the Cartan connection and skew-symmetric tensor$Q_{abdc}$with respect to c and d are equal, then the h-curvature tensor $H_{abcd}$of the Berwald connection is symmetric with respect to first two indices.

Proof:

Since the tensor$Q_{abcd}$is skew-symmetric with respect toc and d then equation (2.8) reduces in the form

(2.10) $H_{abcd} = H_{bacd} + 2(R_{abcd} - Q_{abdc})$

Inserting$R_{abcd} = Q_{abdc}$in the equation (2.10),we get

(2.11) $H_{abcd} = H_{bacd}$.

Hence, the h-curvature tensor $H_{abdc}$of the Berwald connection is symmetric with respect to first two indices.

3. HP-SCALAR CURVATURE IN FINSLER SPACE:

Definition 3.1:

A Finsler space F^n(n>3) satisfying the condition[6]:

(3.1) $p^*R_{abcd} = R(h_{ac}h_{bd} - h_{ad}h_{bc})$,

wherein R is p-sectional curvature, is called Finsler space of p-scalar curvature.

Definition 3.2:

AFinsler spaceF^n($n \geq 4$) satisfying the relation[7]:

(3.2) $p^*H_{abcd} = K(h_{ac}h_{bd} - h_{ad}h_{bc})$,

wherein K is HP-scalar curvature, is termed as a Finsler space of HP-scalar curvature.

Definition 3.3:

If K is constant, then the space is termed as Finsler space of HP-constant curvature.

In the Finsler space F^n of HP-scalar curvature satisfy the following relations[7]:

(3.3) $p^*H_{abcd} + p^*H_{bacd} = 0$,

(3.4) $H_{abcd} + H_{bacd} = 2F^{-1}\{(C_{abe}H^e_c + P_{abc/0})l_d - (C_{abe}H^e_d + P_{abd/0})l_c\}$

and

(3.5) $H_{ab} = F^{-1}(l_aH_b + l_bH_{a0}) + F^{-2}(H_{a0b} - l_al_bH_0) + (n-2)Kh_{ab}$.

In this regard, we have the following theorems:

Theorem 3.1:

In Finsler space, if $H_{abcd}$is skew-symmetric with respect to indices a andb then HP-scalar curvature vanishes identically.

Proof:

By virtue of equations (3.2) and (3.3), we get

(3.6) $K = 0$.

Therefore, HP-scalar curvature vanishes.

Theorem 3.2:

In Finsler space, H_{abcd} holds the relation $p^*H_{bacd}h^{ac}h^{bd} = p^*H - 2(n-1)(n-2)K$.

Proof:

In view of equation (3.2), we observe that

(3.7) $(p^*H_{abcd} - p^*H_{bacd}) = 2K(h_{ac}h_{bd} - h_{ad}h_{bc})$

Transvecting equation (3.7) with h^{ac} and using equations (1.3), (1.4), we obtain

(3.8) $p^*H_{bd} - p^*H_{bacd}h^{ac} = 2(n-2)Kh_{bd}$

Transvecting equation (3.8) with h^{bd} and using equation (1.3), we get

(3.9) $p^*H_{bacd}h^{ac}h^{bd} = p^*H - 2(n-1)(n-2)K.$

Now, in a Finsler space F^n of HP-scalar curvature K and p-sectionalcurvature R, the tensor Q_{abcd} is givenby[7]:

(3.10) $Q_{abcd} = (K-R)(h_{ac}h_{bd} - h_{ad}h_{bc})$

Transvecting equation (3.10) by h^{ac} and using equations (1.3), (1.4), (2.6), we obtain

(3.11) $Q_{bd} = (n-2)(K-R)h_{bd}$,

Transvecting equation (3.11) with h^{bd} and using equations (1.3), (2.7), we get

(3.12) $Q = (n-2)(K-R)$.

In this regard, we have the following theorems:

Theorem 3.3:

In a Finsler space $F^n(n>3)$, HP-scalar curvature K and p-sectional curvature R are equaliff Q = 0.

Theorem 3.4:

In a Finsler space $F^n(n>3)$, if Q = 0 and HP-scalar curvature K be constant, then p-sectional curvature R is also constant.

Theorem 3.5:

InR3-like Finsler space $F^n(n \geq 4)$, if Q = 0 thenHP-scalar curvature K admits the relation K = 2m.

Proof:

AnR3-like Finsler space $F^n(n \geq 4)$, the curvature tensor R_{abcd} is defined as[7]:

(3.13) $R_{abcd} = g_{ac}L_{bd} + g_{bd}L_{ac} - g_{ad}L_{bc} - g_{bc}L_{ad}$

Wherein

(3.14) $L_{ab} = m_{ab} + \alpha_a l_b + \beta_b l_a + \gamma l_a l_b$

and

(3.15) $h^{ab}m_{ab} = (n-1)m.$

Operating p* to equation (3.13), we obtain

(3.16) $p^*R_{abcd} = h_{ac}m_{bd} + h_{bd}m_{ac} - h_{ad}m_{bc} - h_{bc}m_{ad}$

Solving the equations (2.10), (3.2) and (3.16) then transvectingwith h^{ac} and using equations (1.3), (1.4) yield

(3.17) $Q_{bd} = \{(n-2)K - (n-1)m\}h_{bd} - (n-3)m_{bd}$

Contracting equation (3.17) with h^{bd} and using equations (1.3), (2.7), we get

(3.18) $Q = (n-2)(K-2m)$

Inserting $Q = 0$ in equation (3.18), we obtain

(3.19) $K = 2m$.

Theorem 3.6:

InR3-like Finsler space $F^n(n \geq 4)$, if $Q = 0$ thenp-sectionalcurvature R hold the relation $R = 2m$.

Proof:

From equations (3.12) and (3.18), we obtain

(3.20) $R = 2m$.

REFERENCES

[1]. C. Shibata: On the curvature tensor $R_{hijk}$of Finslerspaces of scalar curvature, Tensor, N.S., P.(311-317),32, (1978).

[2]. D. Bao, S.S. Chern and Z. Shen: An introduction to Riemann-FinslerGeometry, Springer-Verlag, (2000).

[3]. Dr. A.K. Singh and Mani Gupta: A note on stretch curvature tensor of Finsler space, ShodhHastakshep, Vol. I, No. 1, P.(209-213), (2011).

[4]. H. Izumi: On *P-Finslerspaces,I, Memoirs of Defense Acad., Japan, P.(133-138), 16, (1976).

[5]. H. Izumi: On *P-Finslerspaces,II, Memoirs of Defense Acad., Japan, P.(01-09),17, (1977).

[6]. Hideo Izumi and Mamoru Yoshida: On Finsler spaces of perpendicular scalar curvature, Tensor, N.S., P.(219-224), 32, (1978).

[7]. Mamoru Yoshida: On Finsler spaces of HP-scalar curvature, Tensor, N.S., P.(205-210), Vol. 38, (1982).

[8]. Z. Shen: Lectures on Finsler Geometry, World Scientific, Singapore, 307 pages, (2001).

Received: May, 2015