Mathematica Aeterna, Vol. 7, 2017, no. 2, 131 - 145

The Effects of Viscous Dissipation on Convection in a Porus Medium

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Abstract

The aim of this paper is to study of the effects of variable physical properties and viscous dissipation on a free convective flow over a vertical plate with a variable temperature embedded in a porous medium. We study the effects of varying physical properties on heat transfer and on flow when the medium is filled with some commonly used experimental fluids, in particular, Glycerin, Water and Methyl chloride (a commonly refrigerant). A similarity transformation technique is used to reduce the partial differential equations governing the flow. The resulting system of non-linear coupled ordinary differential equations is solved numerically with appropriate boundary conditions using the Runge-Kutta-Gill method coupled with a shooting technique. Using this approach, a study is conducted on both hot and cold plates and results presented using a combination of graphical illustrations and tables of the effect of changing a variety of physical parameters, in particular, the temperature and viscosity of the fluid.

Mathematics Subject Classification:
76R10, 76S05, 76DXX, 76BXX, 76N20

Keywords:
Porous medium, free convection, porous medium, viscous dissipation, experimental fluids.

1 Introduction

The study of heat transfer in porous media is a classical problem that has drawn the attention from researchers for many decades. As this study helps in understanding various phenomena prevalent to this problem, this paper represents a contribution to the various scientific and engineering fields in general. The problem considered has a number of industrial applications. For example, the problem of heat flow in a vertical flat plate finds application in heat exchangers and the problem of heat transfer to a fluid flowing in circular pipes is of interest in the design of solar water heaters. More recently, another class of heat transfer problem has emerged, namely, the study of heat transfer in porous media. This is due to the wide applications associated with solutions to this problem from chemical particle beds to debris fields and from investigation into soil erosion to modern heat exchanger technology. Theoretical and experimental work on the problem of the free convection flows in porous medium can be found in recent monographs by Nield and Bejan [1] and Ingham and Pop [2] which cover several problems such as the heat transfer through a porous medium, heat transfer in a vertical plate and inclined plate.

2 Viscous Dissipation

It is well known that viscous dissipation is a factor that plays a vital role in temperature distribution especially when the fluid is highly viscous. This problem of viscous dissipation was first studied by Gebhart [3] who found that a non-dimensional parameter $g\beta x/c_p$ where g is the acceleration due to gravity, $\beta$is the coefficient of thermal expansion, x is the distance along the plate and c_p is the specific heat under constant pressure. Known as as the 'Dissipation Parameter', this parameter is a measure of the extent of the viscous dissipation of a medium.

The present paper deals with the free convective flow of three experimental fluids whose viscosity ranges from low values such as that of water to high values such as that of glycerin along a vertical plate embedded in a porous medium. The model developed takes into account the effect of viscous dissipation and the problem is solved for two cases: (i) where the temperature on the plate is more than that of the ambient temperature (the hot plate problem); (ii) where the temperature on the plate is less than that of the ambient temperature (the cold plate problem). In this context, the citations given below provides a background to the need for the study given in this paper and the originality of its content.

Pantokratoras [4] studied the effect of viscous dissipation on natural convection along a heated vertical plate in a non-porous medium and obtained results that show a strong interaction between the viscous heating and the buoyancy force. Christian et al [5] studied the Magneto Hydro-Dynamic (MHD) thermal boundary layer flow over a flat plate with internal heat generation, viscous dissipation and convective surface boundary conditions. Sharma et al [6] analysed the viscous dissipation and mass transfer effects on an unsteady MHD free convective flow along a moving vertical porous plate in the presence of internal heat generation and variable suction. Hossain [7] considered the viscous dissipation effects on natural convection from a vertical plate with uniform surface heat flux embedded in a thermally stratified media.

It is to be noted, that all these studies, though the considered effects of viscous dissipation on the temperature of a fluids, do not take into account the variable physical properties. In many practical heat transfer problems, due to the large differences in the temperatures of the surface and the fluid, the constant fluid properties may result in significant errors, and, hence, variable fluid properties need to be considered. In this context, the few related works that may be associated with the present study are as follows. Carey and Mollendorf [8] study the effects of variable viscosity on natural convection viscous flows (non-porous) of different fluids at an isothermal vertical plate using a similarity transformation by taking a linear variation of viscosity. Vajravelu [9] studied the effect of linearly varying viscosity and thermal conductivity on the laminar free convective flow (non-porous) of air/water confined between two parallel vertical isothermal plates. Cheng and Chang [10] present similarity solutions for free convection in a porous medium adjacent to a horizontal plate when the wall temperature varies as a power function of distance along the plate. Lai and Kulacki [11] discussed the effect of variable viscosity on convection heat transfer in three different cases, that of natural, mixed and forced convection, taking fluid viscosity to vary inversely with temperature. Raja Rani et al. [12] study an application of a HAM for MHD Heat Source problem with variable fluid properties for hot and cold plate.

To the best knowledge of the authors, very little work has been reported on the heat transfer problems taking into account viscous dissipation and variable fluid properties. Although Singh [13] studied the effect of variable fluid properties and viscous dissipation, the study was on mixed convection flows. Thus, the present paper focuses on understanding the effects of variable physical properties in presence of viscous dissipation for both the hot and cold plate problems. The fluids taken for the present study are the most commonly used experimental fluids, namely, Glycerin, Water and Methyl Chloride (a refrigerant).

3 Formulation of the Mathematical Model

Let the x-axis be taken to be the vertical along the plate and y-axis to be perpendicular to it. The temperature of the plate is denoted by T_w, the temperature of the fluid is denoted by T and T_∞ is taken to denote the ambient temperature of the fluid. The orientation of the both hot and cold plates, for free convection, is presented in the Figure 1.

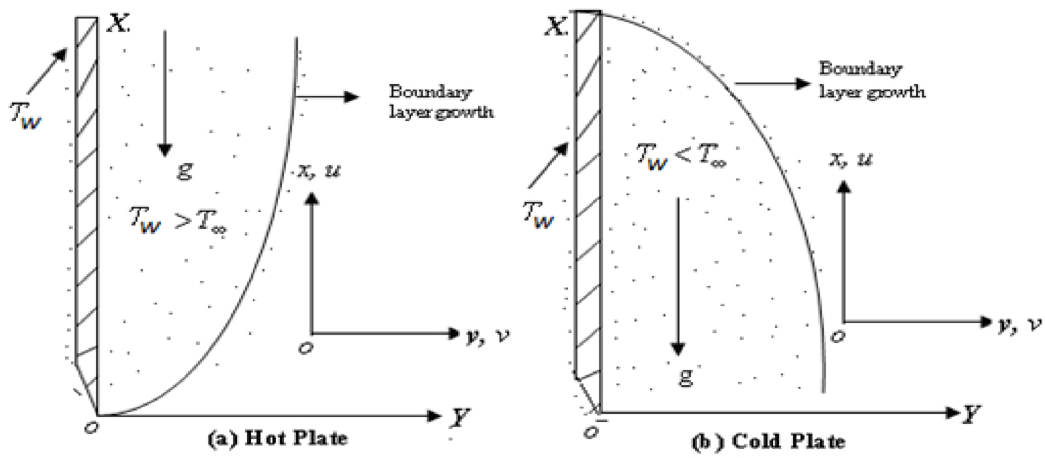

Figure 1: A schemetic of the coordinate system for free convection.

Using the Darcy model, under the Boussinesq approximation, the equations governing the steady free convection boundary layer flow are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$\frac{\partial p}{\partial x} + \rho g + \frac{\mu}{K} u = 0 \tag{2}$$

$$\frac{\partial p}{\partial y} + \frac{\mu}{K} v = 0 \tag{3}$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left(k_m \frac{\partial T}{\partial y} \right) + \mu \frac{u^2}{K} \tag{4}$$

where p is the fluid pressure, ρ is the density of the fluid, μ is its viscosity, g is acceleration due to gravity, u and v are the components of the fluid velocity in x and y directions, respectively, T is the temperature of the medium, K is Permeability, c_p is the specific heat at constant pressure and k_m is effective thermal conductivity of the porous medium.

The boundary conditions on the fluid velocity and the fluid temperature are taken as:

$$\text{at } y = 0;\ v = 0,\ T = T_w + Ax^\lambda \ \text{ and } \ \text{as } y \to \infty;\ u \to 0,\ T \to T_\infty \tag{5}$$

where the expression for temperature indicates that the plate is a non-isothermal plate for values of $\lambda \neq 0$ and isothermal for $\lambda = 0$.

In view of Equation (1), the velocity components u and v can be expressed in terms of stream function ψ as

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Using the above expressions for the velocity components, by eliminating the pressure term, equations (2) and (3) take the form

$$\mu \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial y}\frac{\partial \mu}{\partial y} = K g \rho_\infty \beta \frac{\partial T}{\partial y} \tag{7}$$

$$\rho c_p \left(\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y}\left(k_m \frac{\partial T}{\partial y} \right) + \frac{\mu}{K}\left(\frac{\partial \psi}{\partial y} \right)^2 \tag{8}$$

and the boundary conditions on T and ψ given in (5) become

$$\text{at } y = 0; \ \frac{\partial \psi}{\partial x} = 0, \ T = T_w + A x^\lambda \ \text{ and } \ \text{as } y \to \infty; \ \frac{\partial \psi}{\partial y} \to 0, \ T \to T_\infty \tag{9}$$

Consider the Boussinesq approximation, $\rho = \rho_\infty[1 - \beta(T - T_\infty)]$ as a body force term in Equation (2) and the viscosity and thermal conductivity as varying quantities as defined in [3] and [11],

$$\mu = \mu_f \left[1 + \gamma_\mu \left(\theta - \frac{1}{2} \right) \right] \ \text{ and } \ k_m = k_f \left[1 + \gamma_k \left(\theta - \frac{1}{2} \right) \right] \tag{10}$$

where γ_μ is the viscosity variation coefficient, γ_k is the thermal conductivity variation coefficient, θ is the non-dimensional temperature function defined in Equation (10) and μ_f and k_f are viscosity and thermal conductivity evaluated at film temperature, respectively.

Introducing Rayleigh number Ra_x, the viscous dissipation parameter Ge_x and the non-dimensional functions f and θ together with a similarity variable η through the relations

$$Ra_x = \frac{\rho_\infty K g \beta \mid T_w - T_\infty \mid x}{\alpha_m \mu_f}; \ Ge_x = \frac{g \beta x}{c_p}; \ f(\eta) = \frac{\psi}{\alpha \sqrt{Ra_x}};$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}; \ \eta = \frac{y}{x}\sqrt{Ra_x}; \ \alpha \frac{k_m}{\rho c_p} \tag{11}$$

where the film temperature is given by $T_f = (T_w + T_\infty)/2$, Equations (7) and (8) become, respectively, the linear equation

$$\left[1 + \gamma_\mu \left(\theta - \frac{1}{2} \right) \right] f'' + \gamma_\mu \theta' - \theta' = 0 \tag{12}$$

and the nonlinear equation

$$\lambda\theta f' - \frac{(1+\lambda)}{2}\theta' f - \left[1 + \gamma_k\left(\theta - \frac{1}{2}\right)\right]\theta'' - \gamma_k(\theta')^2$$
$$-Ge_x(f')^2 - Ge_x\gamma_\mu\left(\theta - \frac{1}{2}\right)(f')^2 = 0 \tag{13}$$

The corresponding boundary conditions given by Equation (9) now become become

$$\text{at } \eta = 0;\ f = 0,\ \theta = 1 \ \text{ and } \ \text{as } \eta \to \infty;\ f' \to 1,\ \theta \to 0 \tag{14}$$

The linear Equation (12) can be integrated once using the condition f' at $\eta \to \infty$ to give

$$f' = \frac{\theta}{\left[1 + \gamma_\mu\left(\theta - \frac{1}{2}\right)\right]} \tag{15}$$

the evaluation of Equation (15) at $\eta = 0$ giving the 'slip velocity' $f'(0)$.

4 Parameter Analysis: The Effect of Parameters on Flow and Heat Transfer

In the problem as specified and modelled in the previous section, the flow and heat transfer depend is presented in the following sub-sections.

4.1 Power Index of the Plate Temperature λ

To determine the possible values of λ, the total heat convected at any downstream x is considered. This is governed by the law that states that the total heat convected, i.e.

$$Q(x) = \int_0^\infty \rho c_p \beta (T - T_\infty) u dy$$

is proportional to $x^{(1+3\lambda)/2}$. In this context:

(i) For uniform heat flux surface, $Q(x)$ should vary linearly with x and thus $\lambda = \frac{1}{3}$.

(ii) For an adiabatic surface, $Q(x)$ should be independent of x and thus $\lambda = -\frac{1}{3}$.

(iii) For the isothermal case, $\lambda = 0$.

In this study, solutions have been found for all these values of.

Table 1: Numerical Values of γ_μ amd γ_k for three fluids

Fluid	**Case**	T_0(°C)	T_∞(°C)	T_f(°C)	γ_μ	γ_k
Glycerin - $C_3H_5(OH)_6$	$T_w < T_\infty$	10	30	20	1.4	-0.01
	$T_w > T_\infty$	30	10	20	-1.4	0.01
Water - H_2O	$T_w < T_\infty$	0	40	20	0.67	-0.13
	$T_w > T_\infty$	40	0	20	-0.67	0.13
Methyl Chloride - CH_3Cl	$T_w < T_\infty$	0	40	20	0.4	0.21
(Refrigerant 40)	$T_w > T_\infty$	40	0	20	-0.4	-0.21

4.2 The Viscosity variation Coefficient γ_μ and the Thermal Conductivity variation Coefficient γ_k

The values of γ_μ and γ_k are calculated using the expressions given in Equation (10) for the three fluids considered in this study using the data published in [14]. The values are shown in Table 1.

4.3 The Viscous Dissipation Parameter Ge_x

The viscous dissipation parameter Ge_x is assumed to take on the values 0, 0.1 and 0.2. $Ge_x = 0$ indicates zero viscous dissipation and $Ge_x > 0$ indicates the presence of viscous dissipation with an increase in magnitude indicating increase in the intensity of the parameter.

5 Numerical Solutions

To compute the numerical values of f and θ, Equations (12) and (13) are integrated numerically subject to the bounday conditions given by Equation (14) using the Runge-Kutta-Gill method together with shooting technique [15]. The results are compared with those of [11] for the isothermal case, for constant variations of viscosity and thermal conductivity and the case of zero viscous dissipation in order to validate the model considered. Substituting the values of $\lambda = 0$, $\gamma_\mu = 0$, $\gamma_k = 0$ and $Ge_x = 0$ into Equations (12) and (13) it is observed that, $f''(0) = -\theta'(0)$ and the numerical values obtained are $f''(0) = -0.4437483$, $-\theta'(0) = 0.4437483$ and $f'(0) = 1$ which are in excellent agreement with those provided in [11].

6 Numerical Results

The numerical results are compounded in Tables 2, 3 and 4 from which the following observation are apparent:

(i) Values of $f''(0)$, $f'(0)$ and $-\theta'(0)$ depend significantly on the nature of the viscosity of the fluid (which is to be expected).

(ii) For adiabatic conditions, and, in presence of viscous dissipation, the skin friction $f''(0)$ is positive for all the fluids (for both hot and cold plates) but for the isothermal and uniform heat flux cases, $f''(0) < 0$ can result the boundary layer separation.

(iii) In case of adiabatic, isothermal and a uniform heat flux, the slip velocity for a hot plate is greater than that for a cold plate.

(iv) The flow of the fluid and rate of heat transfer for a hot plate is greater than for a cold plate for all the fluids.

(v) When the viscous dissipation parameter Ge_x increases, the rate of heat transfer $-\theta'(0)$ decreases. This may be due to the buoyancy force that works against the flow as a result of which, retardation may have occurred in the heat transfer process.

6.1 Temperature Profiles

The temperature profiles for the three fluids are shown in Figure 2. From these results it can be seen that:

(i) The temperature and the thermal boundary layer thickness are greater on the cold plate than for the hot plate for all three fluids.

(ii) As viscous dissipation increases, temperature rises in cases (for all three fluids) and the rise in temperature is greater in case of Glycerin (the refrigerant) than that of water whosing that an increase in temperature of the fluid due to viscous dissipation is significantly greater in fluids with high viscosities.

(iii) For all the fluids, a rise in temperature at the plate for the case of $\lambda = -0.3$ (adiabatic surface) is higher and further, the rise in temperature is greater for a cold plate than that a hot plate.

Table 2: Numerical Values of $f''(0)$, $f'(0)$ and $-\theta'(0)$ for the Fluid Glycerin

Case	γ_μ	γ_k	Ge_x	λ	$f''(0)$	$f'(0)$	$\theta'(0)$
$T_w < T_\infty$	1.4	-0.01	0.0	-0.3	-0.040756	0.588235	0.0692852
				0.0	-0.236369	0.588235	0.4018267
				0.3	-0.348517	0.588235	0.5924789
			0.1	-0.3	0.04611923	0.588235	-0.0785268
				0.0	-0.201496	0.588235	0.3425428
				0.3	-0.323794	0.588235	0.5504504
			0.2	-0.3	-0.162489	0.588235	-0.3158094
				0.0	-0.162489	0.588235	0.2762306
				0.3	-0.297539	0.588235	0.505817
$T_w > T_\infty$	1.4	-0.01	0.0	-0.3	-0.3116454	3.3333	0.0934936
				0.0	-1,943619	3.3333	0.5830858
				0.3	-2.891587	3.3333	0.5924789
			0.1	-0.3	0.385659	3.3333	-0.1156977
				0.0	-1.646296	3.3333	0.4938888
				0.3	-2.678996	3.3333	0.8036989
			0.2	-0.3	0.3986572	3.3333	-0.1195972
				0.0	-1.315731	3.3333	0.3947192
				0.3	-2.453663	3.3333	0.736099

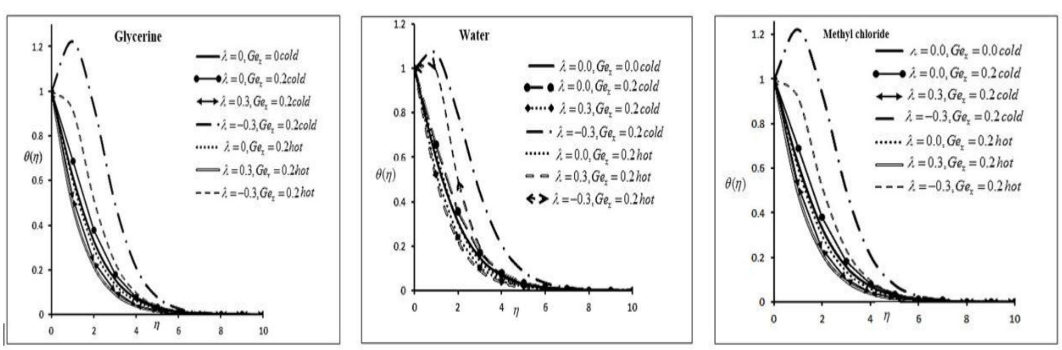

Figure 2: Temperature distributions for three different fluids: Glycerine (left), Water (centre) and Methyl Chloride (right).

Table 3: Numerical Values of $f''(0)$, $f'(0)$ and $-\theta'(0)$ for water.

Case	γ_μ	γ_k	Ge_x	λ	$f''(0)$	$f'(0)$	$\theta'(0)$
$T_w < T_\infty$	0.67	-0.13	0.0	-0.3	-0.070015	0.749037	0.0934703
				0.0	-0.331759	0.749037	0.4428979
				0.3	-0.485964	0.749037	0.6487622
			0.1	-0.3	0.037615	0.749037	-0.0502166
				0.0	-0.284626	0.749037	0.379975
				0.3	-0.452145	0.749037	0.6036139
			0.2	-0.3	0.203852	0.749037	-0.2721425
				0.0	-0.232345	0.749037	0.3101799
				0.3	-0.416334	0.749037	0.5558063
$T_w > T_\infty$	-0.67	0.13	0.0	-0.3	-0.0802313	1.50376	-0.0802313
				0.0	-0.6967391	1.50376	0.4633332
				0.3	-1.044636	1.50376	0.694683
			0.1	-0.3	0.2057257	1.50376	-0.1368076
				0.0	-0.5858193	1.50376	0.3895700
				0.3	-0.966342	1.50376	0.642618
			0.2	-0.3	0.6953732	1.50376	-0.4624232
				0.0	-0.4614025	1.50376	0.306833
				0.3	-0.8830979	1.50376	0.5872600

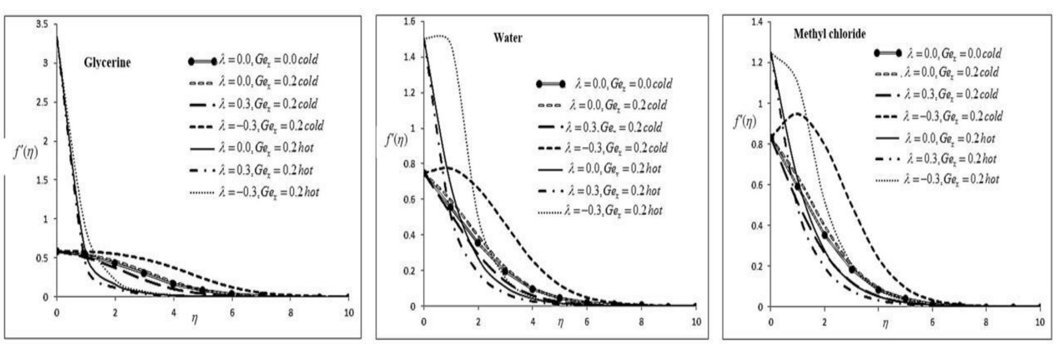

Figure 3: Velocity profiles for three different fluids: Glycerine (left), Water (centre) and Methyl Chloride (right).

6.2 Velocity Profiles

The velocity profiles for the three fluids are shown in Figure 3.

The following observations are noted:

(i) From Figure 2 and Figure 3, it is observed that hydrodynamic boundary layer thickness is more than the thermal boundary layer thickness irrespective of the fluid considered. Further, the hydrodynamic boundary layer thickness is greater for the cold plate than for the hot plate.

(ii) From Figure 3 it can be seen that, the velocity of flow is greater for the hot plate than at the cold plate. This result may be due to the continuous interaction between viscous heating and the buoyancy force. In the case of the hot plate (where $T_w > T_\infty$), the fluid on the plate is warmer than the ambient surroundings and an extra viscous heat added to the initial heat makes it warmer; hence, the fluid velocity increases. In the cold plate case (where $T_w < T_\infty$), the temperature of the fluid at the plate is cooler than the ambient surroundings, and, hence, the velocity at the cold plate decreases.

(iii) The velocity in the vicinity of the hot plate for the three fluids observed as follows: the velocity of Glycerin is more than the velocity of Water which, in turn, is greater than the velocity of the refrigerant. In the vicinity of the cold plate, the results are the opposite to that of the above observations.

(iv) For all the three fluids, it is observed that the flow for an adiabatic surface ($\lambda = -0.3$) is greater than that for the isothermal case ($\lambda = 0$) and for a uniform heat surface ($\lambda = 0.3$).

6.3 Heat Transfer

Variations of the heat transfer $-\theta'(0)$ with different values of λ, for the three fluids is illutrated in Figure 4.

Here, it is observed that:

(i) The heat transfer rate is greater for the hot plate than for the cold plate.

(ii) The heat transfer rate increases as λ increases from -0.3 to 0 and then from 0 to 0.3, i.e. the heat transfer rate is greater for a uniform heat flux surface than for the isothermal case which in turn is higher for an adiabatic surface.

(iii) Negative values of the heat transfer rate have been obtained in the case of adiabatic surface ($\lambda = -0.3$) with an increase in the viscous dissipation parameter.

Table 4: Numerical Values of $f''(0)$, $f'(0)$ and $\theta'(0)$ for Methyl Chloride.

Case	γ_μ	γ_k	Ge_x	λ	$f''(0)$	$f'(0)$	$\theta'(0)$
$T_w < T_\infty$	0.4	0.21	0.0	-0.3	-0.0281981	0.83333	0.0338377
				0.0	-0.3251878	0.83333	0.3902253
				0.3	-0.4890674	0.83333	0.5868807
			0.1	-0.3	0.1197385	0.83333	-0.1436862
				0.0	-0.2724358	0.83333	0.3269229
				0.3	-0.452181	0.83333	0.5426171
			0.2	-0.3	0.4137573	0.83333	-0.4965015
				0.0	-0.2128568	0.83333	0.2554281
				0.3	-0.4890674	0.83333	0.5868807
$T_w > T_\infty$	-0.4	-0.21	0.0	-0.3	-0.1523867	1.25	0.1219094
				0.0	-0.6413627	1.25	0.5130902
				0.3	-0.9354319	1.25	0.7483456
			0.1	-0.3	0.0380087	1.25	-0.030407
				0.0	-0.5525966	1.25	0.4420773
				0.3	-0.8710959	1.25	0.6968768
			0.2	-0.3	0.2977827	1.25	-0.2382262
				0.0	-0.4547971	1.25	0.3638377
				0.3	-0.8031383	1.25	0.6425106

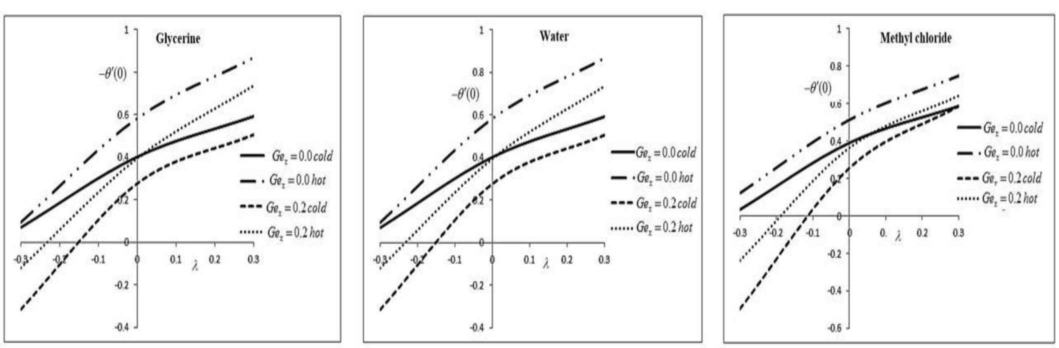

Figure 4: Variations in the heat transfer with respect to varying wall temperature for: Glycerine (left), Water (centre) and Methyl Chloride (right).

6.4 Skin Friction

Figure 5 shows the variations of skin friction $f''(0)$ with respect to varying wall λ for the three fluids.

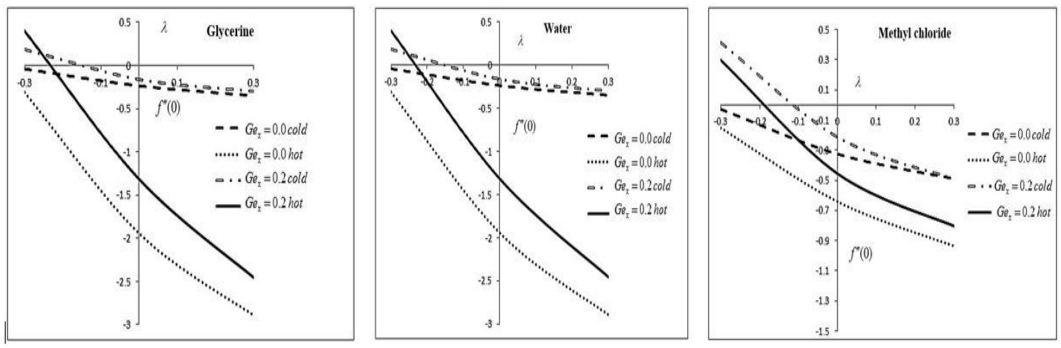

Figure 5: Variations in the Skin Friction with respect to a varying wall parameter λ for: Glycerine (left), Water (centre) and Methyl Chloride (right).

It is observed that:

(i) The Skin Friction is greater for a cold plate than for a hot plate.

(ii) The Skin Friction increases for increasing values of viscous dissipation.

(iii) The Skin Friction decreases as λ increases from -0.3 to 0 and then from 0 to 0.3, i.e. the Skin Friction is larger for an adiabatic surface than for an isothermal and uniform heat flux surface.

7 Conclusions

In this paper, the effects of variable physical properties and viscous dissipation on a free convective flow over a vertical plate is studied. The plate, which has a varying wall temperature, is embedded in a porous medium and the effects of variable physical properties on heat transfer and flow is studied when the medium is filled with some commonly used experimental fluids, namely, Glycerin, Water and Methyl chloride. The following conclusions are drawn basing on the numerical computations undertaken.

(i) As viscous dissipation increases, temperature increases for all three fluids and the rise in temperature is greater in the case of Glycerin and the refrigerant compared Water. This shows that the effect of viscous dissipation is significantly greater in fluids with high viscosities.

(ii) The flow of the fluid and the rate of heat transfer for a hot plate is greater than that for a cold plate.

(iii) As Ge_x (the viscous dissipation parameter) increases, the rate of heat transfer decreases. This may be due to the buoyancy force that works against the flow as a result of which retardation may occur in the heat transfer process.

(iv) The velocity of Glycerin is greater than the velocity of water and the velocity of water is greater than the velocity of the refrigerant in the vicinity of the hot plate. In then case of a cold plate the velocity of the refrigerant is greater than the velocity of water and the velocity of water is greater than the velocity of the Glycerin.

(v) The Skin Friction is greater for a cold plate than for a hot plate and it increases for increasing values of viscous dissipation.

Acknowledgements

The authors would like to acknowledge the support of the Military Technological College, Oman.

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Received: April 17, 2017