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Propagation of Rayleigh waves in non-homogeneous elastic half-space of orthotropic material under initial compression and influence of gravity

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Abstract

The aim of the present paper is to investigate the propagation of surface waves in a non-homogeneous, orthotropic elastic solid medium under the influence of initial compression and gravity. The wave velocity equation has been obtained. When the gravity field and non-homogeneity of the material medium vanishes, the medium is isotropic and unstressed, the frequency equation is in complete agreement with the corresponding classical results.

Mathematics Subject Classification: 74B10

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1. Introduction

The earth has a layered structure, and this exerts a significant influence on the propagation of elastic waves. The simplest cases of influence exerted on the propagation of seismic waves by a single plane boundary which separates two half-spaces with different properties, and by two parallel plane boundaries forming a layer. Earth is being treated as an elastic body in which three types of waves can occur. 1. Dilatational and equivoluminal waves in the interior of the earth. 2. In the neighborhood of its surface known as Rayleigh waves (1885). 3. Third type of waves occurs near the surface of contact of two layers of the earth known as love waves (1911). The Rayleigh waves are observed far from the disturbance source near the surface. Since the energy carried by these waves is concentrated over the surface, its dissipation is slower than the Dilatational and equivoluminal waves where the energy is dissipated over the volume of the disturbed region. Therefore, during earth quakes for an observer remote from the source of disturbance, the Rayleigh waves represent the greatest danger. In the case of Love waves, the energy is concentrated near the interface; hence they are dissipated more slowly.

The effect of gravity on wave propagation in an elastic solid medium was first considered by [4] Bromwich, treating the force of gravity as a type of body force. [13] Love extended the work of Bromwich investigated the influence of gravity on superfacial waves and showed that the Rayleigh wave velocity is affected by the gravity field. [17] Sezawa studied the dispersion of elastic waves propagated on curved surfaces.

[11] Haskell studied the dispersion of surface waves in multilayered media. A source on elastic waves is the monograph of [9] Ewing, Jardtezky and Press.

[3] Biot studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible. Taking into account, the effect of initial stresses and using Biot's theory of incremental deformations, [12] Jones, [8] De and Sengupta studied many problems of elastic waves and vibrations under the influence of gravity field. [16] Sengupta PR and Acharya D studied the influence of gravity on the propagation of waves in a thermoelastic layer. [5] Brunelle studied the surface wave propagation under initial tension of compression. [19, 20] Willson W and Yu CP and Tang S studied the propagation of megnato-thermo, elastic plane waves. Wave propagation in a thin two-layered laminated medium with stress couples under initial stresses was studied by [15] Roy. [7] Datta studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium.[10] Goda studied the effect of inhomogeneity and anisotropy on Stoneley waves. [1, 2, 14] Abd-Alla, El-Naggar and Ahmed studied the Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. Recently Rayleigh waves in rotating thermo-elastic solids with voids were studied by [18] Sharma JN and Kaur D.[6] A. Chattopadhyay, S. Gupta, V.K. Sharma and P. Kumari, studied the effect of point source and heterogeneity on the propagation of SH- waves.

In the present paper, the influence of gravity and initial stress on the propagation of Rayleigh type waves in a non-homogeneous, orthotropic elastic solid medium has been discussed. The Dispersion equation so obtained is in well agreement with the corresponding classical results.

2. Formulation of the problem

Let us consider an orthotropic, non-homogeneous elastic solid under an initial compression P along x-direction further it is also under the influence of gravity. Here we consider Oxyz cartesian coordinates system where O be any point on the plane boundary and Oz be normal to the medium and Rayleigh wave propagation is taken in the +ve direction of x-axis. It is also assumed that at a great distance from center of disturbance, the wave propagation is two dimensional and is polarised in (x, z) plane. So displacement components along x and z directions.i.e. u and w are non-zero while v = 0.
Also it is assumed that wave is surface wave as the disturbance is extensively confined to the boundary. let g be the acceleration due to gravity and ρ be the density of the material medium.
Here states of initial stresses are given by

$$\left.\begin{aligned} \sigma_{ij} &= \sigma \; ; \; i = j \\ &= 0 \; ; \; i \neq j \end{aligned}\right\} \text{, where i, j = 1, 2, 3} \tag{1}$$

Further σ is a function of z
$\therefore$ equation of equilibrium of initial compression are

$$\frac{\partial \sigma}{\partial x} = 0 = \frac{\partial \sigma}{\partial y}, \tag{2}$$

$$\frac{\partial \sigma}{\partial z} - \rho g = 0.$$

3. Solution of problem

Considering eq (1) and eq (2) and conditions for compressibility, the dynamical equations in three dimensions of an elastic medium under initial compression and gravity are given by

$$\sigma_{11,x} + \sigma_{12,y} + \sigma_{13,z} + P(w_{z,y} - w_{y,z}) - \rho g\, u_{3,x} = \rho u_{,tt}, \tag{3}$$

$$\sigma_{12,x} + \sigma_{22,y} + \sigma_{23,z} - P w_{z,x} = \rho v_{,tt}, \tag{4}$$

$$\sigma_{13,x} + \sigma_{23,y} + \sigma_{33,z} - P w_{y,x} + \rho g\, u_{1,x} = \rho w_{,tt}, \tag{5}$$

where u, v, w are displacement components in x, y and z direction and w_x, w_y, w_z are rotational components and are given by

$$w_x = \frac{1}{2}(w_{,y} - v_{,z}) \; ; \; w_y = \frac{1}{2}(u_{,z} - w_{,x})$$

$$w_z = \frac{1}{2}(v_{,x} - u_{,y}). \tag{6}$$

Further dynamical eqs in (x, z) directions are given by

$$\sigma_{11,x} + \sigma_{13,z} - P w_{y,z} - \rho g\, u_{3,x} = \rho u_{,tt},$$

$$\sigma_{13,x} + \sigma_{33,z} - P w_{y,x} + \rho g\, u_{1,x} = \rho w_{,tt}, \tag{7}$$

where stress components are given by

$$\sigma_{11} = (C_{11} + P)\, u_{1,x} + (C_{13} + P)\, u_{3,z},$$

$$\sigma_{33} = C_{31}\, u_{1,x} + C_{33}\, u_{3,z},$$

$$\sigma_{13} = C_{44}\,(u_1{,}_z + u_3{,}_x), \tag{8}$$

where C_{ij} are elastic constants.

Let us take the assumption that $C_{44} = \frac{1}{2}(C_{11} - C_{13})$.

Substituting equation (6) and equation (8) in equation (7) ; we have

$$(C_{11} + P)\,(2u_1,_{xx} + u_1,_{zz} + u_3,_{xz}) + C_{13}\,(u_3,_{xz} - u_1,_{zz}) + (u_1,_z + u_3,_x)(C_{11} - C_{13}),_z + 2u_1,_x\,(C_{11} + P),_x + 2u_3,_z\,(C_{13} + P),_x - 2\rho g\,u_3,_x = 2\rho u_1,_{tt}, \tag{9}$$

$$C_{11}\,(u_1,_{xz} + u_3,_{xx}) + (C_{13} + P)\,(u_1,_{xz} - u_3,_{xx}) + 2\,C_{33}\,u_3,_{zz} + 2\rho g\,u_1,_x + (u_1,_z + u_3,_x)\,(C_{11} - C_{13}),_x + u_1,_x\,C_{13},_z + u_3,_z\,C_{33},_z = 2\rho u_3,_{tt}\,. \tag{10}$$

Now we assume the non-homogeneity for the elastic half space, density and compression are given by

$$C_{ij} = \alpha_{ij}\,e^{mz},\ \rho = \rho_0\,e^{mz},\ P = P_0\,e^{mz}, \tag{11}$$

where $\lambda_{i,j}$, ρ_0, P_0 and m are constants.

Substituting eq (11) in eqs (9) and (10) we get

$$e^{mz}\,(\alpha_{11} + P_0)\,(2u_1,_{xx} + u_1,_{zz} + u_3,_{xz}) + \alpha_{13}\,(u_3,_{xz} - u_1,_{zz})\,e^{mz} + (u_1,_z + u_3,_x)\,(\alpha_{11} - \alpha_{13})\,m\,e^{mz} - 2\rho_0\,g\,u_3,_x\,e^{mz} = 2\rho_0 u_1,_{tt}\,, \tag{12}$$

$$\alpha_{11}\,(u_1,_{xz} + u_3,_{xx}) + (\alpha_{13} + 2P_0)\,(u_1,_{xz}) - (\alpha_{13} + 2P_0)\,u_3,_{xx} + 2\alpha_{33}\,u_3,_{zz} + 2\rho_0 g\,u_1,_x + 2\,\alpha_{13}\,m\,u_1,_x + 2\alpha_{33}\,m u_3,_z = 2\rho u_3,_{tt}\,. \tag{13}$$

To investigate the surface wave propagation along Ox, we introduce displacement potentials in terms of displacements components are given by

$$u = \phi,_x - \psi,_z\ ;\ w = \phi,_z + \psi,_x \tag{14}$$

Introducing eq (14) in eqs (12) and (13) we get

$$2\,(\alpha_{11} + P_0)\,\nabla^2\phi - 2\rho_0\,g\,\psi,_x + m\,(\alpha_{11} - \alpha_{13})\,(2\phi,_z + \psi,_x) = 2\rho_0\,\phi,_{tt}\,, \tag{15}$$

$$(\alpha_{11} + P_0 - \alpha_{13})\,\nabla^2\psi + 2\rho_0\,g\phi,_x - m\,(\alpha_{11} - \alpha_{13})\,\psi,_z = 2\rho_0\,\psi,_{tt}\,, \tag{16}$$

and

$$\alpha_{11}\,\phi,_{xx} + \alpha_{33}\,\phi,_{zz} - \rho_0 g\,\psi,_x - 2\alpha_{13}\,m\,\psi,_x + 2\alpha_{33}\,m\phi,_z = 2\rho_0\phi,_{tt}\,, \tag{17}$$

$$(\alpha_{11} - \alpha_{13} - 2P_0)\,\psi,_{xx} + (2\,\alpha_{33} - \alpha_{13} - \alpha_{11} - 2P_0)\,\psi,_{zz} + (2\,\rho_0 g + 2\alpha_{13}\,m)\,\phi,_x + 2\alpha_{33}\,m\,\psi,_z = 2\rho_0\,\psi,_{tt}\,, \tag{18}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

Since the velocity of waves are different in x and z direction. Now eq (15) and (16) represent the compressive wave along x and z-direction while eq (17) and (18) represents the shear waves along these directions. Since we consider the propagation of Rayleigh waves in x-direction $\therefore$ we consider only equation (15) and equation (18).

To solve equation (15) and equation (18) we introduce

$$\phi(x, y, z) = f(z)\, e^{i\alpha(x-ct)},$$

and $$\psi(x, y, z) = h(z)\, e^{i\alpha(x-ct)}. \tag{19}$$

putting eq (19) in eq (15) and eq (18) we get

$$f_{,zz} + A f_{,z} + Bf + Ch = 0, \tag{20}$$

$$h_{,zz} + A' h_{,z} + B'h + C'f = 0, \tag{21}$$

where

$$A = \frac{m(\alpha_{11} - \alpha_{13})}{\alpha_{11} + P_0},\ B = \frac{\alpha^2(\rho_0 c^2 - \alpha_{11} - P_0)}{\alpha_{11} + P_0},\ C = \frac{[-2\rho_0 g + m(\alpha_{11} - \alpha_{13})]i\alpha}{2(\alpha_{11} + P_0)},$$

$$A' = \frac{2m\alpha_{33}}{2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0},\ B' = \frac{\alpha^2(2c^2\rho_0 - \alpha_{11} + \alpha_{13} + 2P_0)}{2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0},$$

$$C' = \frac{(2\rho_0 g + 2m\alpha_{13})i\alpha}{2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0}. \tag{22}$$

Now eq (20) and eq (21) have exponential solution in order that f (z) and h (z) describe surface waves and also they varnish as $z \to \infty$ hence eq (15) takes the form,

$$\phi(x, z, t) = \left[C_1 e^{-\lambda_1 z} + C_2 e^{-\lambda_2 z}\right] e^{i\alpha(x-ct)},$$

and $$\psi(x, z, t) = \left[C_3 e^{-\lambda_1 z} + C_4 e^{-\lambda_2 z}\right] e^{i\alpha(x-ct)}, \tag{23}$$

where C_1, C_2, C_3, C_4 are arbitrary constants and λ_1, λ_2 are the roots of the equation

$$\lambda^4 + \left[\frac{m(\alpha_{11} - \alpha_{13})}{\alpha_{11} + P_0} + \frac{2m\alpha_{33}}{2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0}\right]\lambda^3$$

$$+\ \alpha^2\left[\frac{2\rho_0 c^2 - \alpha_{11} + \alpha_{13} + 2P_0}{2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0} + \frac{\rho_0 c^2 - \alpha_{11} - P_0}{\alpha_{11} + P_0}\right]\lambda^2$$

$$+\ m\alpha^2\left[\frac{(\alpha_{11} - \alpha_{33})(2\rho_0 c^2 - \alpha_{11} + \alpha_{13} + 2P_0) + (\rho_0 c^2 - \alpha_{11} - P_0)2\alpha_{33}}{(\alpha_{11} + P_0)(2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0)}\right]\lambda$$

$$+\left[\frac{\alpha^4(\rho_0 c^2 - \alpha_{11} - P_0)(2\rho_0 c^2 - \alpha_{11} + \alpha_{13} + 2P_0)}{(\alpha_{11} + P_0)(2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0)}\right.$$

$$\left. +\alpha^2 \frac{\{m(\alpha_{11}-\alpha_{13})-2\rho_0 g\}\,(2\rho_0 g+2m\alpha_{13})}{2(\alpha_{11}+P_0)\,(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0)} \right] = 0. \tag{24}$$

Here we consider only real roots of eq (24). Now the constants C_1, C_2 and C_3, C_4 are related by the eqs (20) and eq (21).

By equating the co-efficients of $e^{-\lambda_1 z}$ and $e^{-\lambda_2 z}$ to zero, eq (20) gives,

$$C_3 = \gamma_1 C_1,\ C_4 = \gamma_2 C_2, \tag{25}$$

where

$$\gamma_j = \frac{2i[(\alpha_{11}+P_0)\lambda_j^{\,2} - m\,(\alpha_{11}-\alpha_{13})\lambda_j + (\rho_0 c^2 - \alpha_{11} - P_0)]}{\alpha\left[m(\alpha_{11}-\alpha_{13})-2\rho_0 g\right]} \quad ; j = 1, 2. \tag{26}$$

4. Boundary Conditions

The plane z = 0 is free from stresses i.e. $\sigma_{13} = \sigma_{33} = 0$ at z = 0, (27)

where

$$\sigma_{13} = \frac{1}{2}(\alpha_{11}-\alpha_{13})\,[2\,\phi_{,xz} - \psi_{,zz} + \psi_{,xx}]\,e^{mz}, \tag{28}$$

$$\sigma_{33} = \alpha_{31}\,[\phi_{,xx} - \psi_{,xz}]\,e^{mz} + \alpha_{33}[\phi_{,zz} + \psi_{,zx}]\,e^{mz}. \tag{29}$$

Introducing eq (28) and (29) in eq (27) we have

$$C_1\,(2\lambda_1 i\alpha + \gamma_1\lambda_1^{\,2} + \alpha^2\gamma_1) + C_2\,[2\,\lambda_2 i\alpha + \gamma_2\lambda_2^{\,2} + \alpha^2\gamma_2] = 0, \tag{30}$$

$$C_1\,[-\alpha^2\lambda_{13} + \lambda_1^{\,2}\alpha_{33} - \alpha_1\gamma_1 i\alpha\,(\alpha_{33}-\alpha_{13})] + C_2\,[-\alpha^2\lambda_{13} + \lambda_2^{\,2}\alpha_{33} - \lambda_1\gamma_1 i\alpha\,(\alpha_{33}-\alpha_{13})] = 0. \tag{31}$$

Eliminating C_1 and C_2 from eq (30) and eq (31) ; we have

$$\begin{aligned}&[2\lambda_1 i\alpha + \gamma_1\lambda_1^{\,2} + \alpha^2\gamma_1]\,[-\alpha^2\lambda_{13} + \lambda_2^{\,2}\alpha_{33} - \lambda_1\gamma_1 i\alpha\,(\alpha_{33}-\alpha_{13})]\\ &-[2\,\lambda_2 i\alpha + \gamma_2\lambda_2^{\,2} + \alpha^2\gamma_2]\,[-\alpha^2\lambda_{13} + \lambda_1^{\,2}\alpha_{33} - \alpha_1\gamma_1 i\alpha\,(\alpha_{33}-\alpha_{13})] = 0,\end{aligned} \tag{32}$$

where γ_j (j = 1, 2) are given by equation (26) and λ_j (j = 1, 2) are roots of eq. (24).

Now eq (32) gives the wave velocity equation for Rayleigh waves in a non-homogeneous elastic half space of orthotropic material under the initial compression and influence of gravity.

From eq (32), it follows that Rayleigh waves depends on gravity, initial compression, non-homogeneous character of the medium and nature of the material.

From equation (32),we conclude that if α is large i.e. length of wave i.e. $\frac{2\pi}{\alpha}$ is small then gravity and compression have small effects on Rayleigh waves in non-homogeneous orthotropic half space and if α is small i.e. $\frac{2\pi}{\alpha}$ is large then gravity and compression plays a vital role for finding out the wave velocity c.

When the medium is isotropic, eq (32) becomes

$$[2\lambda_1\, i\alpha + \gamma_1\lambda_1^{\,2} + \alpha^2\gamma_1]\,[K_1^{\,2}\,(\lambda_2^{\,2} - \alpha^2) + 2\,K_2^{\,2}\,(1 - i\alpha\,\gamma_2\lambda_2)]$$
$$- [2\,\lambda_2\, i\alpha + \gamma_2\lambda_2^{\,2} + \alpha^2\,\gamma_2]\,[K_1^{\,2}\,(\lambda_1^{\,2} - \alpha^2) + 2\,K_2^{\,2}\,(1 - i\alpha\,\gamma_1\lambda_1)] = 0, \tag{33}$$

where $K_1^{\,2} = \frac{\lambda + 2\mu + P}{\rho}$, $K_2^{\,2} = \frac{\mu - P/2}{\rho}$, ($\lambda$, μ are Lame's constants). (34)

eq (34) determines the Rayleigh waves in a non-homogeneous isotropic elastic solid under the influence of gravity and compression.

When initial compression is absent i.e. $P_0 = 0$, then equation (33) reduces to,

$$[2\lambda_1\, i\alpha + \gamma_1\lambda_1^{\,2} + \alpha^2\gamma_1]\,[K_1^{\,2}\,(\lambda_2^{\,2} - \alpha^2) + 2\,K_2^{\,2}\,(1 - i\alpha\,\gamma_2\lambda_2)]$$
$$- [2\,\lambda_2\, i\alpha + \gamma_2\lambda_2^{\,2} + \alpha^2\,\gamma_2]\,[K_1^{\,2}\,(\lambda_1^{\,2} - \alpha^2) + 2\,K_2^{\,2}\,(1 - i\alpha\,\gamma_1\lambda_1)] = 0, \tag{35}$$

where $K_1^{\,2} = \frac{\lambda + 2\mu}{\rho}$, $K_2^{\,2} = \frac{\mu}{\rho}$.

eq (35) determines the Rayleigh surface waves in a non-homogeneous isotropic elastic solid under the influence of gravity which is similar to corresponding classical result given by Das et al.

When non-homogeneity of the material is absent, we get same dispersion eq. as (32) with

$$\gamma_j = \frac{-i[(\alpha_{11} + P_0)\lambda_j^{\,2} + (\rho_0\, c^2 - \alpha_{11} - P_0)]}{\alpha\rho_0 g} \quad ; j = 1, 2,$$

where λ_1, λ_2 are the roots of the equation

$$\lambda^4 + \alpha^2\left[\frac{2\rho_0 c^2 - \alpha_{11} + \alpha_{13} + 2P_0}{2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0} + \frac{\rho_0 c^2 - \alpha_{11} - P_0}{\alpha_{11} + P_0}\right]\lambda^2$$
$$+\left[\frac{\alpha^4\left(\rho_0 c^2 - \alpha_{11} - P_0\right)\left(2\rho_0 c^2 - \alpha_{11} + \alpha_{13} + 2P_0\right) - 2\alpha^2\,\rho_0^{\,2}\,g^2}{\left(P_0 + \alpha_{11}\right)\left(2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0\right)}\right] = 0. \tag{36}$$

When gravity field is absent, we get same velocity eq. for Rayleigh waves in non-homogeneous elastic solid under initial compression as eq (32) with

$$\gamma_j = \frac{2i[(\alpha_{11} + P_0)\lambda_j^{\,2} - m\,(\alpha_{11} - \alpha_{13})\lambda_j + (\rho_0\, c^2 - \alpha_{11} - P_0)]}{\left[\alpha m\left(\alpha_{11} - \alpha_{13}\right)\right]} \quad ; j = 1, 2,$$

where λ_1, λ_2 are roots of the equation

$$\lambda^4 + \left[\frac{m\left(\alpha_{11} - \alpha_{13}\right)}{\alpha_{11} + P_0} + \frac{2m\alpha_{33}}{2\alpha_{33} - \alpha_{11} - \alpha_{13} - 2P_0}\right]\lambda^3$$

$$+\alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0}{2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0}+\frac{\rho_0c^2-\alpha_{11}-P_0}{\alpha_{11}+P_0}\right]\lambda^2$$

$$+\,m\alpha^2\left[\frac{(\alpha_{11}-\alpha_{33})(2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0)+(\rho_0c^2-\alpha_{11}-P_0)2\alpha_{33}}{(\alpha_{11}+P_0)(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0)}\right]\lambda$$

$$+\left[\frac{\alpha^4(\rho_0c^2-\alpha_{11}-P_0)(2\rho_0c^2-\alpha_{11}+\alpha_{13}+2P_0)}{(\alpha_{11}+P_0)(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0)}\right.$$

$$\left.+\alpha^2\;\frac{\{m(\alpha_{11}-\alpha_{13})\}\,(m\alpha_{13})}{(\alpha_{11}+P_0)\,(2\alpha_{33}-\alpha_{11}-\alpha_{13}-2P_0)}\right]=0. \tag{37}$$

When medium is initially unstressed i.e. $P_0 = 0$,
we get, velocity equation for Rayleigh waves is similar to equation (32) with

$$\gamma_j=\frac{2i[(\alpha_{11}\lambda_j^{\,2}-\mathrm{m}\,(\alpha_{11}-\alpha_{13})\lambda_j+(\rho_0\,\mathrm{c}^2-\alpha_{11})]}{\alpha\left[m(\alpha_{11}-\alpha_{13})-2\rho_0g\right]}\quad;\ j=1,2,$$

where λ_1, λ_2 are roots of the equation

$$\lambda^4+\left[\frac{m\,(\alpha_{11}-\alpha_{13})}{\alpha_{11}}+\frac{2m\,\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}\right]\lambda^3$$

$$+\,\alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}+\frac{\rho_0c^2-\alpha_{11}}{\alpha_{11}}\right]\lambda^2$$

$$+\,m\alpha^2\left[\frac{(\alpha_{11}-\alpha_{33})\,(2\rho_0c^2-\alpha_{11}+\alpha_{13})+(\rho_0c^2-\alpha_{11})\,2\alpha_{33}}{(\alpha_{11})(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right]\lambda$$

$$+\left[\frac{\alpha^4\,(\rho_0c^2-\alpha_{11})\,(2\rho_0c^2-\alpha_{11}+\alpha_{13})}{(\alpha_{11})(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right.$$

$$\left.+\alpha^2\;\frac{\{m(\alpha_{11}-\alpha_{13})-2\rho_0g\}\;(2\rho_0g+2m\alpha_{13})}{2(\alpha_{11})\,(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right]=0. \tag{38}$$

When the non-homogeneity of the material and gravity field are absent further medium is initially unstressed and isotropic, eq (32) reduces to,

$$4\sqrt{\left(1-\frac{c^2}{K_1^2}\right)\left(1-\frac{c^2}{K_2^2}\right)}=\left(2-\frac{c^2}{K_2^2}\right)^2, \tag{39}$$

where $K_1^{\,2}=\dfrac{\lambda+2\mu}{\rho}$, $K_2^{\,2}=\dfrac{\mu}{\rho}$.

Equation (39) is similar to the equation given by Rayleigh.

5 Discussion and Conclusions

Equation (32) represents the wave velocity equation for the Rayleigh waves in a non-homogeneous, orthotropic elastic solid medium under the influence of gravity and initial compression. It depends upon the wave number and confirming that waves are dispersive. Moreover, the dispersion equation contains terms involving gravity, initial compression and non-homogeneity, so the phase velocity 'c' not only depends upon the gravity field and initial compression but also on the non-homogeneity of the material medium.
The explicit solutions of this wave velocity equation cannot be determined by analytical methods. However, these equations can be solved with the help of numerical method, by a suitable choice of physical parameters involved in medium.

REFERENCES

[1]Abd-Alla AM and Ahmed SM (1996). Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. J. Earth, Moon Planets 75, pp.185-197.

[2]Ahmed SM (1999). Influence of gravity on the propagation of waves in granular medium. Appl.Math.Compu.101, no.2-3. pp. 269-280.

[3]Biot MA (1965). Mechanics of incremental Deformations, J. Willy.

[4]Bromwich TJ (1898). On the influence of gravity on elastic waves, and, in particular, on the vibrations of an elastic globe. Proc. London Math. Soc., 30. pp. 98-120.

[5]Brunelle EJ (1973).Surface wave propagation under initial tension or compression. Bull. Seismol. Soc. Am. 63. pp. 1895-1899.

[6]Chattopadhyay A, Gupta S, Sharma VK and Kumari P (2010). Effect of point source and heterogeneity on the propagation of SH- waves. Int. J. of appl. Math. and Mech. 6(9): pp.76-89.

[7]Datta BK (1986). Some observation on interactions of Rayleigh waves in an elastic solid medium with the gravity field. Rev. Roumaine Sci. Tech. Ser. Mec. Appl. 31, pp. 369-374.

[8]De SK and Sengupta PR (1974). Influence of gravity on wave propagation in an elastic layer, J.Acanst. Soc. Am. 55. pp. 5-13.

[9]Ewing WM, Jardetzky WS, and Press F (1957). Elastic waves in layered media. Mcgraw-Hill, New York.

[10]Goda MA (1992). The effect of inhomogeneity and anisotropy on Stoneley waves. Acta Mech. 93, no.1-4. pp. 89-98.

[11]Haskell NA (1953). The dispersion of surface waves in multilayered media. Bull. Seis. Soc. Amer. 43. pp. 17-34.

[12]Jones JP (1964). Wave propagation in a two layered medium. J. Appl. Mechanics, pp. 213-222.

[13]Love AEH (1965). Some problems of Geodynamics. Cambridge University Press, London.

[14]El-Naggar,Abd-Alla AM and Ahmed SM (1994). Rayleigh waves in a megnatoelastic initially stressed conducting medium with the gravity field. Bull.Calcutta Math. Soc.86, no.3. pp. 243-248.

[15]Roy PP(1984). Wave propagation in a thin two layered medium with stress couples under initial stresses. Acta Mechanics 54. pp. 1-21.

[16]Sengupta PR and Acharya D (1979). The influence of gravity on the propagation of waves in a thermoelastic layer. Rev. Romm. Sci. Techmol. Mech. Appl., Tome 24. pp. 395-406.

[17]Sezawa K (1927). Dispersion of elastic waves propagated on the surface of stratified bodies and on curved surfaces. Bull. Earthq. Res. Inst. Tokyo, 3. pp. 1-18.

[18]Sharma JN and Kaur D (2010). Rayleigh waves in rotating thermo-elastic solids with voids. Int. J. of appl. Math. and Mech. 6(3): pp.43-61.

[19]Willson W (1963). The propagation of megnato-thermo, elastic plane waves Proc.Cambridge Philos. Soc. 59. pp. 483-488.

[20]Yu CP and Tang S (1966). Magneto elastic waves in initially stressed conductors. Z. Angew. Math. Phys.17. pp. 766-775.

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