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SOME PROBLEMS ON KAEHLERIAN SPACE WITH SEMI-SYMMETRIC METRIC F-CONNECTIONS

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ABSTRACT:

This paper delineates the study of Kaehlerianspace with semi-symmetric metric F-connections. We have obtained few importanttheorems.

KEY WORDS:

Riemannian space, Kaehlerian space, semi-symmetric, F-connection.

1. INTRODUCTION:

An 2m-dimensional Kaehlerian space is a Riemannian space if it admits a structure tensorF^i_jsatisfying [2,6]:

(1.1) $F^i_j F^j_k = -\delta^i_k$, (1.2) $F_{ij} = -F_{ji}$, (1.3) $\nabla_k F^i_j = 0$,

(1.4) $F^{ij} = -F^{ji}$, (1.5) $F^i_i = 0$, (1.6) $g^{ik}F^j_k = F^{ij}$.

It is easy to verify that in a totally real subspace M_n ofKaehlerian space M_{2m}, the following equations are satisfied [2,6,7]:

(1.7) $F^a_b B^b_x + f^i_x C^a_i = 0$, (1.8) $F^a_b C^b_i - f^x_i B^a_x - f^j_i C^a_j = 0$

Consequently yields

(1.9) $F_{ba} B^{ba}_{yx} = 0$, (1.10) $F_{ba} B^b_x C^a_i + f_{xi} = 0$, (1.11) $F_{ba} C^{ba}_{ji} = f_{ji}$,

(1.12) $F^a_b B^b_x B^y_a = 0$, (1.13) $g_{ba} B^{ba}_{xy} = g_{xy}$.

By virtue of equations (1.2), (1.7), (1.8) and (1.13), we obtain

(1.14) $f_{xi} = f_{ix}$. Wherein (1.15) $f_{ix} = f^y_i g_{yx}$.

Applying the complex structure tensor to equations (1.7), (1.8) and using equations (1.9), (1.10), (1.11), (1.12) and (1.14), we obtain

(1.16) $f^i_x f^y_i = \delta^y_x$ (1.17) $f^i_x f^j_i = 0$

(1.18) $f^i_j f^a_i = 0$ (1.19) $f^k_j f^i_k = -\delta^i_j + f^x_j f^i_x$.

Let $*R^a_{bcd}$ be the curvature tensor of the connection $*\Gamma^a_{bc}$, then we have [1,7]:

(1.20) $*R^a_{bcd} = M^a_{bcd} - \delta^a_b p_{cd} + \delta^a_c p_{bd} - p^a_b g_{cd} + p^a_c g_{bd} - F^a_b q_{cd} + F^a_c q_{bd} - q^a_b F_{cd} + q^a_b F_{bd}$
$- 2(p_d q^a - q_d p^a)F_{bc}$,

Wherein$M^a_{bcd}$is the Riemannian curvature tensor of a Kaehlerian space M_{2m} and

(1.21) $p_{ab} = \nabla_a p_b - p_a p_b + q_a q_b + (1/2)p_c p^c g_{ab}$,

(1.22) $q_{ab} = \nabla_a q_b - p_a q_b - p_b q_a + (1/2)p_c p^c F_{ab}$,

(1.23) $p^a_b = p_{bc} g^{ca}$, (1.24) $q^a_b = q_{bc} g^{ca}$.

In this regard, we have

(1.25) $p_{ab} = q_{ac} F^c_b$, (1.26) $q_{ab} = -p_{ac} F^c_b$, (1.27) $p_{ab} - p_{ba} = 0$.

2. KAEHLERIAN SPACE WITH SEMI-SYMMETRIC METRIC F-CONNECTIONS:

In n-dimensional totally real subspace$M_n$of a Kaehlerian space M_{2m} admits special semi-symmetric metric F-connection, then we observe that the equations (1.16), (1.17), (1.18) and (1.19), gives the following relations [3,5,6]:

(2.1) $f^i_j = 0$, (2.2) $f^x_j f^i_x = \delta^i_j$ and (2.3) $M_{xyij} f^i_z f^j_w = M_{xyzw}$,

Wherein

(2.4) $M_{xyzw} = M^v_{xyz} g_{vw}$, (2.5) $M_{xyij} = M^k_{xyi} g_{kj}$,

and M^k_{xyi} denotes the curvature tensor of the connection induced in the normal bundle.

Ricci equation is given by [4]:

(2.6) $M_{xyij} = M_{abcd} B^{ab}_{xy} C^{cd}_{ij} - T_{xyij}$,

Wherein

(2.7) $M_{abcd} = M^e_{abc} g_{ed}$ and (2.8) $T_{xyij} = H^v_{xi} H_{yvj} - H^v_{yi} H_{xvj}$.

Contracting the covariant form of equation (1.20) with $B^{ab}_{xy} C^{cd}_{ij}$ and making use of equations (1.2), (1.4), (1.5), (1.7), (1.8), (1.9), (1.10), (1.11), (1.12), (1.21), (1.22), (1.23), (1.24), (1.25), (1.26), (1.27), (2.1) and (2.6), we obtain

(2.9) $^*R_{abcd} B^{ab}_{xy} C^{cd}_{ij} = M_{xyij} + T_{xyij} - f_{xj} p_{bc} B^{bc}_{yv} f^v_i + f_{yj} p_{bc} B^{bc}_{xv} f^v_i - f_{yi} p_{bc} B^{bc}_{xv} f^v_j$
$+ f_{xi} p_{bc} B^{bc}_{yv} f^v_j$.

Definition 2.1:

A Riemannian space M_n is called to be M-Einstein [3] if

(2.10) $M_{xy} = (1/n) M g_{xy}$.

Now, we assume that

(2.11) $^*R^a_{bcd} = \mu_{bc} F^a_d$ and (2.12) $^*R_{bcda} = \mu_{bc} F_{da}$

for some tensor μ_{bc} in Kaehlerian space M_{2m}.

In this regard, we have the following theorems:

Theorem 2.1:

Let M_n be a totally real subspace of a Kaehlerian space M_{2m} with special semi-symmetric F-connection whose curvature tensor assumes the form (2.12). If the second fundamental tensor of M_n commute and n=1, then M_n is conformally flat.

Proof:

Let $^*R_{abcd}$ assume the form (2.12) and the second fundamental tensors of M_n commute i.e. T_{xyij} vanishes, then the equation (2.9) in the view of equations (1.11) and (2.1) reduces to the form

(2.13) $M_{xyij} = f_{xj} p_{bc} B^{bc}_{yv} f^v_i - f_{yj} p_{bc} B^{bc}_{xv} f^v_i + f_{yi} p_{bc} B^{bc}_{xv} f^v_j - f_{xi} p_{bc} B^{bc}_{yv} f^v_j$,

Contracting equation (2.13) with $f^i_z f^j_w$ and making use of equations (1.14), (1.16), (2.1), (2.2), (2.3) and (2.6), we obtain

(2.14) $M_{xyzw} = g_{xw} p_{bc} B^{bc}_{yz} - g_{yw} p_{bc} B^{bc}_{xz} + g_{yz} p_{bc} B^{bc}_{xw} - f_{xz} p_{bc} B^{bc}_{yw}$,

Contracting the equation (2.14) with g^{yz} and using the equation $B^{bc} = B^{bc}_{yz} g^{yz}$, we get

(2.15) $M_{xw} = (n-1) g_{xw} p_{bc} B^{bc}$,

If n=1 then we obtain

(2.16) $M_{xw} = 0$.

This establishes the validity of the theorem.

Theorem 2.2:

Let M_n be a totally real subspace of a Kaehlerian space M_{2m} with special semi-symmetric F-connection whose curvature tensor assumes the form (2.12). If the second fundamental tensor of M_n commute, then the M_n is M-Einstein.

Proof:

Contracting the equation (2.15) with g^{xw}, we get

(2.17) $M = n(n-1) p_{bc} B^{bc}$,

From equations (2.15) and (2.17), we get

(2.18) $M_{xw} = (1/n) M g_{xw}$.

This establishes the validity of the theorem.

Theorem 3.3:

For M_n be a totally real subspace of a Kaehlerian space M_{2m} with special semi-symmetric F-connection whose curvature tensor assumes the form (2.12) and satisfying the condition $F_{bc}M_{xyzw} = 0$.

Proof :

Multiplying equation (2.14) by F_{bc} and using equation (1.9) then we obtain

(2.19) $F_{bc}M_{xyzw} = 0$.

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