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Ruin Probability with Investment Returns and Dependent Structures

Fenglong Guo

School of Mathematical Sciences,
University of Electronic Science and Technology of China,
Chengdu 611731, People's Republic of China.
E-mail: fenglongguo@gmail.com

Dingcheng Wang[1]

Center of Financial Engineering,
Nanjing Audit University,
Nangjing 211815, People's Republic of China.
E-mail: wangdc@nau.edu.cn

Abstract

This paper investigates ruin probabilities in a discrete-time risk model, where the premiums are modelled by a Markov chain, while the claims and interest rates follow two first-order autoregressive processes. Recursive and integral equations are given for ruin probabilities in the risk model. Inequalities for ruin probability are derived by recursive techniques.

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1 Introduction

Modern insurance businesses allow insurers to invest their wealth into financial assets. Since a large part of the surplus of insurance businesses comes from investment income, actuaries have been studying ruin problems under risk models with interest force. For example, Sundt and Teugels [6, 7] studied the effects of constant rate on the ruin probability under the compound Poisson

[1]Corresponding author. Dingcheng Wang.

risk model. Yang [9] established both exponential and non-exponential upper bounds for ruin probabilities in a risk model with constant interest force and independent premiums and claims. Cai [1] investigated the ruin probabilities in two risk models with independent premiums and claims and used a first-order autoregressive process to model the rates of interest. Cai and Dickson [2] obtained Lundberg type inequalities for ruin probabilities in two discrete-time risk processes with a Markov chain interest model and independent premiums and claims.

In classic risk theory, the surplus process of insurance businesses is assumed to have independent and stationary increments. However, because of the increasing complexity of insurance and reinsurance products, this assumption seems more and more unrealistic in insurance practice. Thus, actuaries have been paying more and more attention to the modelling of dependent risks. For example, Gerber [3] assumed that the surplus process could be written as an initial surplus plus the annual gains and used a linear model to model the annual gains. Yang and Zhang [10] investigated a discrete-time risk model with constant interest force and adopted first-order autoregressive processes to model both the premiums and claims.

In this paper, we investigate the ruin probabilities of a discrete-time risk model. In the model, the premiums are modelled by a Markov chain, while both the claims and the rates of interest follow two first-order autoregressive processes. Recursive and integral equations for the finite-time and ultimate ruin probabilities are established by using renewal recursive technique. Generalized Lundberg inequalities for ruin probabilities are derived.

Let $\{X_n, n = 1, 2, \dots\}$ be the amount of premiums collected in each period and modelled by a Markov chain. Denote the state space of the Markov chain $\{X_n, n = 0, 1, \dots\}$ by $\{x_0, x_1, \dots\}$. Suppose that for all $n = 0, 1, 2, \dots$ and all states $x_t, x_s, x_{t_0}, x_{t_1}, \dots, x_{t_{n-1}}$,

$$\begin{aligned}&\mathbb{P}(X_{n+1} = x_t \mid X_n = x_s, X_{n-1} = x_{t_{n-1}}, \dots, X_1 = x_{t_1}, X_0 = x_{t_0})\\ =&\mathbb{P}(X_{n+1} = x_t \mid X_n = x_s) = p_{st} \geq 0, \quad s, t = 0, 1, 2, \dots, \end{aligned}\tag{1}$$

where $\sum_{t=0}^{\infty} p_{st} = 1$ for all $s = 0, 1, 2, \dots$. Relation (1) is called the Markov property of $\{X_n, n = 0, 1, 2, \dots\}$.

Let $\{Y_n, n = 1, 2, \dots\}$ be the amount of claims in each period and modelled by a first-order autoregressive process, namely,

$$Y_n = aY_{n-1} + W_n, \quad n = 1, 2, \dots, \tag{2}$$

where $Y_0 = y_0 \geq 0$ and $0 \leq a < 1$ are two constants and $\{W_n, n = 1, 2, \dots\}$ is a sequence of independent, identically distributed, and nonnegative random variables. One possible interpretation of model (2) is the following: the parameter a can be interpreted as the proportion of the old business, which will remain in the new portfolios; while W_n denotes the uncertainty to the n-th period's claims.

Let $\{I_n, n = 1, 2, \dots\}$ be the rates of interest and also modelled by a first-order autoregressive process, namely,

$$I_n = bI_{n-1} + Z_n, \quad n = 1, 2, \dots, \tag{3}$$

where $I_0 = i_0 \geq 0$ and $0 \leq b < 1$ are two constants and $\{Z_n, n = 1, 2, \dots\}$ is a sequence of independent, identically distributed and nonnegative random variables.

Assume the processes $\{X_n, n = 1, 2, \dots\}$, $\{W_n, n = 1, 2, \dots\}$ and $\{Z_n, n = 1, 2, \dots\}$ are mutually independent, and denote $G(z) = \mathbb{P}(Z_1 \leq z)$ and $F(w) = \mathbb{P}(W_1 \leq w)$ with $F(0) = 0$.

Suppose the claims are paid at the end of each period and the premiums are collected at the beginning of each period. Then, the surplus process $\{U_n, n = 1, 2, \dots\}$ with initial surplus $u \geq 0$ can be written as

$$U_n = (U_{n-1} + X_n)(1 + I_n) - Y_n, \tag{4}$$

which can be rearranged as

$$U_n = u \prod_{k=1}^{n} (1 + I_k) + \sum_{k=1}^{n} \left((X_k(1 + I_k) - Y_k) \prod_{j=k+1}^{n} (1 + I_j) \right), \tag{5}$$

where $\prod_{j=s}^{t} (1 + I_j) = 1$ if $s > t$.

Denote finite-time ruin probability and ultimate ruin probability of model (5) with models (1)-(3) and initial surplus $u \geq 0$, respectively, by

$$\Psi_n(u, x_s, y_0, i_0) = \mathbb{P}\left(\bigcup_{j=1}^{n} \{U_j < 0\} \right), \quad \Psi(u, x_s, y_0, i_0) = \mathbb{P}\left(\bigcup_{j=1}^{\infty} \{U_j < 0\} \right).$$

It is clear that

$$\lim_{n \to \infty} \Psi_n(u, x_s, y_0, i_0) = \Psi(u, x_s, y_0, i_0).$$

2 Recursive and integral equations for ruin probabilities

Throughout this paper, denote the tail of any distribution function B by $\overline{B}(x) = 1 - B(x)$. In this section, we give the recursive equation for Ψ_n and the integral equation for Ψ by using the renewal recursive technique.

Theorem 2.1. *For $n = 1, 2, \ldots,$ we have*

$$\begin{aligned}&\Psi_{n+1}(u, x_s, y_0, i_0)\\&=\sum_{t=0}^{\infty} p_{st}\int_0^{\infty}\left(\overline{F}(\hbar_{t,z})+\int_0^{\hbar_{t,z}}\Psi_n(\hbar_{t,z}-w, x_t, y, i)\mathrm{d}F(w)\right)\mathrm{d}G(z),\end{aligned}$$

and

$$\begin{aligned}&\Psi(u, x^{(1)}, z^{(1)}, y_0, w_0, i_s)\\&=\sum_{t=0}^{\infty} p_{st}\int_0^{\infty}\left(\overline{F}(\hbar_{t,z})+\int_0^{\hbar_{t,z}}\Psi(\hbar_{t,z}-w, x_t, y, i)\mathrm{d}F(w)\right)\mathrm{d}G(z)\end{aligned}$$

with

$$\hbar_{t,z} = (u + x_t)(1 + i) - ay_0, \quad y = ay_0 + w, \quad i = bi_0 + z. \tag{6}$$

Proof. Given $X_1 = x_t$, $W_1 = w$ and $Z_1 = z$, by (5),

$$\begin{aligned}U_1 &= (u + X_1)(1 + I_1) - Y_1\\&= (u + x_t)(1 + bi_0 + z) - ay_0 - w := \hbar_{t,z} - w.\end{aligned}$$

Thus, if $w > \hbar_{t,z}$, then,

$$\mathbb{P}(U_1 < 0 \mid X_1 = x_t, W_1 = w, Z_1 = z) = 1.$$

Hence, $\mathbb{P}\left(\bigcup_{k=1}^{n+1}\{U_k < 0\} \mid X_1 = x_t, W_1 = w, Z_1 = z\right) = 1$.

Let $\{\widetilde{X}_n, n = 0, 1, 2, \ldots\}$, $\{\widetilde{W}_n, n = 1, 2, \ldots\}$ and $\{\widetilde{Z}_n, n = 1, 2, \ldots\}$ be independent copies of $\{X_n, n = 0, 1, 2, \ldots\}$, $\{W_n, n = 1, 2, \ldots\}$ and $\{Z_n, n = 1, 2, \ldots\}$, respectively. Given $W_1 = w$, consider a process $\{\widetilde{Y}_n, n = 1, 2, \ldots\}$ described as

$$\widetilde{Y}_n = a\widetilde{Y}_{n-1} + \widetilde{W}_n,$$

where $\widetilde{Y}_0 = ay_0 + w = y$. Clearly, $\{\widetilde{Y}_n, n = 1, 2, \ldots\}$ has a similar structure to that of $\{Y_n, n = 1, 2, \ldots\}$, but with different initial values. Given $Z_1 = w$, consider a process $\{\widetilde{I}_n, n = 1, 2, \ldots\}$ defined as

$$\widetilde{I}_n = b\widetilde{I}_{n-1} + \widetilde{Z}_n,$$

where $\widetilde{I}_0 = bi_0 + z = i$. Trivially, $\{\widetilde{I}_n, n = 1, 2, \ldots\}$ has a similar structure to that of $\{I_n, n = 1, 2, \ldots\}$ but with different initial values. Thus, if $0 \leq w \leq$

$\hbar_{t,z}$, from (5) we have

$$
\begin{aligned}
&\mathbb{P}\left(\bigcup_{k=1}^{n+1}\{U_k<0\}\,\Big|\,X_1=x_t,W_1=w,Z_1=z\right)\\
&=\mathbb{P}\left(\bigcup_{k=2}^{n+1}\{U_k<0\}\,\Big|\,X_1=x_t,W_1=w,Z_1=z\right)\\
&=\mathbb{P}\left(\bigcup_{k=2}^{n+1}\left\{(\hbar_{t,z}-w)\prod_{j=2}^{k}(1+I_j)\right.\right.\\
&\qquad\left.\left.+\sum_{j=2}^{k}(X_j(1+I_j)-Y_j)\prod_{t=j+1}^{k}(1+I_t)<0\right\}\,\Big|\,X_1=x_t\right)\\
&=\mathbb{P}\left(\bigcup_{k=1}^{n}\left\{(\hbar_{t,z}-w)\prod_{j=1}^{k}(1+\widetilde{I}_j)\right.\right.\\
&\qquad\left.\left.+\sum_{j=1}^{k}(\widetilde{X}_j(1+\widetilde{I}_j)-\widetilde{Y}_j)\prod_{t=j+1}^{k}(1+\widetilde{I}_t)<0\right\}\,\Big|\,\widetilde{X}_0=x_t\right)\\
&=\Psi_n(\hbar_{t,z}-w,x_t,y,i),
\end{aligned}
$$

where in the second step we used the Markov property of $\{X_n, n=0,1,2,\dots\}$ and the independence among $\{X_n, n=0,1,2,\dots\}$, $\{W_n, n=1,2,\dots\}$, and $\{I_n, n=1,2,\dots\}$.

Therefore, by conditioning on X_1, W_1 and Z_1, we can get

$$
\begin{aligned}
&\Psi_{n+1}(u,x_s,y_0,i_0)=\mathbb{P}\left(\bigcup_{k=1}^{n+1}\{U_k<0\}\right)\\
&=\sum_{t=0}^{\infty}p_{st}\int_0^{\infty}\!\!\int_0^{\infty}\mathbb{P}\left(\bigcup_{k=1}^{n+1}\{U_k<0\}\,\Big|\,X_1=x_t,W_1=w,Z_1=z\right)\mathrm{d}F(w)\mathrm{d}G(z)\\
&=\sum_{t=0}^{\infty}p_{st}\int_0^{\infty}\left(\int_{\hbar_{t,z}}^{\infty}\mathrm{d}F(w)+\int_0^{\hbar_{t,z}}\Psi_n(\hbar_{t,z}-w,x_t,y,i)\mathrm{d}F(w)\right)\mathrm{d}G(z)\\
&=\sum_{t=0}^{\infty}p_{st}\int_0^{\infty}\left(\overline{F}(\hbar_{t,z})+\int_0^{\hbar_{t,z}}\Psi_n(\hbar_{t,z}-w,x_t,y,i)\mathrm{d}F(w)\right)\mathrm{d}G(z).
\end{aligned}
$$

The integral equation for Ψ in Theorem 2.1 follows immediately by letting $n\to\infty$ in the equation above and dominated convergence theorem. $\square$

3 Inequality for ruin probability

Using the recursive equation for Ψ_n in Section 2, we can derive a Lundberg-type upper bound for ultimate ruin probability Ψ. To this end, we need the following proposition.

Proposition 3.1. *For all $s = 0, 1, 2, \ldots$, assume that $\mathbb{E}[(1+a)W_1 + ay_0] < \mathbb{E}(X_1 \mid X_0 = x_s)$ and there exists some constant $\tau_s > 0$ such that*

$$\mathbb{E}[\mathrm{e}^{\tau_s[(1+a)W_1 - X_1 + ay_0]} \mid X_0 = x_s] = 1, \tag{7}$$

then, with $\gamma = \min_{0 \le s < \infty}\{\tau_s\}$,

$$\mathbb{E}[\mathrm{e}^{\gamma[(1+a)W_1 - X_1 + ay_0]} \mid X_0 = x_s] \le 1, \quad s = 0, 1, 2, \ldots. \tag{8}$$

Proof. For all $s = 0, 1, 2, \ldots$, consider the following function

$$f_s(r) = \mathbb{E}[\mathrm{e}^{r[(1+a)W_1 - X_1 + ay_0]} \mid X_0 = x_s] - 1.$$

Clearly,

$$f_s''(r) = \mathbb{E}\{[(1+a)W_1 - X_1 + ay_0]^2 \mathrm{e}^{r[(1+a)W_1 - X_1 + ay_0]} \mid X_0 = x_s\} \ge 0,$$

which implies that $f_s(r)$ is a convex function. Notice that $f_s(0) = 0$ and

$$f_s'(0) = \mathbb{E}\left[(1+a)W_1 - X_1 + ay_0 \mid X_0 = x_s\right] < 0.$$

Thus, τ_s is the unique positive root of the equation $f_s(r) = 0$ on $(0, \infty)$. Furthermore, if $0 < \tau < \tau_s$, then $f_s(\tau) \le 0$. Therefore, for all $s = 0, 1, 2, \ldots$, $\gamma = \min_{0 \le t < \infty}\{\tau_t\} \le \tau_s$, which implies that $f_s(\gamma) \le 0$, i.e. (8) holds. □

We now derive a probability inequality for Ψ by an inductive approach.

Theorem 3.2. *Under the conditions of Proposition 3.1, for all $s = 0, 1, 2, \ldots$ and $u \ge 0$,*

$$\Psi(u, x_s, y_0, i_0) \le \beta \mathbb{E}\mathrm{e}^{\gamma(1+a)W_1} \mathbb{E}[\mathrm{e}^{-\gamma[(u+X_1)(1+I_1) - ay_0]} \mid X_0 = x_s] \tag{9}$$

with

$$\beta^{-1} = \inf_{t \ge 0} \frac{\int_t^\infty \mathrm{e}^{\gamma(1+a)w} \mathrm{d}F(w)}{\mathrm{e}^{\gamma(1+a)t}\overline{F}(t)}.$$

Proof. For the case $t \ge 0$, it is trivial that

$$\begin{aligned}\overline{F}(t) &= \left(\frac{\int_t^\infty \mathrm{e}^{\gamma(1+a)w} \mathrm{d}F(w)}{\mathrm{e}^{\gamma(1+a)t}\overline{F}(t)}\right)^{-1} \mathrm{e}^{-\gamma(1+a)t} \int_t^\infty \mathrm{e}^{\gamma(1+a)w} \mathrm{d}F(w) \\ &\le \beta \mathrm{e}^{-\gamma t} \int_t^\infty \mathrm{e}^{\gamma(1+a)w} \mathrm{d}F(w) \le \beta \mathrm{e}^{-\gamma t} \mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}.\end{aligned} \tag{10}$$

For the case $t < 0$, since

$$\left(\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\right)^{-1} = \frac{1}{\int_0^\infty \mathrm{e}^{\gamma(1+a)w}\mathrm{d}F(w)} \le \sup_{t\ge 0} \frac{\mathrm{e}^{\gamma(1+a)t}\overline{F}(t)}{\int_t^\infty \mathrm{e}^{\gamma(1+a)w}\mathrm{d}F(w)} = \beta,$$

we can get that

$$\begin{aligned}\overline{F}(t) = 1 &\le \left(\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\right)^{-1}\mathrm{e}^{-\gamma t}\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1} \\ &\le \beta\mathrm{e}^{-\gamma t}\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1} = \beta\mathrm{e}^{-\gamma t}\int_t^\infty \mathrm{e}^{\gamma(1+a)w}\mathrm{d}F(w). \qquad (11)\end{aligned}$$

Hence, from inequalities (10)-(11), we can derive that

$$\begin{aligned}\Psi_1(u, x_s, y_0, i_0) &= \mathbb{P}(W_1 > (u + X_1)(1 + I_1) - ay_0 \mid X_0 = x_s) \\ &= \sum_{t=0}^\infty p_{st}\int_0^\infty \overline{F}((u + x_t)(1 + bi_0 + z) - ay_0)\mathrm{d}G(z) \\ &\le \beta\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\sum_{t=0}^\infty p_{st}\int_0^\infty \mathrm{e}^{-\gamma[(u+x_t)(1+bi_0+z)-ay_0]}\mathrm{d}G(z) \\ &= \beta\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\mathbb{E}[\mathrm{e}^{-\gamma[(u+X_1)(1+I_1)-ay_0]} \mid X_0 = x_s].\end{aligned}$$

Assume that for all $u, y_0, i_0 \ge 0$ and $\min_{0\le s<\infty}\{x_s\} \ge 0$,

$$\begin{aligned}\Psi_n(u, x_s, y_0, i_0) &\le \beta\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\mathbb{E}[\mathrm{e}^{-\gamma[(u+X_1)(1+I_1)-ay_0]} \mid X_0 = x_s] \qquad (12) \\ &\le \beta\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\mathbb{E}[\mathrm{e}^{-\gamma[(u+X_1)(1+Z_1)-ay_0]} \mid X_0 = x_s]. \qquad (13)\end{aligned}$$

Recall the definitions of y, i and $\hbar_{t,z}$ in (6). For $0 \le w \le \hbar_{t,z}$, from (8), (13), $Z_1 \ge 0$ and $0 \le a < 1$, we can get that

$$\begin{aligned}&\Psi_n(\hbar_{t,z} - w, x_t, y, i) \\ &\le \beta\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\mathbb{E}[\mathrm{e}^{-\gamma[(\hbar_{t,z}-w+X_1)(1+Z_1)-a(ay_0+w)]} \mid X_0 = x_t] \\ &= \beta\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\mathbb{E}[\mathrm{e}^{-\gamma[X_1(1+Z_1)-a^2y_0]}\mathrm{e}^{-\gamma[(\hbar_{t,z}-w)(1+Z_1)-aw]} \mid X_0 = x_t] \\ &\le \beta\mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}\mathbb{E}[\mathrm{e}^{-\gamma(X_1-ay_0)} \mid X_0 = x_t]\mathrm{e}^{-\gamma[\hbar_{t,z}-(1+a)w]} \\ &\le \beta\mathrm{e}^{-\gamma[\hbar_{t,z}-(1+a)w]} = \beta\mathrm{e}^{-\gamma[(u+x_t)(1+bi_0+z)-ay_0-(1+a)w]}. \qquad (14)\end{aligned}$$

Thus, by Theorem 2.1, (10)-(11) and (14), we obtain

$$\begin{aligned}
&\Psi_{n+1}(u, x_s, y_0, i_0)\\
&\leq \beta \sum_{t=0}^{\infty} p_{st} \int_0^{\infty} \mathrm{e}^{-\gamma[(u+x_t)(1+bi_0+z)-ay_0]} \int_{\hbar_{t,z}}^{\infty} \mathrm{e}^{\gamma(1+a)w} \mathrm{d}F(w)\mathrm{d}G(z)\\
&\quad + \beta \sum_{t=0}^{\infty} p_{st} \int_0^{\infty} \mathrm{e}^{-\gamma[(u+x_t)(1+bi_0+z)-ay_0]} \int_0^{\hbar_{t,z}} \mathrm{e}^{\gamma(1+a)w} \mathrm{d}F(w)\mathrm{d}G(z)\\
&= \beta \sum_{t=0}^{\infty} p_{st} \int_0^{\infty} \mathrm{e}^{-\gamma[(u+x_t)(1+bi_0+z)-ay_0]} \int_0^{\infty} \mathrm{e}^{\gamma(1+a)w} \mathrm{d}F(w)\mathrm{d}G(z)\\
&= \beta \mathbb{E}\mathrm{e}^{\gamma(1+a)W_1} \mathbb{E}[\mathrm{e}^{-\gamma[(u+X_1)(1+I_1)-ay_0]} \mid X_0 = x_s].
\end{aligned}$$

Therefore, we can conclude that inequality (12) holds for all $n = 1, 2, \ldots$. The inequality (9) follows immediately by letting $n \to \infty$ in (12). □

The refinement of the upper bound in Theorem 3.2 can be obtained when F is new worse than used in convex ordering (NWUC). A lifetime distribution B is said to be NWUC if for all $x \geq 0$, $y \geq 0$,

$$\int_{x+y}^{\infty} \overline{B}(t)\mathrm{d}t \geq \overline{B}(x) \int_y^{\infty} \overline{B}(t)\mathrm{d}t.$$

The class of NWUC distributions is larger than the class of decreasing failure rate (DFR) distributions. See Shaked and Shanthikumar [5] for properties of NWUC and other classes of lifetime distributions.

Corollary 3.3. *Under the conditions of Theorem 3.2, if F is NWUC, then for all $u \geq 0$,*

$$\Psi(u, x_s, y_0, i_0) \leq \mathbb{E}[\mathrm{e}^{-\gamma[(u+X_1)(1+I_1)-ay_0]} \mid X_0 = x_s]. \tag{15}$$

Proof. From Proposition 6.1.1 of Willmot and Lin [8], we can get that if F is NWUC, then $\beta^{-1} = \mathbb{E}\mathrm{e}^{\gamma(1+a)W_1}$. Thus, by Theorem 3.2, we conclude the proof. □

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