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Promotion and Application of the M/M/1/N Queuing System

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Abstract

The customers do not necessarily get into the system though arriving at the system in the case of a fixed customer number, which influences sales industry enormously. This paper focuses on the influence of queue length on input rate, and sets up a queuing model with variable input rates, errors service and impatient customers, which are related to the queue length, and get the following conclusions: the customers get into the system in Poisson flow; the number of customers in the system is a birth-death process; the stationary queue length distribution of the model ; the loss probability for the customers not entering the system while they arriving at the system, the mean of the customers who leaves the system being for impatient, the service error rate per unit time, the loss probability for the customers not joining the queue due to the limited capacity of the system and so on, and the appropriate service speeds and the capacity the business should keep to make the biggest profit, which provides a valuable reference for the enterprise to improve their sales performance.

Mathematics Subject Classification: 60k25

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1 Introduction

Customers' coming to the enterprise to seek for service may constitute a queuing system[1−2]. The arrival process of the customers is the input process; the enterprise is the service agency, and we rule on the principle of " first come, first served". Customers always hope that the queue length for service is as short as possible when they arrive; otherwise, they may refuse to enter the system or even leave immediately, even customers who are already in the system may also leave the system due to impatience, which requires the system to adjust service speeds flexibly: raise the service speed to reduce the loss of customers when the queue is long and reduce the service speeds to reduce the cost of services. In addition, the capacity of the system does impact on profit[3−4], so the research of input rate, impatient customers and capacity appears to be particularly important for a selling [5−6].

2 Model hypothesis

(1) There is only one service window in the system, and its capacity is $N(N > 0)$, first come first served.

(2)Exponential distribution with the parameter λ is applied to time intervals that the customers arrive at system.

(3)Suppose α_n is the probability of entering the system when customers arrive, among which n is the queue length, $n = 0, 1, 2, \cdots, N$, $0 \leq \alpha_n \leq 1$.

(4) Exponential distribution with the parameter μ_n is applied to the service time T for each customer.

(5)The customers who are already in the system may leave the system due to impatience, and those (who leave the system due to impatience) leave the system at Poisson flow with the parameter δ_n, among which n is the queue length, $n = 0, 1, 2 \cdots$ N, and rules $\delta_0 = \delta_1 = 0, 0 < \delta_n < \delta_{n+1}, \lim\limits_{n\to+\infty} \delta_n = +\infty$

(6)The system may be wrong, the faster the lower accuracy rate. The correct rate of the service is β_n when the service rate is μ_n, and rules $\beta_0 = 1$, $0 < \beta_n \leq 1$.

(7)The arrival process of customers and system service process are independent respectively.

3 Mathematical model

Theorem 3.1 *The customer flow arriving at the system $\{I(t), t \geq 0\}$ is Poisson flow with parameter λ, and the customers enter the system at the probability of α_k, and then the event flow of the customers entering the system $\{X(t), t \geq 0\}$ is a Poisson flow with parameter $\lambda\alpha_k$.*

Proof: $p(X(t) = k|I(t) = n)= C_n^k \alpha_k^k (1 - \alpha_k)^{n-k}$,$n \geq k, n = 0, 1, 2 \cdots N$, $p(X(t) = k) = \sum\limits_{n=k}^{\infty} p(I(t) = n)p(X(t) = k|I(t) = n) = \sum\limits_{n=k}^{\infty} \frac{(\lambda t)^n}{n!} e^{-\lambda t} C_n^k \alpha_k (1 - \alpha_k)^{n-k}$ $= \frac{(\lambda\alpha_k t)^k}{k!} e^{-\lambda t} \sum\limits_{n=k}^{\infty} \frac{[\lambda(1-\alpha_k)t]^{n-k}}{(n-k)!} = \frac{(\lambda\alpha_k t)^k}{k!} e^{-\lambda t} e^{-\lambda(1-\alpha_k)t} = \frac{(\lambda\alpha_k t)^k}{k!} e^{-\lambda\alpha_k t}$. *So,* $\{X(t), t \geq 0\}$ *is a Poisson flow with parameter $\lambda\alpha_k$.*

Theorem 3.2 *The number of customers $\{N(t), t \geq 0\}$ (including being serviced) in the system at the moment of t is a birth-death process in the state space $E = \{0, 1, 2 \cdots N\}$, and its parameters are* $\begin{cases} \lambda_k = \lambda\alpha_k \\ \mu_k = \mu\beta_k + \delta_k \end{cases}$

Proof: Time intervals that the customers arrive at system and the service time of system for each customer present exponential distribution. We know that exponential distribution has the following character: the remaining arrival time and service time distribution are the same as the original distribution. Therefore the number of customers $\{N(t), t \geq 0\}$ *in the system is homogeneous Markov chains.*

If $p_{ij}(\Delta t) = p(N(t+\Delta t) = j|N(t) = i)$, $i \geq 0, j \geq 0$. *The next we will get* $p_{ij}(\Delta t)$*(*$\Delta t \to 0^+$*)*

(1) If $i = 0, j = 1$, *then* $p_{01}(\Delta t) = p(N(t+\Delta t) = 1|N(t) = 0) = p$*(One customer enters the system during the time* Δt*)* $= \lambda_0 \Delta t e^{-\lambda_0 \Delta t} = \lambda_0 \Delta t[1 + \frac{1}{1!}(-\lambda_0 \Delta t) + \frac{1}{2!}(-\lambda_0 \Delta t)^2 + \cdots] = \lambda_0 \Delta t + o(\Delta t)$

(2) If $i \geq 1, j = i+1$, *then* $p_{i,i+1}(\Delta t) = p(N(t+\Delta t = i+1|N(t) = i)$ *=P (During the time* Δt*, one customer enters the system while the service to the previous customer hasn't been finished yet, or k customers enter the system while the service to k-1 customers have just been finished,* $k \geq 2$*)* $= \lambda_i \Delta t e^{-\lambda_i \Delta t} e^{-\mu\beta_k + \delta_k \Delta t} + o(\Delta t) = \lambda \alpha_i \Delta t e^{-\lambda \alpha_i \Delta t} e^{-\mu\beta_k + \delta_k \Delta t} + o(\Delta t) = \lambda_i \Delta t + o(\Delta t)$

(3) If $i \geq 1, j = i-1$, *then* $p_{i,i-1}(\Delta t) = p(N(t+\Delta t) = i-1|N(t) = i)$ *=P (During the time* Δt*, no customer enters the system and the service to one customer has been finished, or k customers enter the system while the service to k+1 customers have been finished,* $k \geq 1$*) =P (During the time* Δt*, no customer enters the system while the service of one customer has been finished) + P (During the time* Δt*, k customers enter the system and the service to k+1 customers have been finished,* $k \geq 1$*) =*$e^{-\lambda_i \Delta t}(\mu\beta_k + \delta_k)\Delta t e^{-(\mu\beta_k + \delta_k)\Delta t} + o(t)$ $= (\mu\beta_k + \delta_k)\Delta t + o(\Delta t)$

(4) If $j > i+1$, *then* $p_{i,j}(\Delta t) = p(N(t+\Delta t = j|N(t) = i)$ *=P(During the time* Δt*, i + k- j customers enter the system while the service to k customers have been finished,* $k \geq 2$*)* $= o(\Delta t)$

(5) If $j < i-1$, *then* $p_{i,j}(\Delta t) = p(N(t+\Delta t = j|N(t) = i)$ *= P (During the time* Δt*, j-i +k customers enter the system while the service to k customers have been finished,* $k \geq 2$*)* $= o(\Delta t)$

From the above five circumstances, we know: $N(t)$ *is a birth-death process in the state space* $E = \{0, 1, 2, \cdots, N\}$ *with parameter* $\begin{cases} \lambda_k = \lambda\alpha_k \\ \mu_k = \mu\beta_k + \delta_k \end{cases}$.

The following we will discuss the stationary distribution of the system. In fact, the stationary distribution of the system $p_n = \lim_{t\to\infty} P(N(t) = n)$ exists due to limited state of the system. From the model hypothesis we can get the Kolmogorov equations ($let \rho_i = \frac{\lambda_{i-1}}{\mu\beta_i + \delta_i}$, $i = 1, 2, \cdots, N-1$) :

For state 0: $\lambda_0 p_0 = \gamma_1 p_1 \Rightarrow p_1 = \frac{\lambda_0}{\gamma_1} p_0 = \rho_1 p_0 = \left(\prod_{i=1}^{1} \rho_i\right) p_0$;

For state 1:$\lambda_0 p_0 + \lambda_2 p_2 = (\lambda_1 + \gamma_1) p_1 \Rightarrow p_2 = \frac{\lambda_0}{\gamma_1}\frac{\lambda_1}{\gamma_2} p_0 = \rho_1 \rho_2 p_0 = \left(\prod_{i=1}^{2} \rho_i\right) p_0$;

For state 2:$\lambda_1 p_1 + \gamma_3 p_3 = (\lambda_2 + \gamma_2)p_2 \Rightarrow p_3 = \frac{\lambda_0}{\gamma_1}\frac{\lambda_1}{\gamma_2}\frac{\lambda_2}{\gamma_3}p_0 = \rho_1\rho_2\rho_3 p_0 = \left(\prod_{i=1}^{3}\rho_i\right)p_0$;

$$\cdots\cdots$$

For state N-1:$\lambda_{N-2}p_{N-2}+\gamma_N p_N = (\lambda_{N-1}+\gamma_{N-1})p_{N-1} \Rightarrow p_N = (\frac{\lambda_0}{\gamma_1}\frac{\lambda_1}{\gamma_2}\cdots\frac{\lambda_{N-1}}{\gamma_N})p_0 = (\rho_1\cdots\rho_N)p_0 = \left(\prod_{i=1}^{N}\rho_i\right)p_0$.

In accordance with the regularity we can get $\sum_{k=0}^{N} p_k = 1 \Rightarrow p_0 = [1 + \sum_{i=1}^{N}\prod_{j=1}^{i}\rho_j]^{-1}$.

We obtain the following theorem based on the above derivation:

Theorem 3.3 *Stationary distribution of the system* $p_n = \lim_{t\to\infty} P(N(t) = n)$ *exists, and* $p_n = \left(\prod_{i=1}^{n}\rho_i\right)p_0$, *among which* $n = 1, 2\cdots N$, $p_0 = [1 + \sum_{i=1}^{N}\prod_{j=1}^{i}\rho_j]^{-1}$, $\rho_i = \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}$

Theorem 3.4 *When a customer enters the system at the moment of t, he can see a queue length (not including the new arrival customer himself), which is* $N^-(t)$, *and* $p_n^- = \lim_{t\to\infty} P(N^-(t) = n)$, *then* $P_k^- = \frac{\lambda_k}{\sum_{l=0}^{N-1}\lambda_l p_l}p_k$.

Proof: If $A(t, t + \Delta t)$ *is an event that one customer arrives from time t to* $t + \Delta t$, *and* $A(t, t + \Delta t)$ *doesn't mean the customer who arrived the system must enter the system, then*

$$p^-{}_k(t) = \lim_{\Delta t\to 0} P(N(t) = k|A(t, t + \Delta t)) = \lim_{\Delta t\to 0}\frac{p_k(t)P(A(t,t+\Delta t)|N(t)=k)}{\sum_{l=0}^{N-1} p_l(t)P(A(t,t+\Delta t)|N(t)=l)}$$

By the theorem 2: $N(t)$ *is a birth-death process, so* $P[A(t, t+\Delta t)|N(t) = k] = \lambda_k\Delta t + o(\Delta t)(\Delta t \to 0)$, *so,* $P_k^-(t) = \frac{\lambda_k}{\sum_{l=0}^{N-1}\lambda_l p_l(t)}p_k(t)$, *further,* $P_k^- = \frac{\lambda_k}{\sum_{l=0}^{N-1}\lambda_l p_l}p_k$ *when* $t \to \infty$.

4 The other relevant indicators of system

(1) When a customer reaches the system and finds that there are n customers in the system, if he enters the system at the probability of α_n , and he leaves the system at the probability of $1 - \alpha_n$, then the loss probability of the system because of the customers not entering the system $p_{loss} = \sum_{n=0}^{N} p(X = n)(1 - \alpha_n) = \sum_{n=0}^{N} p_n - \sum_{n=0}^{N}\alpha_n p_n = 1 - \sum_{n=0}^{N}\left(\alpha_n\prod_{i=1}^{n}\frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}\right)p_0$, among which $p_0 = [1 + \sum_{i=1}^{N}\prod_{j=1}^{i}\rho_j]^{-1}$, $\rho_i = \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}$.

(2) When a customer arrives at the system and find that there are N customers in the system, he can not enter the system. The loss probability of

the system because of limited capacity $p_N = \left(\prod\limits_{i=1}^{N} \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}\right) p_0$, among which $p_0 = [1 + \sum\limits_{i=1}^{N} \prod\limits_{j=1}^{i} \rho_j]^{-1}$, $\rho_i = \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}$.

(3) Some customers may leave the system before being served due to impatience. The mean of those customers who leaves the system because of impatience in unit time $L_I = \sum\limits_{n=0}^{N} \delta_n p_{n+1} = \sum\limits_{n=0}^{N} \left(\delta_n \prod\limits_{i=1}^{n+1} \frac{\lambda_{i-1} p_0}{\mu\beta_i+\delta_i}\right)$, among which $p_0 = [1 + \sum\limits_{i=1}^{N} \prod\limits_{j=1}^{i} \rho_j]^{-1}$, $\rho_i = \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}$.

(4)The average input rate of system$\overline{\lambda} = (\sum\limits_{n=0}^{N-1} \lambda_n P_n)(1-P_N) =(1-\prod\limits_{i=1}^{N} \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i})\times \sum\limits_{n=0}^{N-1} \left(\lambda_n \prod\limits_{i=1}^{n} \frac{\lambda_{i-1} p_0}{\mu\beta_i+\delta_i}\right)$, among which $p_0 = [1 + \sum\limits_{i=1}^{N} \prod\limits_{j=1}^{i} \rho_j]^{-1}$, $\rho_i = \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}$.

(5)The average number of those customers who enter the system and receive the service in unit time $\overline{\lambda'} = [\sum\limits_{n=0}^{N-1} (\lambda_n P_n - \delta_n P_{n+1})](1-P_N) = \left[\sum\limits_{n=0}^{N-1} \left(\lambda_n \prod\limits_{i=1}^{n} \frac{\lambda_{i-1} p_0}{\mu\beta_i+\delta_i} - \delta_n \prod\limits_{i=1}^{n+1} \frac{\lambda_{i-1} p_0}{\mu\beta_i+\delta_i}\right)\right]$ $\times(1 - \prod\limits_{i=1}^{N} \frac{\lambda_{i-1}}{\mu_i\beta_i+\delta_i})$, among which $p_0 = [1 + \sum\limits_{i=1}^{N} \prod\limits_{j=1}^{i} \rho_j]^{-1}$, $\rho_i = \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}$.

(6) The average error rate of the system in unit time $L_I = \sum\limits_{n=0}^{N} \beta_n p_n = \sum\limits_{n=0}^{N} \left(\beta_n \prod\limits_{i=1}^{n} \frac{\lambda_{i-1} p_0}{\mu\beta_i+\delta_i}\right)$, among which $p_0 = [1 + \sum\limits_{i=1}^{N} \prod\limits_{j=1}^{i} \rho_j]^{-1}$, $\rho_i = \frac{\lambda_{i-1}}{\mu\beta_i+\delta_i}$.

5 Application

In the above we have analyzed a queue model with variable input rate with the parameter λ_n ,variable service rate with the parameter μ, errors in service with the correct rate β_n , impatient customers with the impatient strength δ_n and the system capacity N. In the following we will investigate the application of this model in enterprise sales, and try to find the appropriate service speeds and capacity the business should keep to make the biggest profit.

Let G = the average income when system serves one customer, E = the average labor cost when system serves one customer, HN = the depreciation for fixed assets (for example: system prepare one chair for each customer or air conditioning cost etc.), among which H is the depreciation factor, N is the system capacity. Let F = the net income of the system in unit time, which is equal to the system revenue, minus the artificial cost of service, and minus the depreciation of the fixed assets, then $F = \overline{\lambda'}G - \mu E - HN = G\sum\limits_{n=0}^{N-1} (\lambda_n p_n - \delta_n p_{n+1}) - E\mu - HN$

Case 1: Let $\lambda_0 = \lambda_1 = \lambda_2 = 3.6$,$\beta_1 = \beta_2 = \beta_3 = 1$,$\delta_1 = \delta_2 = \delta_3 = 0$, $N = 3, E = 1, G = 2, H = 0.1$, request the average customers who enter the system and receive the service, and net income of the system in unit time when $\mu = 2, \mu = 3, \mu = 6$.

Solution: By depending on the subject data we know the input rate and service rate remains unchanged, the correct rate is 1,and the departure strength is 0, so this system is M/M/1/3. Let $\lambda_n \equiv \lambda, \mu_n \equiv \mu, \beta_n \equiv 1, \delta_n \equiv 0$,and put them into p_n: $p_n = \frac{(1-\rho)\rho^n}{1-\rho^{N+1}}, \rho = \frac{\lambda}{\mu}, n = 0, 1, 2, 3$. Then the input rate is $\overline{\lambda} = \lambda (1 - p_N)$, income of the system is $G\lambda (1 - p_N)$, net income of the system is $F = G\lambda (1 - p_N) - E\mu - HN$.

(1) If $\mu = 2$, then $\rho = 1.8, p_3 = 0.49, \overline{\lambda'} = 1.836, F = 1.372$

(2) If $\mu = 3$, then $\rho = 1.21, p_3 = 0.32, \overline{\lambda'} = 2.448, F = 1.56$

(3) If $\mu = 6$, then $\rho = 0.6, p_3 = 0.0993, \overline{\lambda'} = 3.242, F = 0.16$

From above we know: If $\mu = 2$, then $p_3 = 0.49$, $\overline{\lambda'} = 1.836$, $F = 1.372$; If $\mu = 3$, then $p_3 = 0.32 < 0.49$, $\overline{\lambda'} = 2.448 > 1.836$, $F = 1.56 > 1.372$, and we can get: when the input rate becomes larger, the net income may be more; If $\mu = 6$, then $p_3 = 0.0993 < 0.32, \overline{\lambda'} = 3.242 > 2.448, F = 0.16 < 1.56$,and we can get: when the input rate becomes larger, the net income may be less. Further, we can get:

Conclusion 1: When the input rate becomes larger, the net income may be more or may be less; the system serves more, the net income may be more or may be less, so a most profitable business strategy may not necessarily be the highest input rate or the largest number of customers to be served.

Case 2: In the case 1, let $\overline{\gamma} = \sum\limits_{n=1}^{N} \gamma_n p_n, \gamma_n = \mu\beta_n + \delta_n$, request $\overline{\gamma}$ when $\mu = 2, \mu = 3, \mu = 6$

Solution: (1) If $\mu = 2$, then $p_1 = 0.15, p_2 = 0.27, p_3 = 0.491$, $\overline{\gamma} = 1.83$

(2)If $\mu = 3$, then $p_1 = 0.223, p_2 = 0.268, p_3 = 0.32, \overline{\gamma} = 2.433$,

(3)If $\mu = 6$, then $p_1 = 0.276, p_2 = 0.165, p_3 = 0.0993, \overline{\gamma} = 3.24$

From above we know: If $\mu = 2$, then $\overline{\gamma} = 1.83$, $F = 1.36$; If $\mu = 3$, then $\overline{\gamma} = 2.433 > 1.83$, $F = 1.56 > 1.36$; and we can get: when $\overline{\gamma}$ becomes larger, the net income may be more; If $\mu = 6$, then $\overline{\gamma} = 3.24 > 2.433$, $F = 0.16 < 1.56$, and we can get: when $\overline{\gamma}$ becomes larger, the net income may be less. Further, we can get: when $\overline{\gamma}$ becomes larger, can not result the net income be more or be less inevitably. If one of β_n, δ_n is fixed, let the other becomes larger, $\overline{\gamma}$ will becomes larger. So when one of β_n, δ_n is fixed, let the other becomes larger, the net income may be more or less. That is to say,

Conclusion 2: Any one of β_n, δ_n becoming larger independently, can not produce the result the net income will inevitably increase or decrease , from which we can know that the following are not correct: the bigger the correct service rate , the more net income; the bigger the impatience of customers, the less net income.

How to make the biggest profit is related with practical application. Among $\lambda_n, \mu, \beta_n, \delta_n, N$,

λ_n, δ_n are related with customers, and they can not be controlled by the enterprise, though β_n is related with enterprise, it is hard to be controlled by the enterprise , N and μ can be adjusted by the enterprise. The following we will discuss how to get to the biggest value of F when N changes separately. The optimal N should satisfy $\begin{cases} F(N-1) \le F(N) \\ F(N+1) \le F(N) \end{cases} \Leftrightarrow \begin{cases} \overline{\lambda'}(N-1) - (N-1)H \le \overline{\lambda'}(N) - NH \\ \overline{\lambda'}(N+1) - (N+1)H \le \overline{\lambda'}(N) - NH \end{cases}$ $\Leftrightarrow \overline{\lambda'}(N+1) - \overline{\lambda'}(N) \le H \le \overline{\lambda'}(N) - \overline{\lambda'}(N-1)$ (*), on which depended, we can get the optimal N.

Case 3: Let $\lambda_n = 30 - 2n, \mu = 25, \beta_n = 1 - 0.01n, \delta_n = 0.6n, G = 5, E = 1, H = 2.5$, request the optimal N to make F biggest.

Solution:Let $T_n = \overline{\lambda'}(N+1) - \overline{\lambda'}(N)$, then (*)$\Leftrightarrow$ $T_N \le H \le T_{N-1}$. Put above data into p_0, p_N, T_N, F, the result are shown in the table 1.

N	2	3	4	5	6	7	8	9	10
p_0	0.28	0.21	0.16	0.14	0.13	0.12	0.12	0.11	0.11
p_N	0.37	0.27	0.19	0.14	0.09	0.05	0.03	0.01	0.006
T_N	28.7	23.4	17.1	11.1	6.3	3.1	1.2	0.4	0.1
F	229	357	463	539	585	607	612	608	599

Table 1:

From the data of table 1, we can get: $1.2 = T_8 \le H = 2.5 \le T_7 = 3.1$, so when $N = 8$ F will get the maximum value. In fact, from the value of F, we can find F=612 is the biggest, and the corresponding N is 8.

In the following, we will discuss how to get the maximum income when both N and μ are changeable. Considering $\mu < \infty, N < \infty$, we can put all the possible value of (μ, N) into F, then compare all the value of F to find out the biggest one and the corresponding (μ_0, N_0).

Case 4: Let $\lambda_n = 30 - 2n, \beta_n = 1 - 0.01n, \delta_n = 0.6n, G = 5, E = 1, H = 2.5$, request the biggest value of F and the corresponding (μ_0, N_0) when $\{(\mu, N) \,|\, \mu = 35, 36, 37, 38, 39, 40; N = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Solution: Put above data into F, the results are shown in the table 2.

	N=2	N=3	N= 4	N=5	N=6	N= 7	N=8	N=9	N=10	N=11
$\mu = 35$	37.7	55.3	65.1	70.2	72.2	72.2	70.9	69.0	66.7	64.3
$\mu = 36$	38.5	56.3	66.0	70.9	72.7	72.5	71.2	69.2	66.9	64.4
$\mu = 37$	39.2	57.2	66.9	71.5	73.1	72.8	71.3	69.3	66.9	64.5
$\mu = 38$	39.9	58.0	67.6	72.1	73.4	72.9	71.4	69.3	66.9	64.5
$\mu = 39$	40.6	58.5	68.2	72.5	73.7	73.0	71.4	69.2	66.8	64.4
$\mu = 40$	41.2	59.4	68.7	72.8	73.8	73.0	71.3	69.1	66.7	64.2

Table 2:

Comparing all the value of F, we can get the biggest value of F is 73.8 , the corresponding (μ_0, N_0) is $(40, 6)$.

Summary. This paper is based on the study that how to maximize sales profits of an enterprise, and a queuing model with variable input rates, errors in system and impatient customers is established. Proceeding from the stationary distribution, this paper analyzes through specific examples that the loss probability for the customers not entering the system while they arriving at the system, the mean of the customers who leaves the system being for impatient, the loss probability for the customers not joining the queue due to the limited capacity of the system and many other indicators, and find out that any of $\lambda_n, \beta_n, \delta_n$ does impact on the net income, but if any one of $\lambda_n, \beta_n, \delta_n$ becomes larger independently, it can not result the net income be more or be less inevitably. This paper also indicates that a most profitable business strategy may not necessarily be the largest number customer served, nor the highest correct rate of system, nor the least inpatient customers. At last, this paper points out how to make F biggest when N changes independently or both N and μ are changeable. In short, this paper is helpful for the enterprise to maximize their net income .

References

[1] Yu-min LIU ; Xiao-fen LI; Wen-xia WANG; An Ordinary Differential Equation on the M/M/n Model with Variable Import Rate ; Journal of Taiyuan University of Science and Technology 2006,27: 438-440.

[2] Wen-zhi TAI; An explanation of M/M/1 queuing model with variable input rate; Journal of Southwest University for Nationalities(Natural Science Edition; 2009,35: 485-488

[3] Wen-zhi TAI; GAO Shi-ze; A M/M/1 Queuing Model with Variable Input Rate; Journal of Chongqing Normal University(Natural Science; 2009,26: 69-72

[4] Chang TAN; M/M/1 Queuing System of Changeable Input Rates, Journal of Guizhou University(Natural Science); 2008, 25: 445-448

[5] Guanghui Xu ; Stochastic Service System; Science Press; 1984,9

[6] Xu Xiuli, Gao Hong, Tian Naishuo. Analysis for the M/M/1 Vacation Queue with Setup Times and Variable Service Rate(J).Acta Mathematicae Applicatae Sinica. 2008,31(4):692-701.

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