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# Relation between S-Metric And $\mathcal{M}$-Fuzzy Metric Spaces

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**Abstract**

In this work we have considered several common fixed point results in S-metric spaces for weak compatible mappings. By applications of these results we establish some fixed point theorems in fuzzy S-metric spaces.

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## 1 Introduction

In this paper we establish some fixed point results in a fuzzy S-metric space by applications of certain fixed point theorems in S-metric spaces. Also we prove some fixed point results in S-metric spaces. Fuzzy metric space was first introduced by Kramosil and Michalek [13]. Subsequently, George and Veeramani had given a modified definition of fuzzy metric spaces [5] . Fixed point results in such spaces have been established in a large number of works. Some of these works are noted in [6, 17, 15, 23, 24, 25].

**Definition 1.1** [5] *A binary operation* $* : [0,1] \times [0,1] \longrightarrow [0,1]$ *is a continuous t-norm if it satisfies the following conditions:*

*(1)* $*$ *is associative and commutative,*
*(2)* $*$ *is continuous,*
*(3)* $a * 1 = a$ *for all* $a \in [0,1]$,
*(4)* $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$, *for each* $a, b, c, d \in [0,1]$.

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$.

Here we have considered definition of fuzzy metric space (non-Archimedean).

**Definition 1.2** [16] *A 3-tuple* $(X, M, *)$ *is called a fuzzy metric space if* $X$ *is an arbitrary (non-empty) set,* $*$ *is a continuous* $t-$*norm and* $M$ *is a fuzzy set on* $X^2 \times (0, \infty)$ *satisfying the following conditions for each* $x, y, z \in X$ *and* $t, s > 0$*:*

*(1)* $M(x, y, t) > 0$,
*(2)* $M(x, y, t) = 1$ *if and only if* $x = y$,
*(3)* $M(x, y, t) = M(y, x, t)$,
*(4)* $M(x, z, t) * M(z, y, s) \leq M(x, y, t \vee s)$, *where* $t \vee s = max\{s, t\}$,
*(5)* $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ *is continuous.*

All fuzzy metric in this paper are assumed to be non-Archimedean.

In 1976, Jungck [8] introduced the notion of commuting mappings to find common fixed point results in metric spaces. Later on, in [9] Jungck proposed the notion of compatible mappings which is a generalization of the concept of commuting mapping. Some common fixed point theorems for compatible mappings and their generalizations are addressed in [10, 11, 14, 26]. In this paper we consider weak compatible mappings.

**Definition 1.3** [18] *Let* $A$ *and* $S$ *be mappings from a metric space* $X$ *into itself. Then the mappings are said to be weak compatible if they commute at a coincidence point, that is,* $Ax = Sx$ *implies that* $ASx = SAx$.

# 2 Preliminary Notes

First we recall some notions, lemmas, and examples which will be useful later.

**Definition 2.1** *[21] Let $X$ be a nonempty set. A function $S : X^3 \to [0,\infty)$ is said to be an $S$-metric on $X$, if for each $x, y, z, a \in X$,*

1. $S(x, y, z) \geq 0$,
2. $S(x, y, z) = 0$ *if and only if* $x = y = z$,
3. $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

The pair $(X, S)$ is called an *$S$-metric space.*

**Example 2.2** *[21] We can easily check that the following examples are $S$-metric spaces.*

1. *Let $X = R^n$ and $||\cdot||$ a norm on $X$. Then $S(x, y, z) = ||y + z - 2x|| + ||y - z||$ is an $S$-metric on $X$.*

   *In general, if $X$ is a vector space over $R$ and $||\cdot||$ a norm on $X$. Then it is easy to see that*

   $$S(x, y, z) = ||\alpha y + \beta z - \lambda x|| + ||y - z||,$$

   *where $\alpha + \beta = \lambda$ for every $\alpha, \beta \geq 1$, is an $S$-metric on $X$.*

2. *Let $X$ be a nonempty set and $d_1$, $d_2$ be two ordinary metrics on $X$. Then*

   $$S(x, y, z) = d_1(x, z) + d_2(y, z),$$

   *is an $S$-metric on $X$.*

**Lemma 2.3** *[19] Let $(X, S)$ be an $S$-metric space. Then, we have $S(x, x, y) = S(y, y, x)$, $x, y \in X$.*

For more detail of $S$-metric see the reference [20].

**Definition 2.4** *[20] Let $(X, S)$ be an $S$-metric space and $A \subset X$.*

1. *A sequence $\{x_n\}$ in $X$ converges to $x$ if $S(x_n, x_n, x) \to 0$ as $n \to \infty$, that is for every $\varepsilon > 0$ there exists $n_0 \in N$ such that for $n \geq n_0$, $S(x_n, x_n, x) < \varepsilon$. This case, we denote by $\lim_{n\to\infty} x_n = x$ and we say that $x$ is the limit of $\{x_n\}$ in $X$.*
2. *A sequence $\{x_n\}$ in $X$ is said to be Cauchy sequence if for each $\varepsilon > 0$, there exists $n_0 \in N$ such that $S(x_n, x_n, x_m) < \varepsilon$ for each $n, m \geq n_0$.*
3. *The $S$-metric space $(X, S)$ is said to be complete if every Cauchy sequence is convergent.*

**Lemma 2.5** *[20] Let $(X, S)$ be an $S$- metric space. If there exist sequences $\{x_n\}$ and $\{y_n\}$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then*

$$\lim_{n\to\infty} S(x_n, x_n, y_n) = S(x, x, y).$$

# 3 Main Results

Next we establish the following result in S-metric spaces.

**Theorem 3.1** *Let $A, B, C$ and $T$ be self maps on a complete S-metric space $(X, S)$ satisfying:*

(i) *$A(X) \subseteq T(X), B(X) \subseteq C(X)$ and $T(X)$ or $C(X)$ is a closed subset of $X$;*

(ii) *there exist positive real numbers $a, b, c, e$ such that $a + b + c + 3e < 1$ and for each $x, y, z \in X$,*

$$\begin{aligned} S(Ax, Ay, Bz) &\leq aS(Cx, Cx, Tz) + bS(Cx, Cx, Ax) + cS(Tz, Tz, Bz) \\ &+ e(S(Cx, Cx, Bz) + S(Tz, Tz, Ay)); \end{aligned}$$

(iii) *the pairs $(A, C)$ and $(B, T)$ are weakly compatible.*

*Then $A, B, C$ and $T$ have a unique common fixed point in $X$.*

**Proof 1** *Let $x_0$ be an arbitrary point in $X$. By (i), we can choose a point $x_1$ in $X$ such that $y_0 = Ax_0 = Tx_1$ and $y_1 = Bx_1 = Cx_2$. In general, there exists a sequence $\{y_n\}$ such that, $y_{2n} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Cx_{2n+2}$, for $n = 0, 1, 2, \cdots$. We claim that the sequence $\{y_n\}$ is a Cauchy sequence.*

*By (ii), we have,*

$$\begin{aligned} S(y_{2n}, y_{2n}, y_{2n+1}) &= S(Ax_{2n}, Ax_{2n}, Bx_{2n+1}) \\ &\leq aS(Cx_{2n}, Cx_{2n}, Tx_{2n+1}) + bS(Cx_{2n}, Cx_{2n}, Ax_{2n}) \\ &\quad +cS(Tx_{2n+1}, Tx_{2n+1}, Bx_{2n+1}) \\ &\quad +e(S(Cx_{2n}, Cx_{2n}, Bx_{2n+1}) + S(Tx_{2n+1}, Tx_{2n+1}, Ax_{2n})) \\ &= aS(y_{2n-1}, y_{2n-1}, y_{2n}) + bS(y_{2n-1}, y_{2n-1}, y_{2n}) \\ &\quad +cS(y_{2n}, y_{2n}, y_{2n+1}) + e(S(y_{2n-1}, y_{2n-1}, y_{2n+1}) + S(y_{2n}, y_{2n}, y_{2n})). \end{aligned}$$

*If we put $d_n = S(y_n, y_n, y_{n+1})$, then by above inequality we have,*

$$\begin{aligned} d_{2n} &\leq ad_{2n-1} + bd_{2n-1} + cd_{2n} + e(S(y_{2n-1}, y_{2n-1}, y_{2n+1}) + 0) \\ &\leq ad_{2n-1} + bd_{2n-1} + cd_{2n} + e(2S(y_{2n-1}, y_{2n-1}, y_{2n}) + S(y_{2n}, y_{2n}, y_{2n+1})). \end{aligned}$$

*Hence,*

$$d_{2n} \leq ad_{2n-1} + bd_{2n-1} + cd_{2n} + 2ed_{2n-1} + ed_{2n}. \tag{1}$$

*Hence we have,*

$$\begin{aligned} d_{2n} &\leq \frac{a + b + 2e}{1 - c - e} d_{2n-1} \\ &= td_{2n-1}, \end{aligned}$$

*where* $0 < t = \frac{a+b+2e}{1-c-e} < 1$.

*Similarly, it follows that*

$$\begin{aligned} d_{2n+1} &\leq \frac{a+c+2e}{1-e-b} d_{2n} \\ &= t' d_{2n}, \end{aligned}$$

*where* $0 < t' = \frac{a+c+2e}{1-e-b} < 1$. *If we set* $k = \max\{t, t'\} < 1$, *then for every* $n \in \mathbf{N}$ *by above inequalities we get* $d_n \leq k d_{n-1}$.

*Hence,*

$$d_n \leq k d_{n-1} \leq k^2 d_{n-2} \leq \cdots \leq k^n d_0. \tag{2}$$

*That is,*

$$S(y_n, y_n, y_{n+1}) \leq k^n S(y_0, y_0, y_1) \tag{3}$$

*If* $m \geq n$, *then*

$$\begin{aligned} S(y_n, y_n, y_m) &\leq 2S(y_n, y_n, y_{n+1}) + 2S(y_{n+1}, y_{n+1}, y_{n+2}) + \cdots + 2S(y_{m-1,m-1}, y_m) \\ &\leq 2k^n S(y_0, y_0, y_1) + 2k^{n+1} S(y_0, y_0, y_1) \cdots + 2k^{m-1} S(y_0, y_0, y_1) \\ &\leq \frac{2k^n}{1-k} S(y_0, y_0, y_1) \to 0 \end{aligned}$$

*as* $n \to \infty$. *It follows that, the sequence* $\{y_n\}$ *is Cauchy sequence and by the completeness of* $X$, $\{y_n\}$ *converges to* $y \in X$. *Then*

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Cx_{2n+2} = \lim_{n\to\infty} Tx_{2n+1} = y. \tag{4}$$

*Let* $T(X)$ *be a closed subset of* $X$, *then there exists* $v \in X$ *such that* $Tv = y$.

*We now prove that* $Bv = y$. *By (ii), we get*

$$\begin{aligned} \lim_{n\to\infty} S(Ax_{2n}, Ax_{2n}, Bv) &\leq \lim_{n\to\infty} [aS(Cx_{2n}, Cx_{2n}, Tv) + bS(Ax_{2n}, Ax_{2n}, Cx_{2n}) \\ &\quad + cS(Bv, Bv, Tv) + e(S(Bv, Bv, Cx_{2n}) + S(Ax_{2n}, Ax_{2n}, Tv))] \end{aligned}$$

*and so*

$$\begin{aligned} S(y, y, Bv) &\leq aS(y, y, Tv) + bS(y, y, y) + cS(Bv, Bv, y) + e(S(Bv, Bv, y) + S(y, y, Tv)) \\ &< S(y, y, Bv). \end{aligned}$$

*It follows that* $Bv = y = Tv$. *Since* $B$ *and* $T$ *are two weakly compatible mappings, we have* $BTv = TBv$ *and so* $By = Ty$.

*Next, we prove that* $By = y$. *By (ii), we get*

$$\begin{aligned} \lim_{n\to\infty} S(Ax_{2n}, Ax_{2n}, By) &\leq \lim_{n\to\infty} [aS(Cx_{2n}, Cx_{2n}, Ty) + bS(Ax_{2n}, Ax_{2n}, Cx_{2n}) \\ &\quad + cS(By, By, Ty) + e(S(By, By, Cx_{2n}) + S(Ax_{2n}, Ax_{2n}, Ty))]. \end{aligned}$$

*Hence,*

$$\begin{aligned} S(y,y,By) &\leq aS(y,y,Ty)+bS(y,y,y)+cS(By,By,Ty)+e(S(By,By,y)+S(y,y,Ty)) \\ &< S(y,y,By) \end{aligned}$$

*and so* $By = y$.

*Since* $B(X) \subseteq C(X)$, *there exists* $w \in X$ *such that* $Cw = y$. *We prove that* $Aw = y$. *By (ii) we have*

$$\begin{aligned} S(Aw,Aw,By) &\leq aS(Cw,Cw,Ty)+bS(Aw,Aw,Cw)+cS(By,By,Ty) \\ &\quad +e(S(By,By,Cw)+S(Aw,Aw,Ty)) \end{aligned}$$

*and it follows that*

$$\begin{aligned} S(Aw,Aw,y) &\leq aS(y,y,y)+bS(Aw,Aw,y)+cS(y,y,Ty) \\ &+ e(S(y,y,y)+S(Aw,Aw,Ty)) \\ &< S(Aw,Aw,y). \end{aligned}$$

*This implies that* $Aw = y$ *and hence* $Aw = Cw = y$. *Since* $A$ *and* $C$ *are weakly compatible, then* $ACw = CAw$ *and so* $Ay = Cy$.

*Now, we prove that* $Ay = y$. *From (ii), we have*

$$\begin{aligned} S(Ay,Ay,By) &\leq aS(Cy,Cy,Ty)+bS(Ay,Ay,Cy)+cS(By,By,Ty) \\ &\quad +e(S(By,By,Cy)+S(Ay,Ay,Ty)) \end{aligned}$$

*it follows that*

$$\begin{aligned} S(Ay,Ay,y) &\leq aS(Cy,Cy,y)+bS(Ay,Ay,Cy)+cS(y,y,y) \\ &+ e(S(y,y,Cy))+S(Ay,Ay,y)) \\ &< S(Ay,Ay,y) \end{aligned}$$

*and hence* $Ay = y$ *and therefore* $Ay = Cy = By = Ty = y$. *That is* $y$ *is a common fixed point for* $A, B, C, T$.

*The proof is similar when* $C(X)$ *is assumed to be a closed subset of* $X$.

*Now to prove the uniqueness. Assume that* $x$ *is another common fixed point of* $A, B, C$ *and* $T$. *Then*

$$\begin{aligned} S(x,x,y) &= S(Ax,Ax,By) \\ &\leq aS(Cx,Cx,Ty)+bS(Ax,Ax,Cx)+cS(By,By,Ty) \\ &+ e(S(Cx,Cx,By)+S(Ax,Ax,Ty)) \end{aligned}$$

*and so*

$$\begin{aligned} S(x,x,y) &\leq aS(x,x,y)+bS(x,x,x)+cS(y,y,y)+e(S(x,x,y)+S(x,x,y)) \\ &< S(x,x,y). \end{aligned}$$

*Thus it follows that* $x = y$.

**Corollary 3.2** *Let $A$ and $B$ be self maps on a complete S-metric space $(X, S)$ satisfying: there exist positive real numbers $a, b, c, e$ such that $a + b + c + 3e < 1$ and for each $x, y, z \in X$,*

$$S(Ax, Ay, Bz) \leq aS(x, x, z)+bS(x, x, Ax)+cS(z, z, Bz)+e(S(x, x, Bz)+S(z, z, Ay));$$

*Then $A$ and $B$ have a unique common fixed point in $X$.*

**Proof 2** *It is enough set $T = C = I$, identity map in Theorem 3.1.*

Here we introduce $\mathcal{M}$-fuzzy metric. We describe the space along with some associated concepts in the following.

**Definition 3.3** *A 3-tuple $(X, \mathcal{M}, *)$ is called a $\mathcal{M}$-fuzzy metric space if $X$ is an arbitrary (non-empty) set, $*$ is a continuous $t-$norm and $\mathcal{M}$ is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions for each $x, y, z, a \in X$ and $t, s, r > 0$:*

*(1) $\mathcal{M}(x, y, z, t) > 0$,*

*(2) $\mathcal{M}(x, y, z, t) = 1$ if and only if $x = y = z$,*

*(3) $\mathcal{M}(x, y, z, \vee\{t, s, r\} \geq \mathcal{M}(x, x, a, t) * \mathcal{M}(y, y, a, s) * \mathcal{M}(z, z, a, r)$ where $\vee\{t, s, r\}) = max\{t, s, r\}$,*

*(4) $\mathcal{M}(x, y, z, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.*

**Example 3.4** *Let $a * b = ab$ for all $a, b \in [0, 1]$, we define*

$$\mathcal{M}(x, y, z, t) = \exp^{-}\frac{S(x, y, z)}{t}, \tag{5}$$

*where $S$ is an S-metric on set X. Then $(X, \mathcal{M}, *)$ is a $\mathcal{M}$-fuzzy metric space.*

**Proof 3** *(i) $\mathcal{M}(x, y, z, t) > 0$ for all $x, y, z \in X$ and $t > 0$ is trivial.*

$$\begin{aligned}(ii)\ \mathcal{M}(x, y, z, t) = 1 &\iff S(x, y, z) = 0\\ &\iff x = y = z.\end{aligned}$$

$$(iii)\quad Since\ S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a),$$

*hence,*

$$\begin{aligned}\frac{S(x, y, z)}{t \vee s \vee r} &\leq \frac{S(x, x, a) + S(y, y, a) + S(z, z, a)}{t \vee s \vee r}\\ &\leq \frac{S(x, x, a)}{t} + \frac{S(y, y, a)}{s} + \frac{S(z, z, a)}{r}.\end{aligned}$$

*Thus*

$$
\begin{aligned}
(iii)\ \ \exp^{-}\frac{S(x,y,z)}{t\vee s\vee r} &\geq \exp^{-\{\frac{S(x,x,a)}{t}+\frac{S(y,y,a)}{s}+\frac{S(z,z,a)}{r}\}}\\
&= \exp^{-}\frac{S(x,y,z)}{t}.\exp^{-}\frac{S(x,x,a)}{s}.\exp^{-}\frac{S(z,z,a)}{r},
\end{aligned}
$$

*it follows that,*

$$
\begin{aligned}
(iii)\mathcal{M}(x,y,z,t\vee s\vee r) &\geq \mathcal{M}(x,x,a,t).\mathcal{M}(y,y,a,s).\mathcal{M}(z,z,a,r)\\
&= \mathcal{M}(x,x,a,t)*\mathcal{M}(y,y,a,s)*\mathcal{M}(z,z,a,r).
\end{aligned}
$$

$(X,\mathcal{M},*)$ *is a* $\mathcal{M}$*-fuzzy metric space.*

A sequence $\{x_n\}$ in $X$ converges to $x$ if and only if $\mathcal{M}(x_n,x_n,x,t)\to 1$ as $n\to\infty$, for each $t>0$. It is called a Cauchy sequence if for each $0<\varepsilon<1$ and $t>0$, there exits $n_0\in N$ such that $\mathcal{M}(x_n,x_n,x_m,t)>1-\varepsilon$ for each $n,m\geq n_0$. The $\mathcal{M}$-fuzzy metric space $(X,\mathcal{M},*)$ is said to be complete if every Cauchy sequence is convergent.

The following properties of $\mathcal{M}$ noted in the theorem below are easy consequences of the definition.

**Lemma 3.5** *Let* $(X,\mathcal{M},*)$ *be a* $\mathcal{M}$*-fuzzy metric space. Then*

*(1)* $\mathcal{M}(x,x,y,t)=\mathcal{M}(y,y,x,t)$.
*(2)* $\mathcal{M}(x,x,y,t)$ *is nondecreasing with respect to* $t$ *for each* $x,y\in X$.

**Proof 4** *(i) For every* $t\in(0,\infty)$*, we have*

$$
\begin{aligned}
\mathcal{M}(x,x,y,t)=\mathcal{M}(x,x,y,t\vee t\vee t) &\geq \mathcal{M}(x,x,x,t)*\mathcal{M}(x,x,x,t)*\mathcal{M}(y,y,x,t)\\
&= \mathcal{M}(y,y,x,t).
\end{aligned}
$$

*Similarly, we can show that* $\mathcal{M}(y,y,x,t)\geq\mathcal{M}(x,x,y,t)$*. That is* $\mathcal{M}(x,x,y,t)=\mathcal{M}(y,y,x,t)$.

*(ii) For every* $t,s\in(0,\infty)$*, let* $t\geq s$*. Then*

$$
\begin{aligned}
\mathcal{M}(x,x,y,t)=\mathcal{M}(x,x,y,t\vee s\vee s) &\geq \mathcal{M}(x,x,x,t)*\mathcal{M}(x,x,x,s)*\mathcal{M}(y,y,x,s)\\
&= \mathcal{M}(y,y,x,s)=\mathcal{M}(x,x,y,s).
\end{aligned}
$$

**Example 3.6** *Let* $a*b=ab$ *for all* $a,b\in[0,1]$ *and* $M_1$ *and* $M_2$ *be two fuzzy set on* $X\times X\times(0,+\infty)$ *defined by*

$$
\mathcal{M}(x,y,z,t)=M_1(x,z,t)*M_2(y,z,t), \tag{6}
$$

*for all* $x,y,z\in X$*. Then* $(X,\mathcal{M},*)$ *is a* $\mathcal{M}$*-fuzzy metric space.*

**Proof 5** *(i) $\mathcal{M}(x,y,z,t) > 0$ for all $x,y,z \in X$ and $t > 0$ is trivial.*

$$\begin{aligned}(ii)\ \mathcal{M}(x,y,z,t) = 1 &\iff M_1(x,z,t) = M_2(y,z,t) = 1\\ &\iff x = y = z.\end{aligned}$$

*(iii) Let $t \geq s \geq r$, it follows that,*

$$\begin{aligned}&\mathcal{M}(x,y,z,t \vee s \vee r)\\ =\ &\mathcal{M}(x,y,z,t)\\ =\ &M_1(x,z,t) * M_2(y,z,t)\\ \geq\ &M_1(x,a,t) * M_1(a,z,t) * M_2(y,a,t) * M_2(a,z,t)\\ \geq\ &M_1(x,a,t) * M_2(x,a,t) * M_1(y,a,t) * M_2(y,a,t) * M_1(z,a,t) * M_2(z,a,t)\\ =\ &\mathcal{M}(x,x,a,t) * \mathcal{M}(y,y,a,t) * \mathcal{M}(z,z,a,t)\\ \geq\ &\mathcal{M}(x,x,a,t) * \mathcal{M}(y,y,a,s) * \mathcal{M}(z,z,a,r),\end{aligned}$$

*$(X, \mathcal{M}, *)$ is a $\mathcal{M}$-fuzzy metric space.*

**Lemma 3.7** *Let $(X, \mathcal{M}, *)$ be a $\mathcal{M}$-fuzzy metric space. If sequence $\{x_n\}$ in $X$ converges to $x$, then $x$ is unique.*

**Proof 6** *Let $\{x_n\}$ converges to $x$ and $y$, then for each $0 < \varepsilon < 1$ there exist $n_1, n_2 \in N$ such that*

$$\forall\ n \geq n_1 \Longrightarrow \mathcal{M}(x_n, x_n, x, t) > 1 - \varepsilon, \tag{7}$$

*and*

$$\forall\ n \geq n_2 \Longrightarrow \mathcal{M}(x_n, x_n, y, t) > 1 - \varepsilon. \tag{8}$$

*If set $n_0 = \max\{n_1, n_2\}$, then for every $n \geq n_0$ we have:*

$$\begin{aligned}\mathcal{M}(x,x,y,t) &\geq \mathcal{M}(x,x,x_n,t) * \mathcal{M}(x,x,x_n,t) * \mathcal{M}(y,y,x_n,t)\\ &> (1-\varepsilon) * (1-\varepsilon) * (1-\varepsilon)\end{aligned}$$

*By taking the limit when $\varepsilon \to 0$ in above inequality we get $\mathcal{M}(x,x,y,t) \geq 1$. Hence $\mathcal{M}(x,x,y,t) = 1$ so $x = y$.*

**Lemma 3.8** *Let $(X, \mathcal{M}, *)$ be a $\mathcal{M}$-fuzzy metric space. Then the convergent sequence $\{x_n\}$ in $X$ is Cauchy.*

**Proof 7** *Since $\lim_{n\to\infty} x_n = x$ then for each $0 < \varepsilon < 1$ there exists $n_1, n_2 \in N$ such that*

$$n \geq n_1 \Rightarrow \mathcal{M}(x_n, x_n, x, t) > 1 - \varepsilon, \tag{9}$$

*and*

$$m \geq n_2 \Rightarrow \mathcal{M}(x_m, x_m, x, t) > 1 - \varepsilon. \tag{10}$$

*If set* $n_0 = \max\{n_1, n_2\}$, *then for every* $n, m \geq n_0$ *we have:*

$$\begin{aligned} \mathcal{M}(x_n, x_n, x_m, t) &\geq \mathcal{M}(x_n, x_n, x, t) * \mathcal{M}(x_n, x_n, x, t) * \mathcal{M}(x_m, x_m, x, t) \\ &> (1 - \varepsilon) * (1 - \varepsilon) * (1 - \varepsilon). \end{aligned}$$

*By taking the limit when* $\varepsilon \to 0$ *in above inequality we get* $\mathcal{M}(x_n, x_n, x_m, t) \geq 1$. *Hence* $\{x_n\}$ *is a Cauchy sequence.*

**Lemma 3.9** *Let* $(X, \mathcal{M}, *)$ *be a* $\mathcal{M}$*-fuzzy metric space. If there exist sequences* $\{x_n\}$ *and* $\{y_n\}$ *such that* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} y_n = y$, *then*

$$\lim_{n\to\infty} \mathcal{M}(x_n, x_n, y_n, t) = \mathcal{M}(x, x, y, t). \tag{11}$$

**Proof 8** *Since* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} y_n = y$, *then for each* $0 < \varepsilon < 1$ *there exist* $n_1, n_2 \in N$ *such that*

$$\forall\ n \geq n_1 \Rightarrow \mathcal{M}(x_n, x_n, x, t) > 1 - \varepsilon, \tag{12}$$

*and*

$$\forall\ n \geq n_2 \Rightarrow \mathcal{M}(y_n, y_n, y, t) > 1 - \varepsilon. \tag{13}$$

*If set* $n_0 = \max\{n_1, n_2\}$, *then for every* $n \geq n_0$ *we have:*

$$\begin{aligned} &\mathcal{M}(x_n, x_n, y_n, t) \\ \geq\ &\mathcal{M}(x_n, x_n, x, t) * \mathcal{M}(x_n, x_n, x, t) * \mathcal{M}(y_n, y_n, x, t) \\ \geq\ &\mathcal{M}(x_n, x_n, x, t) * \mathcal{M}(x_n, x_n, x, t) * \mathcal{M}(y_n, y_n, y, t) * \mathcal{M}(y_n, y_n, y, t) * \mathcal{M}(x, x, y, t) \\ >\ &(1 - \varepsilon) * (1 - \varepsilon) * (1 - \varepsilon) * (1 - \varepsilon) * \mathcal{M}(x, x, y, t). \end{aligned}$$

*By taking the limit when* $\varepsilon \to 0$ *in above inequality we get*

$$\mathcal{M}(x_n, x_n, y_n, t) \geq \mathcal{M}(x, x, y, t). \tag{14}$$

*On the other hand, we have*

$$\begin{aligned} &\mathcal{M}(x, x, y, t) \\ \geq\ &\mathcal{M}(x, x, x_n, t) * \mathcal{M}(x, x, x_n, t) * \mathcal{M}(y, y, x_n, t) \\ \geq\ &\mathcal{M}(x, x, x_n, t) * \mathcal{M}(x, x, x_n, t) * \mathcal{M}(y, y, y_n, t) * \mathcal{M}(y, y, y_n, t) * \mathcal{M}(x_n, x_n, y_n, t) \\ >\ &(1 - \varepsilon) * (1 - \varepsilon) * (1 - \varepsilon) * (1 - \varepsilon) * \mathcal{M}(x_n, x_n, y_n, t), \end{aligned}$$

*as* $\varepsilon \to 0$ *we have*

$$\mathcal{M}(x, x, y, t) > \mathcal{M}(x_n, x_n, y_n, t). \tag{15}$$

*Therefore by relations (14) and (15) we have*

$$\lim_{n\to\infty} \mathcal{M}(x_n, x_n, y_n, t) = \mathcal{M}(x_n, x_n, y_n, t). \tag{16}$$

**Lemma 3.10** *Let $(X, \mathcal{M}, *)$ be a $\mathcal{M}$-fuzzy metric space with $a * b \geq ab$ for all $a, b \in [0, 1]$.If define $S : X^3 \longrightarrow [0, \infty)$ by $S(x, y, z) = \int_0^1 \log_\alpha(\mathcal{M}(x, y, z, t))dt$, then $S$ is an S-metric on $X$ for $0 < \alpha < 1$.*

**Proof 9** *It is clear from the definition that $S(x, y, z)$ is well defined for each $x, y, z \in X$. (i) $S(x, y, z) \geq 0$ for all $x, y, z \in X$ is trivial.*

$$\begin{aligned} (ii)\ S(x,y,z) = 0 &\iff \log_\alpha(\mathcal{M}(x,y,z,t)) = 0 \text{ for all } t > 0 \\ &\iff \mathcal{M}(x,y,z,t) = 1 \text{ for all } t > 0 \iff x = y = z. \end{aligned}$$

$$\begin{aligned} (iv)\text{Since } \mathcal{M}(x,y,z,t) &\geq M(x,x,a,t) * \mathcal{M}(y,y,a,t) * \mathcal{M}(z,z,a,t) \\ &\geq \mathcal{M}(x,x,a,t).\mathcal{M}(y,y,a,t).\mathcal{M}(z,z,a,t) \end{aligned}$$

*it follows that,*

$$\begin{aligned} & S(x,y,z) \\ = & \int_0^1 \log_\alpha(\mathcal{M}(x,y,z,t))dt \\ \leq & \int_0^1 \log_\alpha(\mathcal{M}(x,x,a,t).\mathcal{M}(y,y,a,t).\mathcal{M}(z,z,a,t))dt \\ \leq & \int_0^1 \log_\alpha(\mathcal{M}(x,x,a,t))dt + \int_0^1 \log_\alpha(\mathcal{M}(y,y,a,t))dt + \int_0^1 \log_\alpha(\mathcal{M}(z,z,a,t))dt \\ = & S(x,x,a) + S(y,y,a) + S(z,z,a) \end{aligned}$$

*This proves that $S$ is an S-metric on X.*

The following lemma plays an important role to give fixed point results on a $\mathcal{M}$-fuzzy metric space.

**Lemma 3.11** *Let $(X, \mathcal{M}, *)$ be a $\mathcal{M}$-fuzzy metric space.*

*(a) $\{x_n\}$ is a Cauchy sequence in $(X, \mathcal{M}, *)$ if and only if it is a Cauchy sequence in the S-metric space $(X, S)$.*

*(b) A $\mathcal{M}$-fuzzy metric space $(X, \mathcal{M}, *)$ is complete if and only if the S-metric space $(X, S)$ is complete.*

**Proof 10** *First we show that every Cauchy sequence in $(X, \mathcal{M}, *)$ is a Cauchy sequence in $(X, S)$. To this end let $\{x_n\}$ be a Cauchy sequence in $(X, \mathcal{M}, *)$. Then $\lim_{n,m\to\infty} \mathcal{M}(x_n, x_n, x_m, t) = 1$. Since*

$$S(x_n, x_n, x_m) = \int_0^1 \log_\alpha(\mathcal{M}(x_n, x_n, x_m, t))dt,$$

*is an $S$-metric. Hence, we have*

$$\begin{aligned}&\lim_{n,m\to\infty} S(x_n, x_n, x_m)\\ = &\lim_{n,m\to\infty} \int_0^1 \log_\alpha(\mathcal{M}(x_n, x_n, x_m, t))dt = 0.\end{aligned}$$

*We conclude that $\{x_n\}$ is a Cauchy sequence in $(X, S)$. Next we prove that completeness of $(X, S)$ implies completeness of $(X, \mathcal{M}, *)$. Indeed, if $\{x_n\}$ is a Cauchy sequence in $(X, \mathcal{M}, *)$ then it is also a Cauchy sequence in $(X, S)$. Since the $S$-metric space $(X, S)$ is complete we deduce that there exists $y \in X$ such that $\lim_{n\to\infty} S(x_n, x_n, y) = 0$. Therefore,*

$$\int_0^1 \log_\alpha(\lim_{n\to\infty} \mathcal{M}(x_n, x_n, y, t))dt = \lim_{n\to\infty} S(x_n, x_n, y) = 0.$$

*Hence we follow that $\{x_n\}$ is a convergent sequence in $(X, \mathcal{M}, *)$.*

*Now we prove that every Cauchy sequence $\{x_n\}$ in $(X, S)$ is a Cauchy sequence in $(X, \mathcal{M}, *)$. Since $\{x_n\}$ is a Cauchy sequence in $(X, S)$, then*

$$\lim_{n,m\to\infty} S(x_n, x_n, x_m) = \lim_{n,m\to\infty} \int_0^1 \log_\alpha(\mathcal{M}(x_n, x_n, x_m, t))dt = 0.$$

*Hence, $\lim_{n,m\to\infty} \mathcal{M}(x_n, x_n, x_m, t) = 1$.*

*That is, $\{x_n\}$ is a Cauchy sequence in $(X, \mathcal{M}, *)$.*

*We shall have established the lemma if we prove that $(X, S)$ is complete if so is $(X, \mathcal{M}, *)$. Let $\{x_n\}$ be a Cauchy sequence in $(X, S)$. Then $\{x_n\}$ is a Cauchy sequence in $(X, \mathcal{M}, *)$, and so it is convergent to a point $y \in X$ with*

$$\lim_{n,m\to\infty} \mathcal{M}(x_n, x_m, y, t) = 1.$$

*As a consequence we have*

$$\lim_{n,m\to\infty} S(x_n, x_m, y) = \lim_{n,m\to\infty} \int_0^1 \log_\alpha(\mathcal{M}(x_n, x_m, y, t))dt = 0.$$

*Therefore $(X, S)$ is complete.*

**Lemma 3.12** *Let $(X, \mathcal{M}, *)$ be a $\mathcal{M}$-fuzzy metric space with $a*b = \min\{a, b\}$ for all $a, b \in [0, 1]$. We define $S : X^3 \longrightarrow [0, \infty)$ by $S(x, y, z) = \int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(x, y, z, t))dt$, then $S$ is an $S$-metric on $X$.*

**Proof 11** *(i) $S(x, y, z) \geq 0$ is trivial.*

$$\begin{aligned}(ii)\qquad S(x, y, z) = 0 &\iff \cot(\frac{\pi}{2}\mathcal{M}(x, y, z, t)) = 0 \text{ for all } t > 0\\ &\iff \mathcal{M}(x, y, z, t) = 1 \text{ for all } t > 0 \iff x = y = z.\end{aligned}$$

*(iii) Since* $$\begin{aligned} \mathcal{M}(x,y,z,t) &\geq \mathcal{M}(x,x,a,t) * \mathcal{M}(y,y,a,t) * \mathcal{M}(z,z,a,t) \\ &= \min\{\mathcal{M}(x,x,a,t), \mathcal{M}(y,y,a,t), \mathcal{M}(z,z,a,t)\}. \end{aligned}$$

*and also since* $0 < \frac{\pi}{2}\mathcal{M}(x,y,z,t) \leq \frac{\pi}{2}$ *it follows that,*

$$\begin{aligned} & S(x,y,z) \\ = & \int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(x,y,z,t))dt \\ \leq & \int_0^1 \cot[\frac{\pi}{2}(\mathcal{M}(x,x,a,t) * \mathcal{M}(y,y,a,t), \mathcal{M}(z,z,a,t))]dt \\ = & \int_0^1 \cot(\frac{\pi}{2}\min\{\mathcal{M}(x,x,a,t), \mathcal{M}(y,y,a,t), \mathcal{M}(z,z,a,t)\})dt \\ = & \min\{\int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(x,x,a,t))dt, \int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(y,y,a,t))dt, \int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(z,z,a,t))dt\} \\ \leq & \int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(x,x,a,t))dt + \int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(y,y,a,t))dt + \int_0^1 \cot(\frac{\pi}{2}\mathcal{M}(z,z,a,t))dt \\ = & S(x,x,a) + S(y,y,a) + S(z,z,a), \end{aligned}$$

*that is $S$ is an S-metric on $X$.*

**Remark 3.13** *Let $a, b \in (0,1]$, then it is a standard result that*

$$Arccot(\min\{a,b\}) \leq Arccot(a) + Arccot(b) - \frac{\pi}{4}$$

**Lemma 3.14** *Let $(X, \mathcal{M}, *)$ be a $\mathcal{M}$-fuzzy metric space with $a*b = \min\{a,b\}$ for all $a, b \in [0,1]$. We define $S : X^3 \longrightarrow [0,\infty)$ by $S(x,y,z) = \int_0^1 (\frac{4}{\pi}Arccot(\mathcal{M}(x,y,z,t)) - 1)dt$, then $S$ is an S-metric on $X$.*

**Proof 12** *(i)* $0 \leq S(x,y,z) < 1$ *is trivial.*

*(ii)* $$\begin{aligned} S(x,y,z) = 0 &\iff \frac{4}{\pi}Arccot(\mathcal{M}(x,y,z,t)) - 1 = 0 \text{ for all } t > 0 \\ &\iff Arccot(\mathcal{M}(x,y,z,t)) = \frac{\pi}{4} \text{ for all } t > 0. \\ &\iff \mathcal{M}(x,y,z,t) = 1 \text{ for all } t > 0 \iff x = y = z. \end{aligned}$$

*(iii) Since*

$$\begin{aligned} \mathcal{M}(x,y,z,t) &\geq \mathcal{M}(x,x,a,t) * \mathcal{M}(y,y,a,t) * \mathcal{M}(z,z,a,t) \\ &= \min\{\mathcal{M}(x,x,a,t), \mathcal{M}(y,y,a,t), \mathcal{M}(z,z,a,t)\}, \end{aligned}$$

*it follows that,*

$$
\begin{aligned}
& Arccot(\mathcal{M}(x,y,z,t)) \\
\leq\ & Arccot[\mathcal{M}(x,y,z,t) * \mathcal{M}(y,y,a,t) * \mathcal{M}(z,z,a,t)] \\
=\ & Arccot(\min\{\mathcal{M}(x,x,a,t), \mathcal{M}(y,y,a,t), \mathcal{M}(z,z,a,t)\}) \\
\leq\ & Arccot(\mathcal{M}(x,x,a,t)) + Arccot(\mathcal{M}(y,y,a,t)) + Arccot(\mathcal{M}(z,z,a,t)) - \frac{\pi}{2}
\end{aligned}
$$

*Hence,*

$$
\begin{aligned}
S(x,y,z) &= \int_0^1 (\frac{4}{\pi} Arccot(\mathcal{M}(x,y,z,t)) - 1)dt \\
&\leq \int_0^1 (\frac{4}{\pi} Arccot(\mathcal{M}(x,x,a,t)) - 1)dt + \int_0^1 (\frac{4}{\pi} Arccot(\mathcal{M}(y,y,a,t)) - 1)dt \\
&+ \int_0^1 (\frac{4}{\pi} Arccot(\mathcal{M}(z,z,a,t)) - 1)dt \\
&= S(x,x,a) + S(y,y,a) + S(z,z,a),
\end{aligned}
$$

*that is $S$ is an $S$-metric on $X$.*

We now apply the theorem 3.1 to prove the following fixed point result in $\mathcal{M}$-fuzzy metric spaces.

**Theorem 3.15** *Let $(X, \mathcal{M}, *)$ be a complete $\mathcal{M}$-fuzzy metric space with $d * f \geq df$ for all $d, f \in [0,1]$. Let $A, B, C$ and $T$ be self maps on $X$ satisfying:*

(i) *$A(X) \subseteq T(X), B(X) \subseteq C(X)$ and $T(X)$ or $C(X)$ is a closed subset of $X$;*

(ii) *there exists positive real numbers $a, b, c, e$ such that $a + b + c + 3e < 1$ and for each $x, y, z \in X$,*

$$
\mathcal{M}(Ax,Ay,Bz,t) \geq \begin{array}{l} \mathcal{M}^a(Cx,Cx,Tz,t) * \mathcal{M}^b(Cx,Cx,Ax,t) \\ *\mathcal{M}^c(Tz,Tz,Bz,t) * [\mathcal{M}(Cx,Cx,Bz,t) * \mathcal{M}(Tz,Tz,Ay,t)]^e \end{array};
$$

(iii) *the pairs $(A, C)$ and $(B, T)$ are weakly compatible.*

*Then $A, B, C$ and $T$ have a unique common fixed point in $X$.*

**Proof 13** *From inequality $(ii)$ above, we get,*

$$
\begin{aligned}
& \int_0^1 \log_\alpha^{\mathcal{M}(Ax,Ay,Bz,t)} dt \\
\leq\ & \int_0^1 \log_\alpha^{\left( \begin{array}{l} \mathcal{M}^a(Cx,Cx,Tz,t) * \mathcal{M}^b(Cx,Cx,Ax,t) \\ *\mathcal{M}^c(Tz,Tz,Bz,t) * [\mathcal{M}(Cx,Cx,Bz,t) * \mathcal{M}(Tz,Tz,Ay,t)]^e \end{array} \right)} dt
\end{aligned}
$$

$$\begin{aligned}
&\leq \int_0^1 \log_\alpha \begin{pmatrix} \mathcal{M}^a(Cx,Cx,Tz,t)\mathcal{M}^b(Cx,Cx,Ax,t) \\ \mathcal{M}^c(Tz,Tz,Bz,t)[\mathcal{M}^e(Cx,Cx,Bz,t)\mathcal{M}^e(Tz,Tz,Ay,t)] \end{pmatrix} dt \\
&= \begin{array}{l} a\int_0^1 \log_\alpha^{\mathcal{M}(Cx,Cx,Tz,t)} dt + b\int_0^1 \log_\alpha^{\mathcal{M}(Cx,Cx,Ax,t)} dt \\ +c\int_0^1 \log_\alpha^{\mathcal{M}(Tz,Tz,Bz,t)} dt + e(\int_0^1 \log_\alpha^{\mathcal{M}(Cx,Cx,Bz,t)} dt + \int_0^1 \log_\alpha^{\mathcal{M}(Tz,Tz,Ay,t)} dt) \end{array}.
\end{aligned}$$

*If set $S(x,y,z) = \int_0^1 \log_\alpha^{\mathcal{M}(x,y,z,t)} dt$ for every $x,y,z \in X$ and $0 < \alpha < 1$. Then it follows that,*

$$\begin{aligned}
S(Ax,Ay,Bz) \leq\ & aS(Cx,Cx,Tz) + bS(Cx,Cx,Ax) + cS(Tz,Tz,Bz) \\
& +e(S(Cx,Cx,Bz) + S(Tz,Tz,Ay)).
\end{aligned}$$

*Hence by Lemma 3.14 all of conditions Theorem 3.1 hold. Thus $A,B,C$ and $T$ have a unique common fixed point in $X$.*

**Corollary 3.16** *Let $(X,\mathcal{M},*)$ be a complete $\mathcal{M}$-fuzzy metric space with $d*f \geq df$ for all $d,f \in [0,1]$. Let $A$ and $B$ be self maps on $X$ satisfying: there exists positive real numbers $a,b,c,e$ such that $a+b+c+3e < 1$ and for each $x,y,z \in X$,*

$$\mathcal{M}(Ax,Ay,Bz,t) \geq \begin{array}{l} \mathcal{M}^a(x,x,z,t) * \mathcal{M}^b(x,x,Ax,t) \\ *\mathcal{M}^c(z,z,Bz,t) * [\mathcal{M}(x,x,Bz,t) * \mathcal{M}(z,z,Ay,t)]^e \end{array},$$

*Then $A$ and $B$ have a unique common fixed point in $X$.*

**Proof 14** *It is enough set $T = C = I$, identity map in Theorem 3.15.*

# References


[1] R. P. Agarwal, M. A. El-Gebeily, D. O'regan, *Generalized contractions in partially ordered metric spaces*, Appl. Anal. 87 (1)(2008) 109-116.

[2] Lj. Ćirić, D. Mihet, R. Saadati, *Monotone generalized contractions in partiality ordered probabilistic metric spaces*, Topology and its Applications (2009) doi: 10. 1016/j. na. 2009. 08. 029.

[3] M. Demmaa, R. Saadati, P. Vetro, Fixed Point Results on b-Metric Space via Picard Sequences and b-Simulation Functions, Iranian Journal of Mathematical Sciences and Informatics Vol. 11, No. 1 (2016), pp 123-136.

[4] Fang JX. On fixed point theorems in fuzzy metric spaces. Fuzzy Sets Sys 46(1992), 107-113.

[5] George A, Veeramani P. On some result in fuzzy metric space. Fuzzy Sets Syst 64(1994), 395–399.

[6] Gregori V, Sapena A. On fixed-point theorem in fuzzy metric spaces. Fuzzy Sets and Sys 125(2002), 245–252.

[7] Hadgic O. Common fixed point theorems for families of mapping in complete metric space, Math. Japon.29 (1984); 127-134.

[8] Jungck G. Commuting mappings and fixed points, Amer Math. Monthly., 83, (1976), 261-263.

[9] Jungck G. Compatible mappings and common fixed points, Internat,J, Math. Math. Sci., 9, (1986), 771-779.

[10] Jungck G. Common fixed points of commuting and compatible maps on compacta, Proc.Amer. Math. Soc., 103(1988), 977-983.

[11] Kang S.M., Cho Y.J. and Jungck G. Common fixed point of compatible mappings, Internet. J. Math. Math. Sci., 13(1990), 61-66.

[12] Jong Kyu Kim, Shaban Sedghi, A. Gholidahneh, M. Mahdi Rezaee, Fixed point theorems in $S$-metric spaces, East Asian Math. J., Vol. 32 (2016), No. 5, pp. 677-684.

[13] Kramosil I, Michalek J. Fuzzy metric and statistical metric spaces. Kybernetica 11(1975), 326–34.

[14] Mishra S.N. Common fixed points of compatible mappings in PM-spaces, Math. Japonica.36(2)(1991); 283-289.

[15] Miheţ D. A Banach contraction theorem in fuzzy metric spaces. Fuzzy Sets Sys 144(2004), 431–439.

[16] Miheţ D. Fuzzy $\psi$-contractive mappimgs in non-Archimedean fuzzy metric spaces. Fuzzy Sets and Systems 159(2008)739-744.

[17] Rodríguez López J, Ramaguera S. The Hausdorff fuzzy metric on compact sets. Fuzzy Sets Sys 147(2004), 273–283.

[18] Rhoades B.E. Common fixed point theorems for nonself Quasi-Contraction Mappings Varohmihir Journal of Mathematical Sciences Vol. 2 No.I (2002), 11-14

[19] S. Sedghi, I. Altun, N. Shobe, M. A. Salahshour, Some Properties of S-metric Spaces and Fixed Point Results, Kyungpook Math. J. 54(2014), 113-122 http://dx.doi.org/10.5666/KMJ. 2014. 54. 1. 113.

[20] S. Sedghi, NV. Dung: Fixed point theorems on S-metric spaces. Mat. Vensnik 66, 113-124 (2014).

[21] S. Sedghi, N. Shobe, A. Aliouche, A generalization of fixed point theorems in S-metric spaces, Mat. Vanik 64 (2012), 258-266.

[22] Shaban Sedghi, Nabi Shobe and Tatjana Dosenovic, Fixed point results in $S$-metric spaces, Nonlinear Functional Analysis And Applications, Vol. 20, No. 1 (2015), pp. 55-67.

[23] Sedghi S., Choudhury B. S. and Shobe N., Unique common fixed point theorem for four weakly compatible mappings in complete fuzzy metric spaces, J. Fuzzy. Math., (To appear).

[24] Vasuki R. Common fixed points for $R$-weakly commuting maps in fuzzy metric spaces. Indian J Pure Appl Math. 30(1999), 419–423.

[25] Vasuki R, Veeramani P. Fixed point theorems and Cauchy sequences in fuzzy metric spaces. Fuzzy Sets Sys 135(2003), 409–413.

[26] Xieping D. Common fixed point theorem of commuting mappings in PM-spaces, Kexue Tongbao 29(1984), 147-150.

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