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Rota Baxter operators of the simple 3-Lie algebra II

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Abstract

This paper investigates the existence of Rota-Baxter operators on the simple 3-Lie algebra over the complex field. It provides the classification of all Rota-Baxter operators of weight zero with the rank $1, 2, 3$, respectively. And it gives the concrete expression of every Rota-Baxter operators.

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Keywords: 3-Lie algebra, Rote-Baxter operator, Rote-Baxter 3-Lie algebra.

1 Introduction

In recent years, kinds of multiple algebras are studied [1-3]. For example, Rota Baxter 3-Lie algebra was introduced in paper [3], and the structure of Rota Baxter 3-Lie algebras is discussed. Rota-Baxter (associative) algebras, originated from the work of G. Baxter [4] in probability and populated by the work of Cartier and Rota have connections with many areas of mathematics and physics. The paper [5] proved that there does not exist Rota-Baxter operators with rank 3 of weight zero over the simple 3-Lie algebras.

In this paper we investigate the existence of Rota-Baxter operators of the weight zero with rank $1, 2, 3$ on the simple 3-Lie algebras over the complex field. First we introduce some basic notions.

Let A be a 3-Lie algebra, $\lambda \in F$, if a linear mapping $P : A \to A$ satisfies

$$[P(x_1), P(x_2), P(x_3)] = P([P(x_1), P(x_2), x_3] + [P(x_1), x_2, P(x_3)] + [x_1, P(x_2), P(x_3)]) \quad (1)$$

for any $x_1, x_2, x_3 \in A$, then P is called a Rota-Baxter operator of weight zero, and $(A, [,,], P)$ is called a Rota-Baxter 3-Lie algebra.

2 Classification of Rota-Baxter operators

In this section we study the Rota-Baxter operators on the simple 3-Lie algebras over the complex field F. From paper [5], there does not exist Rota-Baxter operators P with $R(P) = 3$ of weight zero on the simple 3-Lie algebra. In the following we will study the Rota-Baxter operators P of weight zero with $R(P) \neq 3$ on the simple 3-Lie algebra. Ling in [6] proved that there exists only one simple 3-Lie algebra, that is, the 4-dimensional 3-Lie algebra A in the following multiplication

$$[e_1, e_2, e_3] = e_4, [e_1, e_2, e_4] = e_3, [e_1, e_3, e_4] = e_2, [e_2, e_3, e_4] = e_1, \quad (2)$$

where e_1, e_2, e_3, e_4 is a basis of the 3-Lie algebra A.

Let $P : A \to A$ be a linear mapping. Set $P(e_i) = \sum_{j=1}^{4} a_{ij} e_j$, $a_{ij} \in F, 1 \leq i, j \leq 4$. Then the matrix form of P in the basis e_1, e_2, e_3, e_4 is

$$M(P) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}.$$

The rank of the matrix $M(P)$ is called the rank of P and is denoted by $R(P)$.

Theorem *Let A be the 4-dimensional 3-Lie algebra with the multiplication (2) in the basis e_1, e_2, e_3, e_4. And linear mapping P of A is a Rota-Baxter operator of weight zero. Then P is one and only one of the following possibilities:*

1. If $R(P) = 1$, for any $P(e_1) = \sum_{j=1}^{4} a_{1j} e_j \neq 0$, $P(e_2) = \lambda P(e_1), P(e_3) = \mu P(e_1), P(e_4) = \nu P(e_1)$, $\forall a_{11}, a_{12}, a_{13}, a_{14}, \lambda, \mu, \nu \in F$.

2. If $R(P) = 2$, $P(e_1) = \sum_{j=1}^{4} a_{1j} e_j$, $P(e_2) = \sum_{j=1}^{4} a_{2j} e_j$ are linearly independent, and

$$M(P)=\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ \sum\limits_{j=1}^{2}\lambda_j a_{j1} & \sum\limits_{j=1}^{2}\lambda_j a_{j2} & \sum\limits_{j=1}^{2}\lambda_j a_{j3} & \sum\limits_{j=1}^{2}\lambda_j a_{j4}\\ \sum\limits_{j=1}^{2}\mu_j a_{j1} & \sum\limits_{j=1}^{2}\mu_j a_{j2} & \sum\limits_{j=1}^{2}\mu_j a_{j3} & \sum\limits_{j=1}^{2}\mu_j a_{j4} \end{pmatrix}, \tag{3}$$

where $a_{i1}, a_{i2}, a_{i3}, a_{i4}, \lambda_i, \mu_i, \in F, i=1,2$ *and satisfy*

$$D_1=\begin{vmatrix} 1+\lambda_1^2-\mu_1^2 & -\mu_1 & \lambda_1\lambda_2-\mu_1\mu_2\\ a_{11}+\lambda_1+\mu_1 & a_{14} & a_{12}+\lambda_2+\mu_2\\ a_{21}+\lambda_1+\mu_1 & a_{24} & a_{22}+\lambda_2+\mu_2 \end{vmatrix}=0, \tag{4}$$

$$D_2=\begin{vmatrix} \mu_1\mu_2-\lambda_1\lambda_2 & \mu_2 & 1+\mu_2^2-\lambda_2^2\\ a_{11}+\lambda_1+\mu_1 & a_{14} & a_{12}+\lambda_2+\mu_2\\ a_{21}+\lambda_1+\mu_1 & a_{24} & a_{22}+\lambda_2+\mu_2 \end{vmatrix}=0, \tag{5}$$

$$D_3=\begin{vmatrix} 1+\lambda_1^2-\mu_1^2 & \lambda_1 & \lambda_1\lambda_2-\mu_1\mu_2\\ a_{11}+\lambda_1+\mu_1 & a_{13} & a_{12}+\lambda_2+\mu_2\\ a_{21}+\lambda_1+\mu_1 & a_{23} & a_{22}+\lambda_2+\mu_2 \end{vmatrix}=0, \tag{6}$$

$$D_4=\begin{vmatrix} \mu_1\mu_2-\lambda_1\lambda_2 & -\lambda_2 & 1+\mu_2^2-\lambda_2^2\\ a_{11}+\lambda_1+\mu_1 & a_{13} & a_{12}+\lambda_2+\mu_2\\ a_{21}+\lambda_1+\mu_1 & a_{23} & a_{22}+\lambda_2+\mu_2 \end{vmatrix}=0. \tag{7}$$

4. If $R(P)=4$, *then* $M(P)=\begin{pmatrix} 0 & a_{12} & a_{13} & a_{14}\\ a_{12} & 0 & a_{23} & a_{24}\\ -a_{13} & a_{23} & 0 & a_{34}\\ a_{14} & -a_{24} & a_{34} & 0 \end{pmatrix}$, (8)

where $a_{1i}, a_{23}, a_{24}, a_{34} \in F, i=2,3,4$ *and satisfy* $a_{12}a_{34}+\begin{vmatrix} a_{13} & a_{14}\\ a_{23} & a_{24}\end{vmatrix}\neq 0.$

Proof By Eq.(1) and (2), for $1\le l<m<n\le 4$, we have

$$\begin{aligned}
&[P(e_l),P(e_m),P(e_n)]\\
=&[\sum_{j=1}^{4}a_{lj}e_j,\sum_{j=1}^{4}a_{mj}e_j,\sum_{j=1}^{4}a_{nj}e_j]\\
=&(a_{l1}a_{m2}a_{n3}-a_{l1}a_{m3}a_{n2})e_4+(a_{l1}a_{m2}a_{n4}-a_{l1}a_{m4}a_{n2})e_3+(a_{l1}a_{m3}a_{n4}-a_{l1}a_{m4}a_{n3})e_2\\
&+(a_{l2}a_{m3}a_{n4}-a_{l2}a_{m4}a_{n3})e_1+(a_{l2}a_{m4}a_{n1}-a_{l2}a_{m1}a_{n4})e_3+(a_{l2}a_{m3}a_{n1}-a_{l2}a_{m1}a_{n3})e_4\\
&+(a_{l3}a_{m4}a_{n2}-a_{l3}a_{m2}a_{n4})e_1+(a_{l3}a_{m4}a_{n1}-a_{l3}a_{m1}a_{n4})e_2+(-a_{l3}a_{m2}a_{n1}+a_{l3}a_{m1}a_{n2})e_4\\
&+(a_{l4}a_{m2}a_{n3}-a_{l4}a_{m3}a_{n2})e_1+(a_{l4}a_{m1}a_{n3}-a_{l4}a_{m3}a_{n1})e_2+(-a_{l4}a_{m2}a_{n1}+a_{l4}a_{m1}a_{n2})e_3\\
=&[a_{l2}(a_{m3}a_{n4}-a_{m4}a_{n3})+a_{l3}(a_{m4}a_{n2}-a_{m2}a_{n4})+a_{l4}(a_{m2}a_{n3}-a_{m3}a_{n2})]e_1\\
&+[a_{l1}(a_{m3}a_{n4}-a_{m4}a_{n3})+a_{l3}(a_{m4}a_{n1}-a_{m1}a_{n4})+a_{l4}(a_{m1}a_{n3}-a_{m3}a_{n1})]e_2\\
&+[a_{l1}(a_{m2}a_{n4}-a_{m4}a_{n2})+a_{l2}(a_{m4}a_{n1}-a_{m1}a_{n4})+a_{l4}(a_{m1}a_{n2}-a_{m2}a_{n1})]e_3\\
&+[a_{l1}(a_{m2}a_{n3}-a_{m3}a_{n2})+a_{l2}(a_{m3}a_{n1}-a_{m1}a_{n3})+a_{l3}(a_{m1}a_{n2}-a_{m2}a_{n1})]e_4.
\end{aligned}$$

$$\begin{aligned}
&P([P(e_l),P(e_m),e_n]+[P(e_l),e_m,P(e_n)]+[e_l,P(e_m),P(e_n)])\\
=&P([\sum_{j=1}^{4}a_{lj}e_j,\sum_{j=1}^{4}a_{mj}e_j,e_n]+[\sum_{j=1}^{4}a_{lj}e_j,e_m,\sum_{j=1}^{4}a_{nj}e_j])\\
&+P([e_l,\sum_{j=1}^{4}a_{mj}e_j,\sum_{j=1}^{4}a_{nj}e_j])
\end{aligned}$$

$= P((a_{l4}a_{m2}-a_{l2}a_{m4}-a_{l3}a_{n4}+a_{l4}a_{n3})e_1+(a_{l4}a_{m1}-a_{l1}a_{m4}+a_{m3}a_{n4}-a_{m4}a_{n3})e_2$
$+(a_{l1}a_{n4} - a_{l4}a_{n1} + a_{m2}a_{n4} - a_{m4}a_{n2})e_3 + (a_{l1}a_{m2} - a_{l2}a_{m1} + a_{l1}a_{m3}$
$-a_{l3}a_{n1} + a_{m2}a_{n3}e_4 - a_{m3}a_{n2})e_4)$
$=(a_{l4}a_{m2} - a_{l2}a_{m4} - a_{l3}a_{n4} + a_{l4}a_{n3}) \sum_{j=1}^{4} a_{1j}e_j + (a_{l4}a_{m1} - a_{l1}a_{n4} + a_{m3}a_{n4}$
$-a_{m4}a_{n3}) \sum_{j=1}^{4} a_{2j}e_j + (a_{l1}a_{n4} - a_{l4}a_{n1} + a_{m2}a_{n4} - a_{m4}a_{n2}) \sum_{j=1}^{4} a_{3j}e_j$
$+(a_{l1}a_{m2} - a_{l2}a_{m1} + a_{l1}a_{n3} - a_{l3}a_{n1} + a_{m2}a_{n3}e_4 - a_{m3}a_{n2}) \sum_{j=1}^{4} a_{4j}e_j.$

For $l = 1, m = 2, n = 3$, comparing coefficients of basis vectors, we get

$$\begin{aligned}
&a_{12}(a_{23}a_{34} - a_{24}a_{33}) + a_{13}(a_{24}a_{32} - a_{22}a_{34}) + a_{14}(a_{22}a_{33} - a_{23}a_{32})\\
&= a_{11}(a_{14}a_{22} - a_{12}a_{24} - a_{13}a_{34} + a_{14}a_{33}) + a_{21}(a_{14}a_{21} - a_{11}a_{24}\\
&+a_{23}a_{34} - a_{24}a_{33}) + a_{31}(a_{11}a_{34} - a_{14}a_{31} + a_{22}a_{34} - a_{24}a_{32}) + a_{41}(a_{11}a_{22}\\
&-a_{12}a_{21} + a_{11}a_{33} - a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32}),
\end{aligned}$$

$$\begin{aligned}
&a_{11}(a_{23}a_{34} - a_{24}a_{33}) + a_{13}(a_{24}a_{31} - a_{21}a_{34}) + a_{14}(a_{21}a_{33} - a_{23}a_{31})\\
&= a_{12}(a_{14}a_{22}-a_{12}a_{24}-a_{13}a_{34}+a_{14}a_{33})+a_{22}(a_{14}a_{21}-a_{11}a_{24}+a_{23}a_{34}-a_{24}a_{33})+\\
&a_{32}(a_{11}a_{34} - a_{14}a_{31} + a_{22}a_{34} - a_{24}a_{32}) + a_{42}(a_{11}a_{22} - a_{12}a_{21}\\
&+a_{11}a_{33} - a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32}),
\end{aligned}$$

$$\begin{aligned}
&a_{11}(a_{22}a_{34} - a_{24}a_{32}) + a_{12}(a_{24}a_{31} - a_{21}a_{34}) + a_{14}(a_{21}a_{32} - a_{22}a_{31})\\
&= a_{13}(a_{14}a_{22}-a_{12}a_{24}-a_{13}a_{34}+a_{14}a_{33})+a_{23}(a_{14}a_{21}-a_{11}a_{24}+a_{23}a_{34}-a_{24}a_{33})+\\
&a_{33}(a_{11}a_{34} - a_{14}a_{31} + a_{22}a_{34} - a_{24}a_{32}) + a_{43}(a_{11}a_{22} - a_{12}a_{21}\\
&+a_{11}a_{33} - a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32}),
\end{aligned}$$

$$\begin{aligned}
&a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})\\
&= a_{14}(a_{14}a_{22}-a_{12}a_{24}-a_{13}a_{34}+a_{14}a_{33})+a_{24}(a_{14}a_{21}-a_{11}a_{24}+a_{23}a_{34}-a_{24}a_{33})+\\
&a_{34}(a_{11}a_{34} - a_{14}a_{31} + a_{22}a_{34} - a_{24}a_{32}) + a_{44}(a_{11}a_{22} - a_{12}a_{21}\\
&+a_{11}a_{33} - a_{13}a_{31} + a_{22}a_{33} - a_{23}a_{32}).
\end{aligned}$$

For $l = 1, m = 2, n = 4$, we get

$$\begin{aligned}
&a_{12}(a_{23}a_{44} - a_{24}a_{43}) + a_{13}(a_{24}a_{42} - a_{22}a_{44}) + a_{14}(a_{22}a_{43} - a_{23}a_{42})\\
&= a_{11}(a_{12}a_{23}-a_{13}a_{22}-a_{13}a_{44}+a_{14}a_{43})+a_{21}(a_{11}a_{23}-a_{13}a_{21}+a_{23}a_{44}-a_{24}a_{43})+\\
&a_{31}(a_{11}a_{22} - a_{12}a_{21} + a_{11}a_{44} - a_{14}a_{41} + a_{22}a_{44} - a_{24}a_{42})\\
&+a_{41}(a_{11}a_{43} - a_{13}a_{41} + a_{22}a_{43} - a_{23}a_{42}),
\end{aligned}$$

$$\begin{aligned}
&a_{11}(a_{23}a_{44} - a_{24}a_{43}) + a_{13}(a_{24}a_{41} - a_{21}a_{44}) + a_{14}(a_{21}a_{43} - a_{23}a_{41})\\
&= a_{12}(a_{12}a_{23}-a_{13}a_{22}-a_{13}a_{44}+a_{14}a_{43})+a_{22}(a_{11}a_{23}-a_{13}a_{21}+a_{23}a_{44}-a_{24}a_{43})+\\
&a_{32}(a_{11}a_{22} - a_{12}a_{21} + a_{11}a_{44} - a_{14}a_{41} + a_{22}a_{44} - a_{24}a_{42})\\
&+a_{42}(a_{11}a_{43} - a_{13}a_{41} + a_{22}a_{43} - a_{23}a_{42}),
\end{aligned}$$

$$\begin{aligned}
&a_{11}(a_{22}a_{44} - a_{24}a_{42}) + a_{12}(a_{24}a_{41} - a_{21}a_{44}) + a_{14}(a_{21}a_{42} - a_{22}a_{41})\\
&= a_{13}(a_{12}a_{23}-a_{13}a_{22}-a_{13}a_{44}+a_{14}a_{43})+a_{23}(a_{11}a_{23}-a_{13}a_{21}+a_{23}a_{44}-a_{24}a_{43})+\\
&a_{33}(a_{11}a_{22} - a_{12}a_{21} + a_{11}a_{44} - a_{14}a_{41} + a_{22}a_{44} - a_{24}a_{42})\\
&+a_{43}(a_{11}a_{43} - a_{13}a_{41} + a_{22}a_{43} - a_{23}a_{42}).
\end{aligned}$$

$a_{11}(a_{22}a_{43}-a_{23}a_{42})+a_{12}(a_{23}a_{41}-a_{21}a_{43})+a_{13}(a_{21}a_{42}-a_{22}a_{41})$
$=a_{14}(a_{12}a_{23}-a_{13}a_{22}-a_{13}a_{44}+a_{14}a_{43})+a_{24}(a_{11}a_{23}-a_{13}a_{21}+a_{23}a_{44}-a_{24}a_{43})+$
$a_{34}(a_{11}a_{22}-a_{12}a_{21}+a_{11}a_{44}-a_{14}a_{41}+a_{22}a_{44}-a_{24}a_{42})$
$+a_{44}(a_{11}a_{43}-a_{13}a_{41}+a_{22}a_{43}-a_{23}a_{42})$.
For $l=1,m=3,n=4$, we get
$a_{12}(a_{33}a_{44}-a_{34}a_{43})+a_{13}(a_{34}a_{42}-a_{32}a_{44})+a_{14}(a_{32}a_{43}-a_{33}a_{42})$
$=a_{11}(a_{12}a_{33}-a_{13}a_{32}-a_{14}a_{42}+a_{12}a_{44})+a_{21}(a_{11}a_{33}-a_{13}a_{31}+a_{11}a_{44}$
$-a_{14}a_{41}+a_{33}a_{44}-a_{34}a_{43})+a_{31}(a_{11}a_{32}-a_{12}a_{31}+a_{32}a_{44}$
$-a_{34}a_{42})+a_{41}(a_{12}a_{41}-a_{11}a_{42}+a_{32}a_{43}-a_{33}a_{42})$,

$a_{11}(a_{33}a_{44}-a_{34}a_{43})+a_{13}(a_{34}a_{41}-a_{31}a_{44})+a_{14}(a_{31}a_{43}-a_{33}a_{41})$
$=a_{12}(a_{12}a_{33}-a_{13}a_{32}-a_{14}a_{42}+a_{12}a_{44})+a_{22}(a_{11}a_{33}-a_{13}a_{31}$
$+a_{11}a_{44}-a_{14}a_{41}+a_{33}a_{44}-a_{34}a_{43})+a_{32}(a_{11}a_{32}-a_{12}a_{31}+a_{32}a_{44}$
$-a_{34}a_{42})+a_{42}(a_{12}a_{41}-a_{11}a_{42}+a_{32}a_{43}-a_{33}a_{42})$,

$a_{11}(a_{32}a_{44}-a_{34}a_{42})+a_{12}(a_{34}a_{41}-a_{31}a_{44})+a_{14}(a_{31}a_{42}-a_{32}a_{41})$
$=a_{13}(a_{12}a_{33}-a_{13}a_{32}-a_{14}a_{42}+a_{12}a_{44})+a_{23}(a_{11}a_{33}-a_{13}a_{31}+a_{11}a_{44}$
$-a_{14}a_{41}+a_{33}a_{44}-a_{34}a_{43})+a_{33}(a_{11}a_{32}-a_{12}a_{31}+a_{32}a_{44}-a_{34}a_{42})$
$+a_{43}(a_{12}a_{41}-a_{11}a_{42}+a_{32}a_{43}-a_{33}a_{42})$,

$a_{11}(a_{32}a_{43}-a_{33}a_{42})+a_{12}(a_{33}a_{41}-a_{31}a_{43})+a_{13}(a_{31}a_{42}-a_{32}a_{41})$
$=a_{14}(a_{12}a_{33}-a_{13}a_{32}-a_{14}a_{42}+a_{12}a_{44})+a_{24}(a_{11}a_{33}-a_{13}a_{31}$
$+a_{11}a_{44}-a_{14}a_{41}+a_{33}a_{44}$
$-a_{34}a_{43})+a_{34}(a_{11}a_{32}-a_{12}a_{31}+a_{32}a_{44}-a_{34}a_{42})$
$+a_{44}(a_{12}a_{41}-a_{11}a_{42}+a_{32}a_{43}-a_{33}a_{42})$.

For $l=2,m=3,n=4$, we get
$a_{22}(a_{33}a_{44}-a_{34}a_{43})+a_{23}(a_{34}a_{42}-a_{32}a_{44})+a_{24}(a_{32}a_{43}-a_{33}a_{42})$
$=a_{11}(a_{22}a_{33}-a_{23}a_{32}+a_{22}a_{44}-a_{24}a_{42}+a_{33}+a_{44}-a_{34}a_{43})+a_{21}(a_{21}a_{33}$
$-a_{23}a_{31}+a_{21}a_{44}-a_{24}a_{41})+a_{31}(a_{21}a_{32}-a_{22}a_{31}+a_{34}a_{41}-a_{31}a_{44})$
$+a_{41}(a_{22}a_{41}-a_{21}a_{42}+a_{33}a_{41}-a_{31}a_{43})$,

$a_{21}(a_{33}a_{44}-a_{34}a_{43})+a_{23}(a_{34}a_{41}-a_{31}a_{44})+a_{24}(a_{31}a_{43}-a_{33}a_{41})$
$=a_{12}(a_{22}a_{33}-a_{23}a_{32}+a_{22}a_{44}-a_{24}a_{42}+a_{33}+a_{44}-a_{34}a_{43})+a_{22}(a_{21}a_{33}$
$-a_{23}a_{31}+a_{21}a_{44}-a_{24}a_{41})+a_{32}(a_{21}a_{32}-a_{22}a_{31}+a_{34}a_{41}-a_{31}a_{44})$
$+a_{42}(a_{22}a_{41}-a_{21}a_{42}+a_{33}a_{41}-a_{31}a_{43})$,

$a_{21}(a_{32}a_{44}-a_{34}a_{42})+a_{22}(a_{34}a_{41}-a_{31}a_{44})+a_{24}(a_{31}a_{42}-a_{32}a_{41})$
$=a_{13}(a_{22}a_{33}-a_{23}a_{32}+a_{22}a_{44}-a_{24}a_{42}+a_{33}+a_{44}-a_{34}a_{43})+a_{23}(a_{21}a_{33}$
$-a_{23}a_{31}+a_{21}a_{44}-a_{24}a_{41})+a_{33}(a_{21}a_{32}-a_{22}a_{31}+a_{34}a_{41}-a_{31}a_{44})+a_{43}(a_{22}a_{41}$
$-a_{21}a_{42}+a_{33}a_{41}-a_{31}a_{43})$,

$a_{21}(a_{32}a_{43}-a_{33}a_{42})+a_{22}(a_{33}a_{41}-a_{31}a_{43})+a_{23}(a_{31}a_{42}-a_{32}a_{41})$
$=a_{14}(a_{22}a_{33}-a_{23}a_{32}+a_{22}a_{44}-a_{24}a_{42}+a_{33}+a_{44}-a_{34}a_{43})+a_{24}(a_{21}a_{33}$
$-a_{23}a_{31}+a_{21}a_{44}-a_{24}a_{41})+a_{34}(a_{21}a_{32}-a_{22}a_{31}+a_{34}a_{41}-a_{31}a_{44})$
$+a_{44}(a_{22}a_{41}-a_{21}a_{42}+a_{33}a_{41}-a_{31}a_{43})$.

If $R(P) = 1$, it is easy to see, that for any linear mapping P satisfying $P(e_1) \neq 0$, $P(e_2) = \lambda P(e_1)$, $P(e_3) = \mu P(e_1)$, $P(e_4) = \nu P(e_1)$ for $\forall \lambda, \mu, \nu \in F$, is a Rota-Baxter operator.

If $R(P) = 2$, without loss of generality suppose $P(e_1), P(e_2)$ are linearly independent. Then $P(e_3) = \lambda_1 P(e_1) + \lambda_2 P(e_2)$, $P(e_4) = \mu_1 P(e_1) + \mu_2 P(e_2)$, $\lambda_1, \lambda_2, \mu_1, \mu_2 \in F$.

Denotes $e_3' = e_3 - \lambda_1 e_1 - \lambda_2 e_2$, $e_4' = e_4 - \mu_1 e_1 - \mu_2 e_2$, then e_1, e_2, e_3', e_4' is a basis of A, and $P(e_3') = P(e_4') = 0$. From $R(P) = 2$, Eq.(2) and the direct computation, we obtain that P is a Rota-Baxter operator on A with $R(P) = 2$ if and only if P satisfies Eqs.(3)-(7).

If $R(P) = 4$, then $M(P)$ is non degenerate. By Eqs.(1) and (2), and the direct computation we obtain Eq.(8). The result follows.

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