Mathematica Aeterna, Vol. 4, 2014, no. 1, 45 - 54

Some cardinal properties of stratifiable spaces and the space of linked systems with compact elements

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Abstract

In the paper we study cardinal properties of stratifiable spaces and the space of linked systems.

Mathematics Subject Classification: 54B20, 54A25

Keywords: Cardinal invariant, linked systems, compact space.

1 Introduction

In the paper we study cardinal properties of stratifiable spaces and the space of linked systems. It is proved that the Lindelof number, the density, the weakly density, the Souslin number, the network weight and the π-network weight of stratifiable spaces are equal. As well as we study the π-weight, the character, the caliber and the pre-caliber of the space of maximal linked systems with compact elements.

2 Preliminary Notes

A collection $\lambda = \{E_\alpha : \alpha \in A\}$ of nonempty subsets of a topological space X is said to be a network of the space X if for each point $x \in X$ and for each neighborhood U of x there exists $E_\alpha \in \lambda$ such that $x \in E_\alpha \subset U$. If a network λ consists of open sets then λ is called a base of the space X[1].

A collection $\lambda = \{E_\alpha : \alpha \in A\}$ of nonempty subsets of a topological space X is said to be a $\pi-$network of the space X if for each open set U of X there exists $E_\alpha \in \lambda$ such that $E_\alpha \subset U$. If a $\pi-$network λ consists of open sets then λ is called a $\pi-$base of the space X.

A set $A \subset X$ is called dense in X if $[A] = X$. The density of a space X is defined as the smallest cardinal number of the form $|A|$, where A is a dense subset of X; this cardinal number is denoted by $d(X)$. If $d(X) \leq \aleph_0$, then we say that the space X is separable.

The smallest cardinal number $\tau \geq \aleph_0$ such that every family of pairwise disjoint nonempty open subsets of X has the cardinality $\leq \tau$, is called the Souslin number of a space X and is denoted by $c(X)$. If $c(X) = \aleph_0$, we say that the space X has the Souslin property.

The Lindelof number is defined as following way:

$l(X) = \min\{\tau :$ for any open cover γ there exists a subcover γ' such that $|\gamma'| \leq \tau\}$. A topological space X is said to be a Lindelof space if every open cover of X has a countable subcover [1].

We say that the weakly density of a topological space X is equal to $\tau \geq \aleph_0$ if τ is the smallest cardinal number such that there exists a $\pi-$base in X coinciding with τ centered systems of open sets, i.e. there exists a $\pi-$base $B = \bigcup\{B_\alpha : \alpha \in A\}$, where B_α is centered system of open sets for every $\alpha \in A$, $|A| = \tau$.

The weakly density of a topological space X is denoted by $wd(X)$. If $wd(X) \leq \aleph_0$, then the topological space X is called weakly separable [2].

Definition 2.1 . *[3]. We say that T_1-topological space is stratifiable provided that to each open set U one can assign a sequence $\{U_n : n \in N\}$ of open subsets of X such that*

a) $[U_n] \subset U$,

b) $\cup\{U_n : n \in N\} = U$,

c) $U_n \subseteq V_n$ *whenever* $U \subseteq V$.

The construction in the definition is called the stratification. It is obvious that every metrizable space is stratifiable.

J.Ceder [3] proved the following statement.

Theorem 2.2 . *[3]. For a stratifiable space X following conditions are equivalent:*

1) X is finally compact;

2) X is separable;

3) X has the Souslin property;

A.V.Arhangel'skii [4] generalized Ceder's result as below

Theorem 2.3 . *[4]. For a stratifiable space X following conditions are equivalent:*
1) X is finally compact;
2) X is separable;
3) X has the Souslin property;
4) X has countable network.

Proposition 2.4 . *[5]. For any T_1-space we have*

$$c(X) \leq wd(X) \leq d(X)$$

In the work [6] R.A.Stoltenberg proved the following theorem:

Theorem 2.5 . *Let τ be an infinite cardinal. Then for any stratifiable space X following conditions are equivalent:*
1) The space X has dense subset of cardinality $\leq \tau$;
2) Every open cover of the space X has a subcover of cardinality $\leq \tau$
3) Every family of pairwise disjoint non-empty open subsets of the space X has cardinality $\leq \tau$;
4) The space X has a network of cardinality $\leq \tau$.

A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called linked if every two elements of ξ have non-empty intersection. Each linked system can be filled up to a maximal linked system (MLS), but such a completion is not unique [7].

Proposition 2.6 . *[7]. A linked system ξ of a space X is MLS iff it has following density property:*
if a closed set $A \subset X$ intersects all the elements of ξ then $A \in \xi$.

The set of all maximal linked systems of a space X we denote by λX.
For a closed set $A \subset X$ we suppose

$$A^+ = \{\xi \in \lambda X : A \in \xi\}.$$

The family of sets in the form A^+ becomes a closed subbase in the space λX.
For an open set $U \subset X$ we get

$$O(U) = \{\xi \in \lambda X : \textit{there exists}\, F \in \xi \,\textit{such that}\, F \subset U\}.$$

The family of all the sets of the form $O(U)$ covers the set λX ($O(X) = \lambda X$), so it becomes a subbase of a topology on X. The set λX with this topology, is called the superextension of X.

The family of all sets in the form

$O(U_1, U_2, ..., U_n) = \{\xi \in \lambda X :\ for\, each\, i = 1, 2, ..., n\, there\, exist\, F_i\, such\, that\, F_i \subset U\}$, where $U_1, U_2, ..., U_n$ are open sets of X, forms a base of topology on λX.

For an arbitrary point $x \in X$ we denote by $\eta(X)$ the family of all closed sets of X containing x. The system $\eta(X)$ is an ultrafilter, and – a fortiori –MLS. Thereby, the mapping $\eta : X \to \lambda X$, which is continuous by the equality

$$U = \eta^{-1} O(U)\ (1)$$

for any open set $U \subset X$. From equality (1) we obtain that for any T_1-space X the mapping η becomes an embedding of the space X into its superextension λX.

Following two definitions are T.Mahmud's [8].

Definition 2.7 . *[8]. Let X be a topological space and λX its superextension. The MLS $\xi \in \lambda X$ is called thin if it contains at least one finite element. We denote the thin MLS by TMLS.*

Definition 2.8 . *[8]. The space $\lambda^* X = \{\xi \in \lambda X : \xi\, is\, TMLS\}$ is called the thin superkernel (or the thin superextension) of X.*

Let $O = O(U_1, U_2, ..., U_n)$ be nonempty element of the base of the superextension. Under the notion of the frame of O in X we will understand the class $K(O) = \{U_1, U_2, ..., U_n\}$. The system $S(O) = \{\nu_1, \nu_2, ..., \nu_l\}$ of all pairwise intersections of elements of the class $K(O)$, is called the pairwise trace of O in X.

A family $\beta(x)$ of neighborhoods of a point x of a space X is called a base at a point x if for any neighborhood $V$of x there exists such an element $U \in \beta(x)$ that $x \in U \subset V$.

The character [1] of a point x of a space X is the smallest cardinal number in the form $|\beta(x)|$, where $\beta(x)$ is a base of X at x; this cardinal number is denoted by $\chi(x, X)$.

The character [1] of a topological space X is the supremum of all numbers $\chi(x, X)$ for $x \in X$; this cardinal number is denoted by $\chi(X)$. If $\chi(X) \leq \aleph_0$, then we say that the space X satisfies the first axiom of countability or is first-countable.

A family $\beta(x)$ of neighborhoods of a point x of a space X is called a π-base of X at a point x if for any neighborhood V of x there exists an element $U \in \beta(x)$ such that $U \subset V$.

The π-character [1] of a point x of a space X is the smallest cardinal number in the form $|\beta(x)|$, where $\beta(x)$ is a π-base of X at x; this cardinal number is denoted by $\pi\chi(x, X)$.

The π-character [1] of a topological space X is the supremum of all numbers $\pi\chi(x, X)$ for $x \in X$; this cardinal number is denoted by $\pi\chi(X)$.

In the work [8] T.Mahmud proved that:

Theorem 2.9 . *[8]. Let X be an infinite T_1-space, then*
1) $\pi w(\lambda X) = \pi w(X)$;
2) $d(\lambda^* X) = d(X)$.

We say that a cardinal number $\tau > \aleph_0$ is a caliber of a space X if every family of cardinality τ consisting of nonempty open subsets of X contains a subfamily of cardinality τ with nonempty intersection. The cardinal number $\min\{\tau : \tau^+ is\, a\, caliber\, of\, X\}$ is called the Shanin number of X and is denoted by $sh(X)$.

A cardinal number $\tau > \aleph_0$ is called a precaliber [1] of a space X if every family of cardinality τ consisting of nonempty open subsets of X contains a subfamily of cardinality τ with the finite intersection property. Suppose $pk(X) = \{\tau : \tau\, is\, a\, precaliber\, of\, X\}$. Now, the cardinal number $psh(X) = \min\{\tau : \tau\, is\, a\, precaliber\, of\, X\}$ is called the predshanin number.

We denote by τ^+ the cardinal number next to τ. Now consider following cardinal numbers:

$sh_0 X = min\{\tau : for\ all\ \tau \leq \sigma,\ where\ \sigma^+\ is\ a\ caliber\ for\ X\}$

$psh_0 X = min\{\tau : for\ all\ \tau \leq \sigma,\ where\ \sigma^+\ is\ a\ pre-caliber\ for\ X\}$

Definition 2.10 . *[8]. Suppose X is a topological space, φ is a cardinal function, and τ is a cardinal number. A maximal linked system is said to be a φ_τ-maximal if it contains at least one element F such that $\varphi(F) \leq \tau$ and is denoted by φ_τ-MLS.*

Definition 2.11 . *[8]. The following subspace is called the φ_τ-superkernel (or τ-superextension with respect to the function φ) of a space X:*

$$\lambda_\tau^\varphi = \{\xi \in \lambda X : \xi\, is\, a\, \varphi_\tau - MLS\}.$$

Definition 2.12 . *[8]. We say that a topological space X has the φ_τ-superkernel if $\lambda_\tau^\varphi = \lambda X$.*

Now, as a function φ, consider the density d and suppose $\tau = \omega$.

From the definition it directly follows that any space X has the d_τ-superkernel, where $\tau = d(X)$, in particular, every separable space has the d_ω-superkernel.

3 Main Results

From proposition 2.4 and theorem 2.5 we obtain following:

Theorem 3.1 . *Let τ be an infinite cardinal. Then for any stratifiable space X following conditions are equivalent:*

1) The space X has dense subset of cardinality $\leq \tau$;

2) The weakly density of the space X is not exceed τ;

3) Every open cover of the space X has a subcover of cardinality $\leq \tau$

4) Every family of pairwise disjoint non-empty open subsets of the space X has cardinality $\leq \tau$;

5) The space X has a network of cardinality $\leq \tau$.

Now we generalize theorem 3.1 as following:

Theorem 3.2 . *Let τ be an infinite cardinal. Then for any stratifiable space X following conditions are equivalent:*

1) The space X has dense subset of cardinality $\leq \tau$;

2) The weakly density of the space X is not exceed τ;

3) Every open cover of the space X has a subcover of cardinality $\leq \tau$

4) Every family of pairwise disjoint non-empty open subsets of the space X has cardinality $\leq \tau$;

5) The space X has a network of cardinality $\leq \tau$;

6) The space X has a π-network of cardinality $\leq \tau$.

Proof. The implication 5)⇒6) is obvious. We show the implication 1)⇒6). Let $M = \{a_s : s \in \Omega\}$ be a dense subset of cardinality $\leq \tau$ in X. We must prove that the family $\mu = \{E_\alpha = \{a_s\} : s \in \Omega\}$ is a π-network of the space X. Let G be any non-empty open subset of X. There exists a point $a_s \in M$ such that $a_s \in G$ from density of the subset M. Then $\{a_s : s \in \Omega\}$. Hence, the family $\mu = \{E_\alpha = \{a_\alpha\} : s \in \Omega\}$ is a π-network of the space X.

Now we show the implication 6)⇒1). Let $\mu = \{E_\alpha : s \in \Omega\}$ be a π-network of the space X. We choose a point a_s from each set E_s. It is obvious that the set $M = \{a_s : s \in \Omega\}$ has cardinality τ. We show that M is dense in X. Let G be an arbitrary non-empty open subset of X. Since $\mu = \{E_\alpha : s \in \Omega\}$ is a π-network of X there exists $E_s \in \mu$ such that $E_s \subset G$. Thence, we have $a_s \in E_s \subset G$, so that M is dense in$X$. Theorem 3.2 is proved.

Corollary 3.3 . *For a stratifiable space X following conditions are equivalent:*

1) X is separable;

2) X is weakly separable;

3) X is finally compact;

4) X has the Souslin property;
5) X has countable network;
6) X has countable π-network.

Proposition 3.4 . *Let X be a T_1-space. Then $O(U_1, U_2, ..., U_n) \subset O(V_1, V_2, ..., V_m)$ iff for each $i \in \{1, 2, ..., m\}$ there exists $k \in \{1, 2, ..., n\}$ such that $U_k \subset V_i$.*

Proof. Necessity. Let inclusion $O(U_1, U_2, ..., U_n) \subset O(V_1, V_2, ..., V_m)$ hold. We suppose opposite, i.e. there exists such $i_0 \in \{1, 2, ..., m\}$ that $U_k \not\subset V_{i_0}$ for any $k \in \{1, 2, ..., n\}$. Then for each $k \in \{1, 2, ..., n\}$ we have $U_k \backslash V_{i_0} \neq \emptyset$. We get a point $x_k \in U_k \backslash V_{i_0}$ for each $k \in \{1, 2, ..., n\}$. From each set in the form $U_i \bigcap U_j$ we choose a point $x_{i\,j} \in U_i \bigcap U_j$, $i, j = 1, 2, ..., n$, $i \neq j$. Consider following sets: $F_1 = \{x_1, x_{12}, x_{13}, ..., x_{1n}\}$, $F_2 = \{x_2, x_{21}, x_{23}..., x_{2n}\}, \dots,$ $F_n = \{x_n, x_{n1}, x_{n2}..., x_{nn-1}\}$ and $F_{n+1} = \{x_1, x_2, x_2..., x_n\}$. It is obvious that the system $\mu = \{F_1, F_2, ..., F_n, F_{n+1}\}$ is linked. We will complete μ to a MLS ξ. For each $j = 1, 2, ..., n$ we have $F_j \subset U_j$ and $F_j \in \xi$. So $\xi \in O(U_1, U_2, ..., U_n)$.

Now we show that $\xi \notin O(V_1, V_2, ..., V_m)$. Suppose opposite, i.e. $\xi \in O(V_1, V_2, ..., V_m)$. Then for each $j = 1, 2, ..., m$ there exists a closed set $M_j \in \xi$ such that $M_j \subset V_j$. For the set V_{i_0} there exists $F \in \xi$ such that $F \subset V_{i_0}$. Since F and F_{n+1} are elements of ξ then they have nonempty intersection: $F \bigcap F_{n+1} \neq \emptyset$. It implies that the set F contains one of the points x_k of F_{n+1} and $x_k \in F \subset V_{i_0}$. From this contradiction it follows that $\xi \notin O(V_1, V_2, ..., V_m)$ and moreover $O(U_1, U_2, ..., U_n) \not\subset O(V_1, V_2, ..., V_m)$ which is opposite to our condition.

Sufficiency. Suppose for each $i \in \{1, 2, ..., m\}$ there exists $k \in \{1, 2, ..., n\}$ such that $U_k \subset V_i$. We show that $O(U_1, U_2, ..., U_n) \subset O(V_1, V_2, ..., V_m)$. Let ξ be an arbitrary element of $O(U_1, U_2, ..., U_n)$. Then for each $k = 1, 2, ..., n$ there exist $F_k \in \xi$ such that $F_k \subset U_k$ and for each $i \in \{1, 2, ..., n\}$, by the condition there exists such $k \in \{1, 2, ..., n\}$ that $U_k \subset V_i$. Thence we have $F_k \subset U_k \subset V_i$. So $\xi \in O(V_1, V_2, ..., V_m)$. Proposition 3.4 is proved.

Definition 3.5 . *Let X be a topological space and λX- its superextension. MLS $\xi \in \lambda X$ is called compact if it contains at least one compact element. We denote the compact MLS by CMLS.*

Definition 3.6 . *The space $\lambda_c X = \{\xi \in \lambda X : \xi\, is\ CMLS\}$ is called the thin compact kernel (or the compact superextension) of X.*

It is obvious that $\lambda^* X \subset \lambda_c X \subset \lambda X$ for a T_1-space X. We have $\lambda_c X = \lambda X$ when X is compact and $\lambda_c X = \lambda^* X$ when X is discrete space.

Note that there exists such MLS that does not contain any compact element.

Let N be the set of all natural numbers with the discrete topology. Consider following sets: $F_n = \{n,\ n+1,\ n+2,\ ...\} \subset N$. It is clear that the system

$\mu = \{F_n : n \in N\}$ is linked. We fill μ up to MLS and denote it by ξ_μ. It is easy to check that ξ_μ does not contain any compact element. In this case $\xi_\mu \in \lambda N$, but $\xi_\mu \notin \lambda_c N$.

There exists a MLS such that it doesn't contain any finite element. Let R be the real line with the natural topology. If we consider following closed sets: $F_1 = [0,1]$, $F_2 = \{\frac{1}{2}\} \bigcup [2, +\infty)$, $\ldots$, $F_k = \{\frac{1}{k}\} \bigcup [k, +\infty)$, $\ldots$, it is clear that the system $\mu = \{F_1, F_2, ..., F_k, ...\}$ is linked. We'll fill it up to MLS ξ. It is easy to check that the system ξ doesn't contain any finite element, but it contains a compact element F_1.

Now we show the following

Theorem 3.7 . *For any infinite T_1-space X we have:*
1) $\pi w(\lambda_c X) = \pi w(X)$;
2) $\pi\chi(X) \leq \pi\chi(\lambda_c X)$.

Proof. 1) It is known that for any topological space X and its dense subset Y we have $\pi w(Y) = \pi w(X)$. Since $\lambda_c X$ is dense in λX, we have $\pi w(\lambda_c X) = \pi w(\lambda X)$. Now, by the theorem 2.1 [8] we have $\pi w(\lambda_c X) = \pi w(X)$.

2) Now we show $\pi\chi(X) \leq \pi\chi(\lambda_c X)$. Let $\pi\chi(\lambda_c X) = \tau$. We shall prove that at each point x of X there exists a π-base B_x of cardinality τ. Suppose $\xi_x = \{F \in \exp X : x \in F\}$. Clear that $\xi_x \in \lambda_c X$, i.e. ξ_x contains a compact element $\{x\} \in \xi_x$. We get a π-base $\beta_\xi = \{O_\alpha = O_\alpha(U_1, U_2, ..., U_n) : \alpha \in A\}$ of cardinality τ at the point ξ_x in $\lambda_c X$. Suppose $S = \bigcup\{S(O_\alpha) : \alpha \in A\}$, then it is clear that $|S| \leq \tau$. We will check that if S is a π-base at the point x of X.

Indeed, let W be an arbitrary neighborhood of x. Since $x \in W$, then it is clear that $\xi_x \in O = O(W)$. There exists at least one element $O_\alpha = O_\alpha(U_1, U_2, ..., U_n)$ from β_ξ in $O = O(W)$, because β_ξ is a π-base of the point ξ_x in $\lambda_c X$. We shall show that there exists at least one element $\nu_i \in S(O_\alpha)$ such that $\nu_i \subset W$. Suppose opposite. We fix a point x_i in each set $\nu_i \backslash W$. Then we obtain the set $\sigma = \{x_1, x_2, ..., x_l\}$. We get $F_i = \{x_j \in \sigma : x_j \in U_j\}$, where $i = 1, 2, ..., n$. It is clear that the system $\mu = \{F_1, F_2, ..., F_n\}$ is linked and $\mu^+ \subset O_\alpha = O_\alpha(U_1, U_2, ..., U_n)$. On the other hand, we have the fact that there is no element of μ^+ in $O(W)$, because $F_i \bigcap W = \emptyset$. The contradiction shows that there exists at least one element $\nu_i \in S(O_\alpha)$ such that $\nu_i \subset W$. Since $\nu_i \in \beta_x$ then β_x is a π-base at the point x in X. So, the point x being arbitrary, we have $\pi\chi(X) \leq \pi\chi(\lambda_c X)$. Theorem 3.7 is proved.

Corollary 3.8 . *a) For any T_1-space we have:*

$$\pi w(X) = \pi w(\lambda_c X) = \pi w(\lambda X)$$

b) if X is compact then $\pi\chi(X) \leq \pi\chi(\lambda X)$.

Proposition 3.9 .*Let Y be dense subspace of a topological T_1-space X. Then λY is dense in λX.*

Proof. Let $O = O(U_1, U_2, ..., U_n)$ be an arbitrary nonempty set of λX. We shall show that $O = O(U_1, U_2, ..., U_n) \bigcap \lambda Y \neq \emptyset$. Consider the pairwise trace $S(O) = \{\nu_1, \nu_2, ..., \nu_l\}$ of the element O. Since sets $\nu_1, \nu_2, ..., \nu_l$ are open in X, there exist elements $x_i \in \nu_i \bigcap Y$, $i = 1, 2, ..., l$. Then we obtain the set $\Phi = \{x_1, x_2, ..., x_l\}$. Suppose $F_i = \{x_j \in \Phi : x_j \in U_i\}$, $i = 1, 2, ..., n$. Then, it is clear that the system $\mu = \{F_1, F_2, ..., F_n\}$ is linked system of closed sets in Y. Since $F_i \subset U_i \bigcap Y$, $i = 1, 2, ..., n$, then we have $\mu^+ = \{\xi \in \lambda Y : \mu \subset \xi\} \subset O(U_1 \bigcap Y, U_2 \bigcap Y, ..., U_n \bigcap Y) \subset O(U_1, U_2, ..., U_n)$. So that subspace λY is dense in λX. Proposition 3.9 is proved.

Corollary 3.10 . *The functor of superextension preserves properties of dense subsets of T_1-spaces.*

With the counterpart of theorem 1.2.19 [8], we can prove following proposition

Proposition 3.11 . *Let X be a infinite T_1-space and τ is any infinite cardinal, then:*
1) if τ is a caliber of X then, τ is a caliber of $\lambda_c X$;
2) $sh(\lambda_c X) \leq sh(X)$;
3) $sh_0(\lambda_c X) \leq sh_0(X)$.

Proposition 3.12 . *Let X be a infinite T_1-space and τ is any infinite cardinal, then:*
1) if τ is a pre-caliber of X then, τ is a pre-caliber of λX;
2) $psh(\lambda_c X) \leq psh(X)$;
3) $psh_0(\lambda_c X) \leq psh_0(X)$.

The following question was put in the work [8]

Question 3.13 .*[8]. Is it true that any T_1-space X has d_ω-superkernel?*

Consider R^2 with the discrete topology and following consequence of subsets of R^2: $A = \{(x, y) : x = 0, y \geq 0\}$, $B = \{(x, y) : x \geq 0, y = 0\}$, and

$F_\alpha = \{(x, y) : x + \alpha^2 y = \alpha, x \geq 0, y \geq 0\}$ for each real number $\alpha \in (0, +\infty)$. Then the system $\eta = \{A, B, F_\alpha : \alpha \in (0, +\infty)\}$ is linked. We fill it up to a maximal linked system ξ, and this system doesn't contain any element of density $\leq \omega$.

ACKNOWLEDGEMENTS. The author's is very grateful for the helpful suggestions of the referee.

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Received: January, 2014