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The Centroid of Symplectic Ternary Algebras

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Abstract

In this paper, we study the centroid of a symplectic ternary algebra and find some elementary properties. In particular, some further results concerning centroid for nilpotent and simple symplectic ternary algebras are obtained. At the same time, we discuss the central derivation of the symplectic ternary algebras.

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1 Introduction

The algebras studied here are a generalization of the class of ternary algebras , a variation of Freudenthal triple systems[1]. The advantage of the latest algebras, which we call Symplectic ternary algebras, is that they are defined by identities and hence admit direct sums. JOHN.R.FAULKENER and JOSEPH .C.FERRAR study the structure of this algebra, discussed the semisimple Symplectic ternary algebras and give a classification of the simple algebras over algebraically closed fields of characteristic 0. They construct a good connection between Lie triple systems and Symplectic ternary algebras, so it is naturally that we can generalize the conclusion from Lie triple system to Symplectic ternary algebras.

A centroid is closely related to the derivation algerba [2-5]. It can be used to character the properties of algebras. Naturally we consider the centriod of symplectic ternary algebras. We then In section 3 give the definition of centroid and establish some elementary properties for centroid, give the relationship between the decomposition and the centroid and some further results concerning centroid for solvable Symplectic ternary algebras. Section 4 is devoted to investigate the central derivations of a Symplectic ternary algebra.

We claim that Lie triple systems, Symplectic ternary algebras and Lie algebras considered will be finite dimensional over a field K of characteristic zero.

We recall some definitions, notations and facts which can be found in [6] .

A symplectic ternary algebra U is a vector space with a trilinear product $< x, y, z >$ and satisfies the following identities:

$$S(x, y) = L(x, y) - L(y, x) = R(x, y) - R(y, x), \tag{1}$$

$$S(x, y)R(z, w) = R(z, w)S(x, y) = R(zS(x, y), w) = R(z, wS(x, y)), \tag{2}$$

$$[R(x, y), R(z, w)] = R(xR(z, w), y) = R(x, yR(w, z)), \tag{3}$$

where $x, y, z, u, v \in U$. Define $L(x, y), R(x, y)$by $< x, y, z >= L(x, y)z = R(y, z)x$.

Define $L(x, y), U(x, y) \in EndU$ by $< x, y, z >= L(x, y)z = R(y, z)x = U(x, z)y$.

Example Let U be a vector space with non-degenerate skew form $<, >$ with product defined by

$$< x, y, z >= \frac{1}{2}(< x, y > z+ < y, z > x+ < x, z > y), x, y, z \in U.$$

We can verify U is a symplectic ternary algebra by direct calculation.

An ideal of a symplectic ternary algebra U is a subspace I satisfying $< U, I, U > \subseteq S$ and $< I, U, U > \subseteq S$ or $< U, U, I > \subseteq I$. An ideal I of U is called nilpotent if there is a positive integer k for which $I^k \neq 0, I^{k+1} = 0$, where $I^0 = I, \cdots, I^{s+1} =< I^s, I, I > + < I, I^s, I > + < I, I, I^s >$.

We define

$$Z(U) = \{x \in U| < x, U, U >=< U, x, U >= 0\}$$

the center of U,clearly if $x \in Z(U)$, we have $< U, U, x >= 0$ and It is easy to see that $Z(U)$ is an ideal of U.

If a symplectic ternary algebra can be decomposed into some nonzero ideals I_i such that $U = I_1 \oplus I_2 \oplus \ldots \oplus I_t$,we call it **decomposable**.

A derivation of U is a linear transformation D of U into U such that

$$D < x, y, z >=< D(x), y, z > + < x, D(y), z > + < x, y, D(z) > .$$

A isomorphism σ of $U \to U$ is called automorphism, if it satisfied

$$\sigma < x, y, z >=< \sigma(x), \sigma(y), \sigma(z) > .$$

A Lie triple system (LTS) is a vector space T over a field K with a ternary composition [, ,] which is trilinear and satisfies the following identities:

$$[x, y, z] = -[x, z, y], \tag{4}$$

$$[x, y, z]+[y, z, x]+[z, x, y] = 0, \tag{5}$$

and

$$[u, v, [x, y, z]] = [[u, v, x], y, z]+[x, [u, v, y], z]+[x, y, [u, v, z]], \tag{6}$$

for all $x, y, z, u, v \in T$.

A **derivation** of T is a linear transformation D of T to T such that

$$D[xyz] = [(Dx)yz] + [x(Dy)z] + [xy(Dz)].$$

Define $L(x, y), R(x, y) \in End_K T$ by $[x, y, z] = L(x, y)z = R(z, y)x$, we can see that (4),(5),(6) shows that all $L(x, y)$ are derivations. A derivation D of the form $D = \sum L(x_i, y_i), \forall x_i, y_i \in T$, is called an **inner derivation**. The set, $DerT$, of all derivation of T is a Lie algebra of linear transformation acting in T, the so-called **derivation algebra** of T.

Note that for the ternary composition in T and the binary bracket in $L(T)$,we have $[x, y, z] = [[x, y], z]$ for $x, y, z \in T$.

A subspace I of T is called a **subsystem** if $[I, I, I] \subseteq I$. A subspace I of T is called an **ideal** (denoted by $I \lhd T$) if $[I, T, T] \subseteq I$. For the same reason , a subspace I is an ideal if and only if $[I, T, T] + [T, T, I] + [T, I, T] \subseteq I$. T is called **simple** if $[T, T, T] \neq 0$ and T has no proper ideals.

An ideal I of T is called **solvable** [8] if there is a positive integer k for which $I^{(k)} = 0$, where $I^{(0)} = I, I^{(1)} = [I, I, I], \cdots, I^{(n+1)} = [T, I^{(n)}, I^{(n)}]$. Notice that, for each n, $I^{(n)}$ is an ideal of T and $I \supseteq I^{(1)} \supseteq \cdots \supseteq I^{(n)}$. T is called **semi-simple** if the radical $R(T)$ (the unique maximal solvable ideal) of T is zero. T is called **nilpotent** if there is a positive integer k such that $T^k = 0$, where $T^0 = T, T^1 = [T, T, T], \cdots, T^{n+1} = [T^n, T, T]$.

T is called abelian if $T^1 = T^{(1)} = [T, T, T] = 0$.

For a subsystem I of T,define the centralizer of I in T,$Z_T(I) := \{x \in T \mid [x, I, T] = [x, T, I] = 0\}$.In particular, $Z(T) := Z_T(T) := \{x \in T \mid [x, T, T] = 0\}$ is the center of T.

A Lie triple system T is said to be decomposable if there are nonzero ideals A_1, A_2 such that

$$A = A_1 \oplus A_2$$

and $[A_1, A_2, T]$ =0. Otherwise we say that T is indecomposable. Clearly if T is a simple Lie triple system then T is indecomposable.

To conclude this section, we record the following facts which will be needed in sequel.

Theorem 1.1 Let T be an indecomposable, nilpotent, non-abelian LTS, then $Z(T) \subseteq T^1$.

Proof For T is a nilpotent LTS,obviously,$Z(T) \neq 0, T^1 \neq T$. If $Z(T)$ is not contain in T^1, denote$M = Z(T) \cap T^1$, Let $u_1, u_2, \cdots, u_t$ is a basis of M,and extend it to a basis of T^1:$u_1, u_2, \cdots, u_t, u_{t+1}, \cdots, u_m$, and $u_1, u_2, \cdots, u_t, v_1, \cdots, v_l$. then $u_1, u_2, \cdots, u_t, u_{t+1}, \cdots, u_m, v_1, \cdots, v_l$ is a basis of $Z(T) + T^1$, extend it to a basis of T:

$$u_1, u_2, \cdots, u_t, u_{t+1}, \cdots, u_m, , u^{m+1}, \cdots, u_s, v_1, \cdots, v_l,$$

then $T = L(v_1, \cdots, v_l) \oplus L(u_1, u_2, \cdot, u_s) = I + J$,where I, J are ideals of T, which leads a contradiction.

2 The centroid Symplectic ternary algebra

We define, in this section, the centroid of a Symplectic ternary algebra U, and enumerate several elementary results concerning the centroid.

Definition 2.1 Let U be a Symplectic ternary algebra, and

$$\Gamma(U) = \{\phi \in End(U) | \phi L(x, y) = L(x, y)\phi, \phi U(x, y) = U(x, y)\phi, \forall x, y, z \in U\}.$$

we call $\Gamma(U)$ the centroid of Symplectic ternary algebra U. If $\phi \in \Gamma(U)$,by def (1)and (1.1), the following is easily to proved.i.e.$\phi < x, y, z >=< \phi x, y, z >$.

If $\phi \in \Gamma(U)$,and ϕ is isomorphism, we have

$$\phi^{-1} < x, y, z >= \phi^{-1} < \phi\phi^{-1} x, y, z >= \phi^{-1}\phi[\phi^{-1} < x, y, z >=< \phi^{-1} x, y, z >$$

similarly we have $\phi^{-1} < x, y, z >=< x, \phi^{-1} y, z >$, which means that $\phi^{-1} \in \Gamma(U)$.

Example 2.1 If Symplectic ternary algebra U has the decomposition $U = U_1 \oplus U_2 \oplus \ldots \oplus U_s$, where U_i are ideals of U. Let π_i be the canonical projection of U on U_i, then $\pi_i, i = 1, 2 \in \Gamma(U)$, and $\pi_1 + \pi_2 = id$.

By direct calculation,the following theorem can de easily got:

Theorem 2.1 $\Gamma(U)$ is an associative algebra and a Lie algebra.

Theorem 2.2 Let U be a Symplectic ternary algebra, then U is indecomposable if and only if $\forall \phi \in \Gamma(U)$, the eigenvablues ϕ are equal.

Proof When $dimU > 1$, suppose U is the direct sum of the root space for such :$U = \Sigma\lambda U_\lambda$,where

$$U_\lambda = \{x \in U | (\phi - \lambda Id)^m = 0\}.$$

For $\lambda Id \in \Gamma(U)$, so $(\phi - \lambda Id)^m \in \Gamma(U)$, So $(\phi - \lambda Id)^m < U_\lambda, U, U >= 0$, which means that $< U_\lambda, U, U > \subseteq U_\lambda$, similarly $< U, U_\lambda, U > \subseteq U_\lambda$, i.e. U_λ is an ideal of U. Since $U_{\lambda_1} \cap U_{\lambda_2} = 0$, then U is decomposable, which leads a contradiction.

Conversely, if the eigenvalues of $\phi \in \Gamma(U)$ are equal. Suppose U is decomposable and $U = U_1 \oplus U_2$, where U_1, U_2 are nonzero ideals of U. Thanks to example 2.1, π_1 is in $\Gamma(U)$ and π_1 is a idempotents $\neq 0, Id$, which completes the proof.

The following two propositions have proved in [7], which gave the relationship between the decomposition and the centroid.

Proposition 2.1 let U be a Symplectic ternary algebra, $\phi \in \Gamma(U)$, then there exists a natural number k such that $U = Ker\phi^k \oplus Im\phi^k$.

Proposition 2.2 Let U be an indecomposable Symplectic ternary algebra, and

$$\phi_1 + \phi_2 + \cdots + \phi_m = Id.$$

where $\phi_1, \phi_2, \cdots, \phi_m \in \Gamma(U)$, Then $\phi_i \in Aut(U)$ for some i.

Theorem 2.3 Let U be an indecomposable Symplectic ternary algebra, N is the nilradical of Lie algebra $\Gamma(U)$,then

$$\Gamma(U) = FId\dot{+}N,$$

where N is constituted by the nilpotent elements.

Proof $\forall \phi \in \Gamma(U)$, the Jordan decomposition is $\phi = \phi_s + \phi_n$, where ϕ_s, ϕ_n are semisimple and nilpotent rspectively. Thanks to theorem 2.2, $\phi_s = kId$, so $\phi_s \in \Gamma(U)$, then $\phi - \phi_s = \phi_n \in \Gamma(U)$ too. Denote N the set of nilpotent elements of $\Gamma(U)$, for $\phi_{n_1}, \phi_{n_2} \in N$ and $\phi_{n_1}^m = \phi_{n_2}^l = 0$, then we have $(\phi_{n_1} + \phi_{n_2})^k = 0$, where $k = max(m, l)$, which shows that N is a subspace of $\Gamma(U)$.

For any $\phi \in \Gamma(U), \phi_n \in N$, since $det(\phi\phi_n) = det(\phi_n\phi) = 0$, which leads to $\phi\phi_n, \phi_n\phi \in N$, so$[\phi, \phi_n] = \phi\phi_n - \phi_n\phi \in N$, hence N is a nilpotent ideals in $\Gamma(U)$. By the definition of N, N is the nilradical of Lie algebra $\Gamma(U)$.

Theorem 2.4 Let T be an indecomposable and semisimple LTS, then $\Gamma(T) = FId$.

Proof By theorem above, $\Gamma(T) = FId\dot{+}N$. If $N \neq 0$, we have a $\phi \in \Gamma(T)$ is nilpotent, suppose $\phi(T) = I$. Firstly, by the definition of centroid, it is obvious that $I \neq 0$ is an ideal of T.

By induction on n,we get $I^{(n+1)} = [T, I^{(n)}, I^{(n)}] \subseteq \phi^{1+2+\cdots+n-1}I$.For ϕ is nilpotent, then there exists $k \in Z$ such that $\phi^k = 0$, then we have $I^{(k)} = 0$, which shows I is a solvable ideal of T, a contradiction.

Since U be an indecomposable and semi-simple Symplectic ternary algebra only and only if $T(U)$ is an indecomposable and semi-simple LTS and the relationship between the centroid of U and $T(U)$, it is easy to get this corollary.

corollary 2.1 Let U be an indecomposable and semi-simple Symplectic ternary algebra, then $\Gamma(U) = FId$.

For Symplectic ternary algebra U and it's associative Lie triple system $T(U)$, by the same method, we can obtain the following theorem.

Proposition 2.3 Let U be a sovalble non-abelian and indecomposable, then $Z(U) \subseteq U^1$.

Proof Because a Symplectic ternary algebra U is solvable non-abelian and indecomposable if and only if the lie triple system associated with it $T(U)$ is solvable non-abelian and indecomposable, so $Z(T(U) \subseteq T(U)^1$ by theorem2.1. And $Z(T(U)) = T(Z(U))$ [7], $T(U)^1 \subseteq T(U^1)$, so $T(Z(U)) \subseteq T(U^1)$, which means that $Z(U) \subseteq U^1$.

Theorem 2.5 Let U be an indecomposable nilpotent Sympectic ternary algebra, then $\Gamma(U) \neq FId$.

Proof If U is abelian, then $\Gamma(U) = gl(U) \neq FId$.

If U is not abelian, By $Z(U) \neq 0, Z(U) \subseteq U^1$,Let $u_1, u_2, \cdots, u_k, u_{k+1}, \cdots, u_l$is a basis of U^1,and $u_1, u_2, \cdots, u_k$ is a basis of $Z(U)$. Extend it to a basis of U:

$$u_1, u_2, \cdots, u_k, u_{k+1}, \cdots, u_l, v_1, \cdots, v_t.$$

Define an endomorphism ϕ as follows :$\phi(u_j) = 0, \phi(v_j) = u_1$, obviously,$\phi^2 = 0$,and $\phi \in \Gamma(U)$,then there is a $\phi \in \Gamma(U)$ and ϕ is nilpotent which means that $\Gamma(U) = gl(U) \neq FId$.

3 Central derivation

Proposition 3.1 let U be a Symplectic ternary algebra and B a subset of U,then

(1) $Z_U(B)$ is invariant under $\Gamma(U)$.

(2) Every perfect ideal of U is invariant under $\Gamma(U)$.

Proof Suppose $x \in Z_U(B)$ and $\phi \in \Gamma(U)$, then $< \phi(x), B, U >= \phi < x, B, U >= 0$,and $< \phi(x), U, B >= \phi < x, U, B >= 0$, and $< B, \phi(x), U >= \phi < B, x, U >= 0$. therefore $\phi(x) \in Z_U(B)$, so (1) is proved.

Let B be a perfect ideal of U, then $B =< B, B, B >$. For $x \in B$, there exsit $y_1^i, y_2^i, y_3^i \in B$ such that $x = \sum < y_1^i, y_2^i, y_3^i >$. For $\phi(x) = \phi(\sum < y_1^i, y_2^i, y_3^i >) = \sum(< \phi(y_1^i), y_2^i, y_3^i >) \in B$,which implies that B is invariant under $\Gamma(U)$.

Definition 3.1 let U be a Symplectic ternary algebra, and $\phi \in End(U)$, if $\phi(U) \subseteq Z(U)$,$\phi < U, U, U >= 0$, then ϕ is called central derivation.If U is indecopsable Symplectic ternary algebra,we call it small if $\Gamma(U)$ is generated by central derivations and scalars.

Example 3.1 Let U be an abelian Symplectic ternary algebra,the centroid of U is small and moreover $\Gamma(U) = gl(U)$.

The set of all central derivations of U is denoted by $C(U)$. By direct calculation, it is easy to see that $C(U) \subseteq \Gamma(U)$. and $C(U)$ is an ideal of $\Gamma(U)$. A more precise relationship is summarized as follows.

Proposition 3.2 let U be a Symplectic ternary algebra, then $C(U) = \Gamma(U) \cap DerU$.

Proof If $\phi \in C(U)$, then $\phi < x, y, z >=< \phi(x), y, z >=< x, \phi(y), z >=< x, y, \phi(z) >= 0$,so $\phi \in DerU \cap \Gamma(U)$, which means that $C(U) \subseteq \Gamma(U) \cap DerU$.

On the other hand, $forallx, y, z \in U$, if $\phi \in \Gamma(U) \cap DerU$,$\phi < x, y, z >=< \phi(x), y, z > + < x, \phi(y), z > + < x, y, \phi(z) >= 3 < \phi(x), y, z >= 3\phi < x, y, z >$, so $\phi < x, y, z >=< \phi(x), y, z >= 0$. It implies $\Gamma(U) \cap DerU \subseteq C(U)$.

Let B be a $\Gamma(U)$-invariant ideal of U, denote $V(B) = \{\phi \in \Gamma(U) | \phi(B) = 0\}$ its vanishing ideal.It is easily senn that $V(B)$ is an ideal of $\Gamma(U)$.

Similar proof yields the following conclusion.

Corollary 3.1 let U be a Symplectic ternary algebra, then $C(U) = V(U^1) = \{\Phi \in DerU | Im\phi \subseteq Z(U)\}$.

Theorem 3.1 Let U be a Symplectic ternary algebra, then ϕD is a derivation for $\phi \in \Gamma(U), D \in DerU$.

Proof If $x, y, z \in U$,then

$$\begin{aligned} \phi D < x, y, z > &= \phi(< D(x), y, z > + < x, D(y), z > + < x, y, D(z) > \\ &= < \phi D(x), y, z > + < x, \phi D(y), z > + < x, y, \phi D(z) > . \end{aligned}$$

the proof is finished.

Theorem 3.2 Let U be a Symplectic ternary algebra, then for any $D \in DerU, \phi \in \Gamma(U)$,

(1) $DerU$ is contained in the normalizer of $\Gamma(U)$ in $gl(U)$.

(2) $D\phi$ is contained in $\Gamma(U))$ if and only if $D\phi$ is a central derivation of U.

(3) $D\phi$ is a derivation of L if and only if $[D, \phi]$ is a central derivation of U.

Proof For any $D \in DerU$,$\phi \in \Gamma(U)$and $x, y, z \in U$

$$\begin{aligned} D\phi < x, y, z > &= < D\phi((x), y, z > + < \phi(x), D(y), z > + < \phi(x), y, D(z) > \\ &= < D\phi(x), y, z > + < x, \phi D(y), z > + < x, y, \phi D(z) > \\ &= < D\phi(x), y, z > +\phi D < x, y, z > - < \phi D(x), y, (z) > . \end{aligned}$$

, then we have $(D\phi - \phi D) < x, y, z >=< (D\phi - \phi D)(x), y, (z) >$, this proves (1).By theorem 3.1 and proposition 3.2, $\phi D(< x, y, z >=< \phi D(x), y, z >$ if and any if $\phi D \in \Gamma(U) \cap DerU$, so(2)holds. Thanks to theorem 3.1 and (2),(1),we get the result (3).

Theorem 3.3 If U is a Symplectic ternary algebra and and $U = U_1 \oplus U_2$ is a decomposition of U, then

$$\Gamma(U) = \Gamma(U_1) \oplus \Gamma(U_2) \oplus C_1 \oplus C_2$$

where

$$C_i = \{\phi_i \in hom(U_i, U_j) \mid \phi_i(U_i) \subseteq Z_U(U_j), \phi_i(U_i^1) = 0, for 1 \leq i, j \leq 2, i \neq j\}.$$

Proof By example3.1, we have for $\varphi \in \Gamma(U)$,

$$\varphi = (\pi_1 + \pi_2)\varphi(\pi_1 + \pi_2) = \pi_1\varphi\pi_1 + \pi_1\varphi\pi_2 + \pi_2\varphi\pi_1 + \pi_2\varphi\pi_2.$$

Then $\Gamma(U) = \pi_1\Gamma(U)\pi_1 + \pi_1\Gamma(U)\pi_2 + \pi_2\Gamma(U)\pi_1 + \pi_2\Gamma(U)\pi_2$. Denote $\Gamma(U)_{ij} = \pi_i\Gamma(U)\pi_j$, then $\Gamma(U) = \Sigma_{i,j=1}^{2}\Gamma(U)_{ij}$. It is easy to see that $\Gamma(U)_{1i} \cap \Gamma(U)_{2j} = 0$, by $U_1 \cap U_2 = 0$ So the result $\Gamma(U) = \oplus_{i,j=1}^{2}\Gamma(U)_{ij}$ is suffice to show that $\Gamma(U)_{i1} \cap \Gamma(U)_{i2} = 0$(other case is similar). Suppose $\varphi \in \pi_1\varphi\pi_2 \cap \pi_1\varphi\pi_1$, then there exist $f_1, f_2 \in \Gamma(U)$ such that $\varphi = \pi_1 f_1 \pi_2 = \pi_2 f_2 \pi_1$. For all $x \in U$, $\varphi(x) = \pi_1 f_1 \pi_2(x) = \pi_1 f_1 \pi_2(\pi_2(x)) = \pi_2 f_2 \pi_1(\pi_2(x)) = 0$, so $\varphi = 0$.

Now we prove that $\Gamma(U)_{11} \cong \Gamma(U_1)$. Since $\varphi(U_1) \subseteq U_1, \varphi(U_2) = 0, for \varphi \in \Gamma(U)_{11}$, On the other hand ,one can regard $\Gamma(U_1)$ as a subalgebra of $\Gamma(U)$ by extending any $\varphi_0 \in \Gamma(U_1)$ by zero:

$$\varphi_0(x_1) = \varphi_0(x_1), \varphi_0(x_2) = 0, forall x_i \in U_i, i = 1, 2.$$

Then $\Gamma(U)_{11} \cong \Gamma(U_1)$. Similarly, we have $\Gamma(U)_{22} \cong \Gamma(U_2)$.

Next, we prove $\Gamma(U_{12}) \cong C_2$. For any $\varphi \in \Gamma(U_{12})$, there is a $\varphi_0 \in \Gamma(U)$ such that $\varphi = \pi_1\varphi_0\pi_2$.For any $x_i = x_i^1 x_i^2 \in U$, where $x_i \in U_i$, and $i = 1, 2$, we have

$$\begin{aligned} \varphi < x_1, x_2, x_3 > &= \pi_1\varphi_0\pi_2 < x_1, x_2, x_3 > \\ &= \pi_1\varphi_0 < x_1^2, x_2^2, x_3^2 > \\ &= \pi_1 < \varphi_0(x_1^2), x_2^2, x_3^2 > \\ &= 0. \end{aligned}$$

and

$$< \varphi(x_1), x_2, x_3 >= \varphi < x_1, x_2, x_3 >= 0,$$

then $\varphi(U) \subseteq Z(U)$ and $\varphi(U^1) = 0$. It follows that $\varphi \mid_{U_2} (U_2) \subseteq Z(U_1)$ and $\varphi \mid_{U_2} (U_2^1) = 0$, so $\varphi \mid_{U_2}$ in C_2. Conversely ,for $\varphi_0 \in C_2$, we can extend it in a natural way :$\varphi_0(x_1) = 0, \varphi_0(x_2) = \varphi_0(x_2)$, for $x_i \in U_i$. then $\varphi_0 \in \Gamma(U)_{12}$ and $\Gamma(U)_{12} \cong C_2$.

Similarly, we can prove $\Gamma(U)_{21} \cong C_1$, from the isomorphism of the algebra above , we get

$$\Gamma(U) = \Gamma(U) = \oplus_{i,j=1}^{2}\Gamma(U)_{ij} \cong \Gamma(U_1) \oplus \Gamma(U_2) \oplus C_1 \oplus C_2.$$

Corollary 3.2 Let Symplectic ternary algebra U be the direct sum of two indecomposable ideals of U ,i.e. $U = U_1 \oplus U_2$, then $\Gamma(U)$ is small if and only if $\varphi < U_i, i = 1, 2$ is small.

Theorem 3.4 Let U be a symplectic ternary algebra and its center $Z(U) =< U, U, U >$ with dimension 1, then U is small.

Proof Firstly, we prove the symplectic ternary algebra U is indecomposable. Suppose U is decomposable such that $U = U_1 \oplus U_2$, since $< U, U, U >=$

$Z(U)$ with dimension 1, so either $< U_1, U_1, U_1 >= 0$ or $< U_2, U_2, U_2 >= 0$, now we suppose $< U_2, U_2, U_2 >= 0$, so $U_2 = Z(U)$, furthermore we have $U_1 = Z(U)$, which leads to a contradiction. Then $\Gamma(U) = FId\dot{+}N$ thanks to theorem 2.3.

If $\varphi \in N$, then there exists a natural number k such that $\varphi^k = 0$. It is suffice to prove that φ is a central derivation. Because $< U, U, U >= Z(U) = Fc$, so

$$\varphi < x_1, x_2, x_3 >= \varphi(lc) =< \varphi(x_1), x_2, x_3 >= hc,$$

so we get that $\varphi(c) = \lambda c$. Since φ is nilpotent, we have $\lambda = 0$ and $\varphi < U, U, U >= 0$. Then $\varphi(U) \subseteq Z(U)$. This proves that $\Gamma(U)$ is small.

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