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Single-machine scheduling with past-sequence-dependent delivery times and deteriorating jobs

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Abstract

This paper addresses some single-machine scheduling problems with past-sequence-dependent (p-s-d) delivery times and deteriorating jobs. By the past-sequence-dependent (p-s-d) delivery times, we mean that the delivery time of any job is proportional to the job's waiting time. It is assumed that the deterioration process reflects a increase in the process time as a function of the job's starting time. This paper shows that the single-machine scheduling problems to minimize the makespan and the total completion time are polynomially solvable under the proposed model. It further shows that the problems to minimize the total weighted completion time, discounted total weighted completion time and total tardiness are polynomially solvable under certain conditions.

Mathematics Subject Classification: 90Bxx

Keywords: Scheduling; Deterioration; Past-sequence-dependent delivery times

1 Introduction

In classical deterministic scheduling models, the processing conditions, including the job processing times, are normally viewed as given constants. However, in many real-life scheduling situations, the processing conditions may vary over time, thereby affecting the actual durations of the jobs. This leads to the study of scheduling models in which the actual processing time of a job in a schedule depends on its position in the schedule. There are two categories of models that address scheduling problems with varying processing times. Informally, in scheduling with learning, the actual processing time of a job gets shorter if the job is scheduled later. These problems have received considerable attention and we refer the reader to the survey paper by Biskup [1] for a state-of-the-art review of the recent results in this area, as well as for references to practical applications of these models. On the other hand, in scheduling with deterioration, we assume that the later a job starts, the longer it takes to process the job. Numerous applications of scheduling with deteriorating jobs are reported in the literature. These include modelling of the forging process in steel plants, manufacturing of pre-heated parts in plastic moulding or in silverware production, finance management, scheduling maintenance or learning activities, and scheduling de-rusting operations.

In many scheduling environments, the job processing times are assumed to be an increasing function of their *starting times* (time-dependent deterioration). Most of the related studies consider linear deterioration. Browne and Yechiali [2] study a single-machine scheduling problem with a deterioration model in which the actual processing time of job J_j, if it is started at time t, is given by $p_j^A = p_j + \alpha_j t$, where p_j and α_j are the normal processing time and deterioration rate of job J_j, respectively. They show that sequencing the jobs in non-decreasing order of p_j/α_j minimizes the makespan. Mosheiov [7] considers a special type of the general linear deterioration model in which the actual processing time of job J_j, if it is started at time t, is given by

$$p_j^A = p_j + \alpha t, \tag{1}$$

where α denotes the deterioration rate with $\alpha \geq 0$. He shows that the problem to minimize the total weighted completion time with weights proportional to the original processing times is polynomially solvable. Similar models have been studied by Toksarı and Güer [8], and Yang and Kuo [9]. For more studies on different scheduling models involving deteriorating jobs, we refer the reader to the well-known survey by Gawiejnowicz [6].

Koulamas and Kyparisis [5] considered a situation in which the delivery times of jobs are past-sequence-dependent. Yin et al. [10] considered some single-machine scheduling problems with past-sequence-dependent delivery times and a linear deterioration simultaneously. As a continuation, this paper addresses some single-machine scheduling problems with past-sequence-dependent delivery times and deterioration effect model (1) simultaneously. The remaining part of this paper is organized as follows. Section 2 formulates the model. Section 3 presents the properties of the optimal schedules of the considered problems. Some conclusions are given in the last section.

2 Model formulation

Assume that there are n jobs $J_1, J_2, \ldots, J_n$ to be processed on a single machine. The machine can handle one job at a time, machine idle and preemption are not allowed. All the jobs are available for processing at some time $t_0 \geq 0$. Let p_j, w_j, d_j denote the normal processing time, the weight, and the due date, respectively, of job J_j, $j = 1, 2, \cdots, n$; also, let $J_{[j]}$ and $p_{[j]}$ denote the job scheduled in the jth position in the sequence and its normal processing time, respectively, ($w_{[j]}$ and $d_{[j]}$ are defined accordingly). For convenience, the jobs are indexed according to the shortest normal processing time (SPT) rule, i.e., $p_1 \leq p_2 \leq \cdots \leq p_n$. Due to the effect of deterioration, the *actual* processing time of job J_j is defined as

$$p_j^A = p_j + \alpha t,$$

if it starts at time t, where α denotes the deterioration index with $\alpha \geq 0$. For convenience, we denote this deterioration effect given in the above equation by DT.

As in Koulamas and Kyparisis [5], we assume that the processing of job $J_{[j]}$ must be followed by a p-s-d delivery time $q_{[j]}$, which can be computed as

$$q_{[j]} = \delta W_{[j]} = \delta \sum_{i=1}^{j-1} p_{[i]}^A,\ j = 2, \cdots, n,\ q_{[1]} = 0,$$

where $\delta \geq 0$ is a normalizing constant, $W_{[j]}$ denotes the waiting time of job $J_{[j]}$, and $p_{[i]}^A$ represents the actual processing time of job $J_{[i]}$. Observe that $W_{[j]} = \sum_{i=1}^{j-1} p_{[i]}^A = \sum_{i=1}^{j-1} (1+\alpha)^{j-i-1} p_{[i]} + t_0(1+\alpha)^{j-1}$, $j = 2, \cdots, n$.

For a given schedule S, let $C_j(S)$ denote the completion time of job J_j and $C_{[j]}(S)$ represent the complete time of the job scheduled in the jth position in S. It is assumed that the post-processing operation of any job $J_{[j]}$ modeled by its delivery time $q_{[j]}$ is performed off-line, consequently, it is not affected by the availability of the machine and it can commence immediately upon completion of the main operation resulting in $C_{[1]} = t_0 + p_{[1]}^A + q_{[1]} = t_0(1+\alpha) + p_{[1]}$, and

$$\begin{aligned} C_{[j]} =& W_{[j]} + p_{[j]}^A + q_{[j]} = W_{[j]} + p_{[j]} + \alpha W_{[j]}) + \delta W_{[j]} \\ =& (1+\alpha+\delta) W_{[j]} + p_{[j]} \\ =& (1+\alpha+\delta) t_0 (1+\alpha)^{j-1} + (1+\alpha+\delta) \sum_{i=1}^{j-1} (1+\alpha)^{j-i-1} p_{[i]} + p_{[j]}, \end{aligned} \quad (2)$$

$j = 2, \cdots, n$.

Using the standard three-field notation introduced by Graham et al. [3], our scheduling problem can be denoted as $1|DT, D_{psd}|\gamma$. In this paper we will consider the problems of minimization the following five functions: the maximum completion time (makespan), the total completion time, the total weighted completion time, the discounted total weighted completion time and the total tardiness. The corresponding scheduling problems are denoted as $1|DT, D_{psd}|C_{\max}$, $1|DT, D_{psd}|\sum C_j$, $1|DT, D_{psd}|\sum w_jC_j$, $1|DT, D_{psd}|\sum w_j(1-e^{-aC_j})$ and $1|DT, D_{psd}|\sum T_j$, respectively.

3 Preliminary results

3.1 The problems $1|DT, q_{psd}|C_{\max}$ and $1|DT, q_{psd}|\sum C_j$

Koulamas and Kyparisis [5] showed that the makespan minimization problem and the total completion time minimization problem without deterioration (i.e., $\alpha = 0$) are polynomially solvable. In what follows, we show that similar results still hold for the problems $1|DT, D_{psd}|C_{\max}$ and $1|DT, D_{psd}|\sum C_j$. Before proceeding, we first formulate a lemma.

Lemma 3.1 *[4] Let there be two sequences of numbers x_i and $y_i (i = 1, 2, \cdots, n)$. The sum $B_i = x_iy_i$ of products of the corresponding elements is the least if the sequences are monotonic in the opposite sense.*

Theorem 3.2 *For the problem $1|DT, D_{psd}|C_{\max}$, there exists an optimal schedule in which the jobs are ordered according to the SPT rule.*

Proof. By Eq. (2), we have

$$C_{\max} = C_{[n]} = (1 + \alpha + \delta)t_0(1 + \alpha)^{n-1} + \sum_{j=1}^{n} w_jp_{[j]} \sum_{i=1}^{j-1} p_{[i]} + p_{[j]},$$

where $w_j = \begin{cases} (1+\alpha+\delta)(1+\alpha)^{n-j-1} & j = 1, 2, \cdots, n-1, \\ 1 & j = n. \end{cases}$ It is clear that $w_1 \geq w_2 \geq \cdots w_n$, thus it follows from Lemma 3.1 that the result holds. $\square$

Theorem 3.3 *For the problem $1|DT, D_{psd}|\sum C_j$, there exists an optimal schedule in which the jobs are ordered according to the SPT rule.*

Proof. Let S and S' be two job schedules where the difference between S and S' is a pairwise interchange of two adjacent jobs J_i and J_j, i.e., $S = (\pi_1\ J_i\ J_j\ \pi_2)$ and $S' = (\pi_1\ J_j\ J_i\ \pi_2)$, where π_1 and π_2 denote the partial sequences. In addition, we assume that there are $r - 1$ jobs in S_1 and $p_i \leq p_j$. Thus, jobs J_i and J_j are the rth and $(r+1)$th jobs in S, whereas jobs J_j and J_i are scheduled in the rth and $(r+1)$th position in S'. To show that S dominates S', it suffices to show that $C_i(S)+C_j(S) \le C_j(S')+C_i(S')$ and $C_j(S) \le C_i(S')$. By Eq. (2), we have

$$C_i(S) = (1+\alpha+\delta)t_0(1+\alpha)^{r-1} + (1+\alpha+\delta)\sum_{l=1}^{r-1}(1+\alpha)^{r-l-1}p_{[l]} + p_i$$

$$C_j(S) = (1+\alpha+\delta)t_0(1+\alpha)^{r} + (1+\alpha+\delta)\sum_{l=1}^{r-1}(1+\alpha)^{r-l}p_{[l]} + (1+\alpha+\delta)p_i + p_j$$

$$C_j(S') = (1+\alpha+\delta)t_0(1+\alpha)^{r-1} + (1+\alpha+\delta)\sum_{l=1}^{r-1}(1+\alpha)^{r-l-1}p_{[l]} + p_j$$

and

$$C_i(S') = (1+\alpha+\delta)t_0(1+\alpha)^{r} + (1+\alpha+\delta)\sum_{l=1}^{r-1}(1+\alpha)^{r-l}p_{[l]} + (1+\alpha+\delta)p_j + p_i.$$

Now, by $p_i \le p_j$, it is easy to see that $C_i(S) \le C_j(S')$ and $C_j(S) \le C_i(S')$, and so $C_i(S)+C_j(S) \le C_j(S')+C_i(S')$. Therefore, S dominates S'. Thus, repeating this interchange argument for all jobs not sequenced according to the SPT sequence will yield the theorem. □

3.2 The problem $1|DT, D_{psd}|\sum w_jC_j$

In this section, we consider the problem $1|DT, q_{psd}|\sum w_jC_j$. We show that the problem is polynomially solvable under certain agreeable conditions.

Theorem 3.4 *For the problem $1|DT, D_{psd}|\sum w_jC_j$, if jobs have reversely agreeable weights, i.e., $p_i \le p_j$ implies $w_i \ge w_j$ for all jobs J_i and J_j, then there exists an optimal schedule in which the jobs are ordered according to the non-decreasing order of p_j/w_j (the WSPT rule).*

Proof. We still adopt the same notations as in the proof of Theorem 3.3. Suppose that $p_i/w_i \le p_j/w_j$. Since jobs have reversely agreeable weights, we have $p_i \le p_j$ and $w_i \ge w_j$. By the proof of Theorem 3.3, we have $C_i(S) \le C_j(S')$, $C_j(S) \le C_i(S')$, and $C_l(S) = C_l(S')$ for each job $J_l \notin \{J_i, J_j\}$. To show that S dominates S', it suffices to show that $w_iC_i(S)+w_jC_j(S) \le w_iC_i(S')+w_jC_j(S')$. In fact, since $C_i(S) \le C_j(S')$ and $C_j(S) \le C_i(S')$, we have

$$\begin{aligned} w_iC_i(S')+w_jC_j(S')-w_iC_i(S)-w_jC_j(S) &\ge w_iC_j(S)+w_jC_i(S)-w_iC_i(S)-w_jC_j(S) \\ &= (w_i-w_j)(C_j(S)-C_i(S)). \end{aligned}$$

From $w_i \ge w_j$ and $C_j(S) \ge C_i(S)$, we have $(w_i-w_j)(C_j(S)-C_i(S)) \ge 0$ and so $w_iC_i(S)+w_jC_j(S) \le w_iC_i(S')+w_jC_j(S')$. Therefore, S dominates S'. Thus, repeating this interchange argument for all jobs not sequenced according to the WSPT sequence will yield the theorem. □

3.3 The problem $1|DT, D_{psd}|\sum w_j(1-e^{-aC_j})$

In this section, we consider the problem $1|DT, D_{psd}|\sum w_j(1-e^{-aC_j})$. We show that the problem is polynomially solvable under certain agreeable conditions.

Theorem 3.5 *For the problem $1|DT, q_{psd}|\sum w_j(1-e^{-aC_j})$, if jobs have reversely agreeable weights, i.e., $p_i \leq p_j$ implies $w_i \geq w_j$ for all jobs J_i and J_j, then there exists an optimal schedule in which the jobs are ordered according to the non-decreasing order of $\frac{1-e^{-ap_j}}{w_j e^{-ap_j}}$ (the WDSPT rule).*

Proof. We still adopt the same notations as in the proof of Theorem 3.3. Suppose that $p_i/w_i \leq p_j/w_j$. Since jobs have reversely agreeable weights, we have $p_i \leq p_j$ and $w_i \geq w_j$. By the proof of Theorem 3.3, we have $C_i(S) \leq C_j(S')$, $C_j(S) \leq C_i(S')$, and $C_l(S) = C_l(S')$ for each job $J_l \notin \{J_i, J_j\}$. To show that S dominates S', it suffices to show that $w_i(1-e^{-aC_i(S)}) + w_j(1-e^{-aC_j(S)}) \leq w_i(1-e^{-aC_i(S')}) + w_j(1-e^{-aC_j(S')})$. In fact, since $C_i(S) \leq C_j(S')$ and $C_j(S) \leq C_i(S')$, we have

$$
\begin{aligned}
&w_i(1-e^{-aC_i(S')}) + w_j(1-e^{-aC_j(S')}) - w_i\sum w_j(1-e^{-aC_i(S)}) - w_j(1-e^{-aC_j(S)})\\
=&w_ie^{-aC_i(S)} + w_je^{-aC_j(S)} - w_ie^{-aC_i(S')} - w_je^{-aC_j(S')}\\
\geq&w_ie^{-aC_j(S')} + w_je^{-aC_i(S')} - w_ie^{-aC_i(S')} - w_je^{-aC_j(S')}\\
=&(w_i-w_j)(e^{-aC_j(S')} - e^{-aC_i(S')}).
\end{aligned}
$$

From $w_i \geq w_j$ and $C_j(S') \leq C_i(S')$, we have $(w_i-w_j)(e^{-aC_j(S')}-e^{-aC_i(S')}) \geq 0$ and so $w_i(1-e^{-aC_i(S)})+w_j(1-e^{-aC_j(S)}) \leq w_i(1-e^{-aC_i(S')})+w_j(1-e^{-aC_j(S')})$. Therefore, S dominates S'. Thus, repeating this interchange argument for all jobs not sequenced according to the WDSPT sequence will yield the theorem.
□

3.4 The problem $1|DT, D_{psd}|\sum T_j$

In this section, we consider the problem $1|DT, D_{psd}|\sum T_j$. We show that the problem is polynomially solvable under certain agreeable conditions.

Theorem 3.6 *For the problem $1|DT, D_{psd}|\sum T_j$ if the job processing times and the due dates are agreeable, i.e., $d_i \leq d_j$ implies $p_i \leq p_j$ for all jobs J_i and J_j, then there exists an optimal schedule in which the jobs are ordered according to the non-decreasing order of d_j (the WDSPT rule)..*

Proof. We still adopt the same notations as in the proof of Theorem 3.3. Assume that $d_i \leq d_j$. Since the job processing times and the due dates are agreeable, we have $p_i \leq p_j$. To show that S dominates S', it suffices to show that $T_i(S) + T_j(S) \leq T_j(S') + T_i(S')$, i.e., $\max\{L_i(S), 0\} + \max\{L_j(S), 0\} \leq \max\{L_j(S'), 0\} + \max\{L_i(S'), 0\}$, since $C_{[l]}(S) \leq C_{[l]}(S')$ for $1 \leq l \leq n$ by the proof of Theorem 3.2. We consider the following cases.

Case 1: $L_j(S') \leq 0$ and $L_i(S') \leq 0$. Then $L_i(S) \leq L_i(S') \leq 0$ and $L_j(S) \leq L_i(S') \leq 0$, hence $T_j(S') + T_i(S') = T_i(S) + T_j(S) = 0$.

Case 2: $L_j(S') \leq 0$ and $L_i(S') > 0$. Then

$$T_j(S') + T_i(S') = \max\{L_i(S'), 0\}$$
$$T_i(S) + T_j(S) = \max\{L_i(S), 0\} + \max\{L_j(S), 0\}.$$

Now since $L_j(S') \leq 0$, i.e., $C_j(S') \leq d_j$, we have

$$\begin{aligned} L_i(S') - L_i(S) - L_j(S) &= C_i(S') - d_i - (C_i(S) - d_i) - (C_j(S) - d_j) = d_j - (C_i(S) + C_j(S) - C_i(\\ &\geq d_j - C_j(S') \geq 0, \end{aligned}$$

this implies $T_j(S') + T_i(S') = \max\{L_i(S'), 0\} = L_i(S') \geq \max\{L_i(S), 0\} + \max\{L_j(S), 0\} = T_i(S) + T_j(S)$.

Case 3: $L_j(S') > 0$. Then $L_i(S') \geq 0$. In fact, if $L_i(S') < 0$, then $C_i(S') < d_i$ and so $C_j(S') \leq C_j(S) \leq C_i(S') < d_i \leq d_j$, which contradicts $L_j(S') > 0$. Thus $T_j(S') + T_i(S') = L_i(S') + L_j(S') \geq L_i(S) + L_j(S) \geq T_i(S) + T_j(S)$.

Thus, in any case, we have $T_j(S') + T_i(S') \geq T_i(S) + T_j(S)$. Therefore, S dominates S'. Hence, repeating this interchange argument for all jobs not sequenced according to the EDD sequence will yield the theorem. □

The following corollaries are direct consequences of Theorem 3.6.

Corollary 3.7 *The EDD sequence leads to an optimal schedule for* $1|DT, D_{psd}, p_j = p|\sum T_j$.

Corollary 3.8 *The EDD sequence leads to an optimal schedule for* $1|DT, D_{psd}, d_j = kp_j|\sum T_j$.

4 Conclusions

This paper investigated some single machine scheduling problems with past-sequence-dependent (p-s-d) delivery times and deteriorating jobs. We showed that the single-machine scheduling problems remain polynomially solvable if the objectives are to minimize the makespan and the total completion time. We also showed that under certain conditions, the problems to minimize the total weighted completion time, discounted total weighted completion time and total tardiness are polynomially solvable. We believe that the model offered here will turn out to be more useful in the theory and applications of scheduling. It is useful to guide the practitioners to choose right scheduling rules and suitable model in practical situations.

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