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OPTIMAL MULTIRATE CONTROL TECHNIQUE APPLIED TO A LINEAR DISCRETE POWER SYSTEM MODEL

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Abstract

In this paper an optimal multirate control method (OMCM) is applied to a linear discrete power system model (derived from its associated linear continuous model), in order to design a suitable excitation controller and thus enhance in a robust manner the dynamic stability characteristics of the physical system under consideration. The relevant computer simulation results, based on a practical power system, demonstrated clearly the validity, suitability, effectiveness and implementability of the so accomplished controller design.

***Key-Words:* -** Discrete system representation, multirate controllers, power systems, synchronous machine.

1. Introduction

The typical control problem has always been to start with a suitable linear (or linearized) open-loop mathematical model of a physical plant (in continuous or discrete form) and attempt to design a proper controller for it, i.e. to obtain an associated closed-loop system with enhanced dynamic stability characteristics (Al-Rahmani and franklin, 1994; Ogata, 1994; Sagfors et al. 1998;Fujimoto and Hori, 2002; Srinivasarao, et al. 2007; QingWei, 2008; Mizumoto, et al. 2007; Chak, et al. 1997). The digital controller applied for the discrete linear systems may be obtained by using a new OMCM (Heidarinejad, et al. 2011; Stoorvogel, 1992; Chen and Qui, 1994; Arvanitis, 1996).

It is pointed out that the used OMCM technique reduced the original LQ regulation problem to an associated discrete-time LQ regulation problem for the performance index with crossed product terms, for which is computer a fictitious static state feedback controller. In addition thus technique offers more flexibility in choosing the sampling rates and provides a power design computed method (Arvanitis, 1996; Polushin, and Marquez, 2004; Cimino and Pagilla, 2010).

In the present work the discrete linear open-loop system model under consideration systematically derived from the associated continuous 6th order MIMO linearized open-loop model of a practical power system, hawing on 160 MVA synchronous machine supplying power to an infinite grid through a step-up transformer and a transmission line (Papadopoulos and Boglou, 1990). The sought digital controller for the enhancement of the dynamic characteristics of the above 8th order discrete model is accomplished by the proper application of the new OMCM to it.

2. Overview of New OMCM for Linear Discrete Systems

Starting with the general linear state space system description in continuous form (Arvanitis, 1996),

$$\dot{x}(t) = Ax(t) + bu(t)$$
$$y(t) = Cx(t) \tag{1}$$

where $x(t) \in R_n$, $u(t) \in R^m$ and $y(t) \in R^p$ are the state, input and output vectors respectively.

The associated discrete system description is obtained by letting $n_i, i \in J_p = 1, 2, ..., p$, be used of observability indices of the pair (A,C), and $T_o \in R^+$ be a sampling period, figure1.

Also, by letting

$$\Phi = \exp(AT_0) \tag{2}$$

and $B_N \in R^{nxp_N}$ be the full rank matrix defined by

$$B_N B_N^T = W_N(T_0, 0) \geq 0 \tag{3}$$

with the generalized reachability Grammian of order N in the interval $[0, T_0]$ being $W_N(T_0,0) = T^{*-1}\sum_{\mu=0}^{N_0-1}\Delta_\mu\Delta_\mu^T$, $p_N = \operatorname{rank} W_N(T_0,0)$ and

$$T^* = T_0/N_0,\ \Delta_\mu = \hat{A}_N^{N_0-\mu-1}\hat{B}_{T^*}$$

$$\hat{A}_N = \exp(AT^*),\ \hat{B}_{T^*} = \int_0^{T^8}\exp(A\lambda)B\mathrm{d}\lambda \tag{4}$$

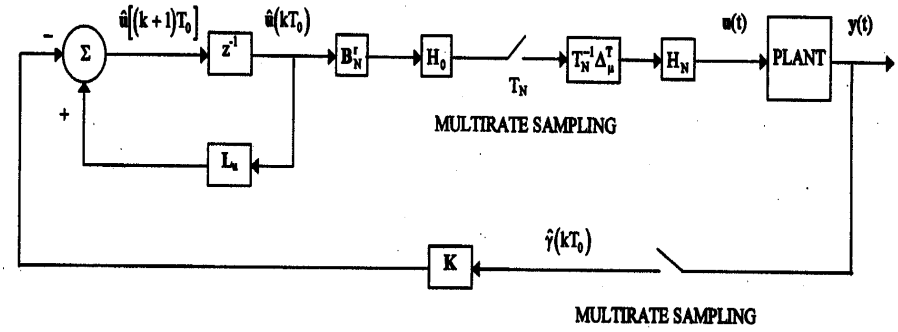


Fig.1. Control of linear systems using OMCM.

Next follows the application of the OMCM technique to the above descriptions. The input of the plant are constrained to the following piecewise constant controls

$$u(kT_0 + \mu T^* + \zeta) = T^{*-1}\Delta_\mu^T B_N^\tau \hat{u}(kT_0),$$
$$\hat{u}(kT_0) \in R^{p_N} \tag{5}$$

for

$$t = kT_0 + \mu T^*,\ \mu = 0,...,N_0 - 1,\ k >> 0$$

and $\zeta \in [0,T^*)$, where $B_N^\tau = B_N(B_N^T B_N)^{-1}$.

The ith plant output $y_i(t)$ is detected at every $T_i = T_0 / M_i$, such that

$$y_i(kT_0 + \rho T_i) = c_i^T x(kT_0 + \rho T_i),\quad \rho = 0,1,...,M_i - 1 \tag{6}$$

where $M_i \in Z^+, i = 1,2,..., p$ are the output multiplicities of the sampling. In general $M_i \neq N.$ The sampled values of the plant outputs obtained over $[kT_0, (k+1)T_0)$ are stored in the M^*-dimentional column vector $\hat{y}(kT_0)$ of the form

$$\hat{y}(kT_0) = [y_1(kT_0)...y_1(kT_0 + (M_1 - 1)T_1)...\,y_p(kT_0)...y_p(kT_0 + (M_p - 1)T_p)]^T$$

where $M^* = \sum_{i=1}^{p} M_i$.

The vector $\hat{y}(kT_0)$ is used in the control law of the form

$$\hat{u}[(k+1)T_0] = L_u\hat{u}(kT_0) - K\hat{y}(kT_0) \tag{7}$$

where $L_u \in R^{p_N x p_N}, K \in R^{p_N x M^*}$.

Finally one scetcs a controller in the form of (5) and (7) which, nowen applied to system (1), minimizes the following performance index

$$J = \frac{1}{2}\int_0^\infty [\mathbf{y}^T(t)\mathbf{Q}y(t) + \mathbf{u}^T(t)\mathbf{Ru}(t)]dt \tag{8}$$

where $Q \in R^{\text{pxp}}$ and $R \in R^{\text{mxm}}$ are symmetric matrices with $Q \geq 0, R > 0$ while $(AC^T QC)$ is an observable pair.

The above problem is equivalent to the problem of designing a control law of the form of equation (8), in order to minimize the following index:

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\begin{bmatrix} \mathbf{x}^T(kT_0) & \hat{\mathbf{u}}^T(kT_0) \end{bmatrix}\begin{bmatrix} \tilde{\mathbf{Q}}_N & \tilde{\mathbf{G}}_N \\ \tilde{\mathbf{G}}_N^T & \tilde{\Gamma}_N \end{bmatrix}\begin{bmatrix} \mathbf{x}(kT_0) \\ \hat{\mathbf{u}}(kT_0) \end{bmatrix}$$

The following basic formula of the multirate sampling mechanism holds

$$\mathbf{Hx}\left[(k+1)T_0\right] = \hat{\gamma}(kT_0) - \mathbf{D}\hat{\mathbf{u}}(kT_0), \quad k \geq 0$$

where, matrices

$$\mathbf{x}(kT_0 + \rho T_i) = \hat{\mathbf{A}}_i^{\rho - M_i}\mathbf{x}\left[(k+1)T_0\right] + \hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}(kT_0)$$

are defined as follows

$$\mathbf{H} = \begin{bmatrix} \mathbf{c}_1^T\left(\hat{\mathbf{A}}_1^{M_1}\right)^{-1} \\ \vdots \\ \mathbf{c}_1^T\hat{\mathbf{A}}_1^{-1} \\ \vdots \\ \mathbf{c}_p^T\left(\hat{\mathbf{A}}_p^{M_p}\right)^{-1} \\ \vdots \\ \mathbf{c}_p^T\hat{\mathbf{A}}_p^{-1} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{c}_1^T\hat{\mathbf{B}}_{1,0} \\ \vdots \\ \mathbf{c}_1^T\hat{\mathbf{B}}_{1,M_1-1} \\ \vdots \\ \mathbf{c}_p^T\hat{\mathbf{B}}_{p,0} \\ \vdots \\ \mathbf{c}_p^T\hat{\mathbf{B}}_{p,M_p-1} \end{bmatrix} \tag{9}$$

and where, in (9)

$$y_i(kT_0 + \rho T_i) = \mathbf{c}_i^T\hat{\mathbf{A}}_i^{\rho - M_i}\mathbf{x}\left[(k+1)T_0\right] + \mathbf{c}_i^T\hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}(kT_0)$$

The ultimate expressions for the control law optimal gain matrices L_{u} and K are as follows

$$\mathbf{L}_{\text{u}} = (\tilde{\mathbf{R}}_{\text{N}} + \mathbf{B}_{\text{N}}^{\text{T}}\mathbf{P}\mathbf{B}_{\text{N}})^{-1}(\tilde{\mathbf{G}}_{\text{N}} + \mathbf{B}_{\text{N}}^{\text{T}}\mathbf{P}\boldsymbol{\Phi})\mathbf{H}^{-1}\mathbf{D} \tag{10}$$

$$\mathbf{K} = (\tilde{\mathbf{R}}_{\text{N}} + \mathbf{B}_{\text{N}}^{\text{T}}\mathbf{P}\mathbf{B}_{\text{N}})^{-1}(\tilde{\mathbf{G}}_{\text{N}} + \mathbf{B}_{\text{N}}^{\text{T}}\mathbf{P}\boldsymbol{\Phi})\mathbf{H}^{-1} \tag{11}$$

where $\tilde{\mathbf{R}}_{\text{N}}, \tilde{\mathbf{G}}_{\text{N}}$ *and* $\mathbf{H}$ are defined in (Ogata, 1994; Sagfors, et al 1998; Arvanitis, 1996). The resulting discrete closed-loop system matrix $\mathbf{A}_{\text{cl/d}}$ takes the following

$$\mathbf{A}_{\text{cl/d}} = \mathbf{A}_{\text{ol/d}} - \mathbf{B}_{\text{N}}\mathbf{KH} \tag{12}$$

where cl=closed-loop, ol=open-loop and d=discrete.

3. Design and Simulations of Open- and Closed-Loop Power System

The system under investigation is shown in block diagram form in Figure 2, and consists of a three-phase 160 MVA synchronous machine with automatic excitation control system supplying power through a step-up transformer and a high-voltage transmission line to an infinite grid. The numerical values of the parameters, which define the total system as well as its operating point, come from (Papadopoulos and Boglou, 1990) and are given in Appendix A.

Based on the state variables figure 2 and the values of the parameters and the operating point (see Appendix A), the system of figure 2 may be described in state-space form, in the form of 1, where

$$\mathbf{x}^T = \begin{bmatrix} E'_q & \omega & \delta & v_1 & v_2 & v_3 & v_4 & v_5 & v_R & E_{fd} \end{bmatrix}$$

$$\mathbf{u}^T = \begin{bmatrix} \Delta E_{\text{Re}f} & \Delta T_m \end{bmatrix}$$

$$\mathbf{y}^T = [\delta \quad v_t$$

and the numerical values of the matrices A, B and C are given in Appendix B.

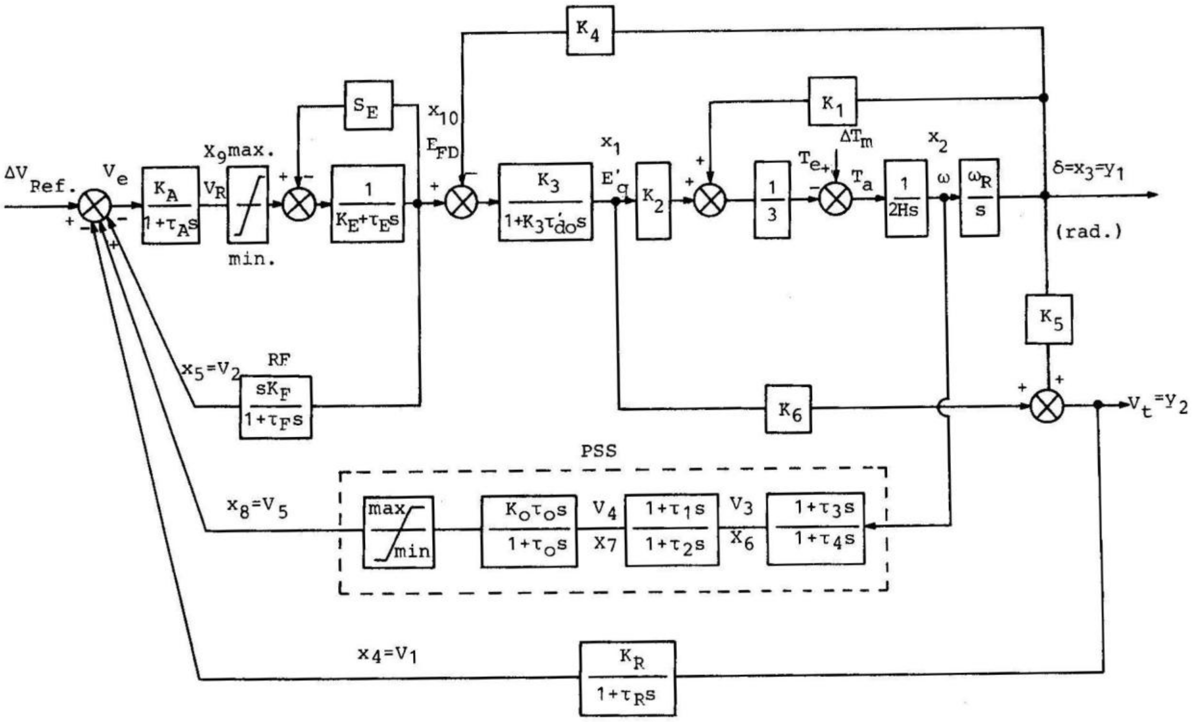


Fig.2, Block diagram representation of regulated synchronous machine supplying power to an infinite grid.

The computed discrete linear open-loop system model for the same operating point corresponding to the linearized continuous transformer open-loop system model of figure 1 is given, in terms of its matrices with sampling period To=0.4 sec., as follows:

$$\mathbf{A}_{ol/d}=\begin{bmatrix} 0.6808 & -5.4034 & -0.0745 & -0.0841 & -0.3513 & -0.3905 & -0.2572 & 0.3942 & 0.0018 & 0.0506 \\ -0.0100 & 0.2719 & -0.0096 & 0.0006 & 0.0018 & 0.0020 & 0.0011 & -0.0020 & 0 & -0.0004 \\ -0.8058 & 92.9580 & 0.2750 & 0.0224 & 0.0579 & 0.0624 & 0.0299 & -0.0619 & -0.0006 & -0.0185 \\ 0.3777 & -5.8500 & -0.0524 & -0.0362 & -0.1360 & -0.1534 & -0.0926 & 0.1506 & 0.0008 & 0.0225 \\ -0.1734 & 1.0489 & 0.0107 & -0.0245 & 0.3595 & -0.2874 & -0.3251 & 0.3800 & 0.0003 & -0.0122 \\ -0.0141 & -0.6437 & -0.0147 & 0.0014 & 0.0048 & 0.0184 & 0.0031 & -0.0053 & 0 & -0.0009 \\ -0.0078 & -2.0973 & -0.0132 & 0.0030 & 0.0120 & -0.1990 & 0.0215 & -0.0134 & -0.0001 & -0.0016 \\ -0.0067 & -2.0913 & -0.0122 & 0.0029 & 0.0119 & -0.1676 & -0.9482 & 0.9475 & -0.0001 & -0.0016 \\ -12.3587 & 122.1028 & 1.4904 & 3.2910 & -14.4844 & 6.4718 & -29.3545 & 24.1441 & -0.0635 & -0.5357 \\ -5.2071 & 30.8392 & 0.3013 & -1.0662 & -9.7173 & -9.3513 & -9.5697 & 11.6895 & 0.0170 & 0.6346 \end{bmatrix}$$

$$\mathbf{B}_{ol/d}=\begin{bmatrix} 0.3995 & -0.0020 & -0.0624 & 0.1524 & 0.3887 & -0.0053 & -0.0136 & -0.0134 & 25.4486 & 11.9382 \\ -0.0480 & 0.0273 & 2.0098 & -0.0856 & 0.0421 & 0.0431 & 0.0428 & 0.0399 & 4.0183 & 1.2487 \end{bmatrix}^T$$

$$\mathbf{C}_{ol/d}=\mathbf{C}=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4777 & 0 & -0.0433 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Due to space limitation, the numerical description of the resulting discrete closed-loop system model is not presented here, but it depends on time following derived weight matrices

$$\mathbf{Q}=\begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix}$$

$$\mathbf{R}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the chosen output multiplicities of the sampling

$\mathbf{M}_t = 4 \quad 8$, while the input multiplicity of the sampling was taken as No=2.

The further application of OMCM leads to the computation of the numerical values of the gain feedback matrices **K**, $\mathbf{L}_u$ and **F** with dimensions (4x12), (4x4) and (4x10), respectively.

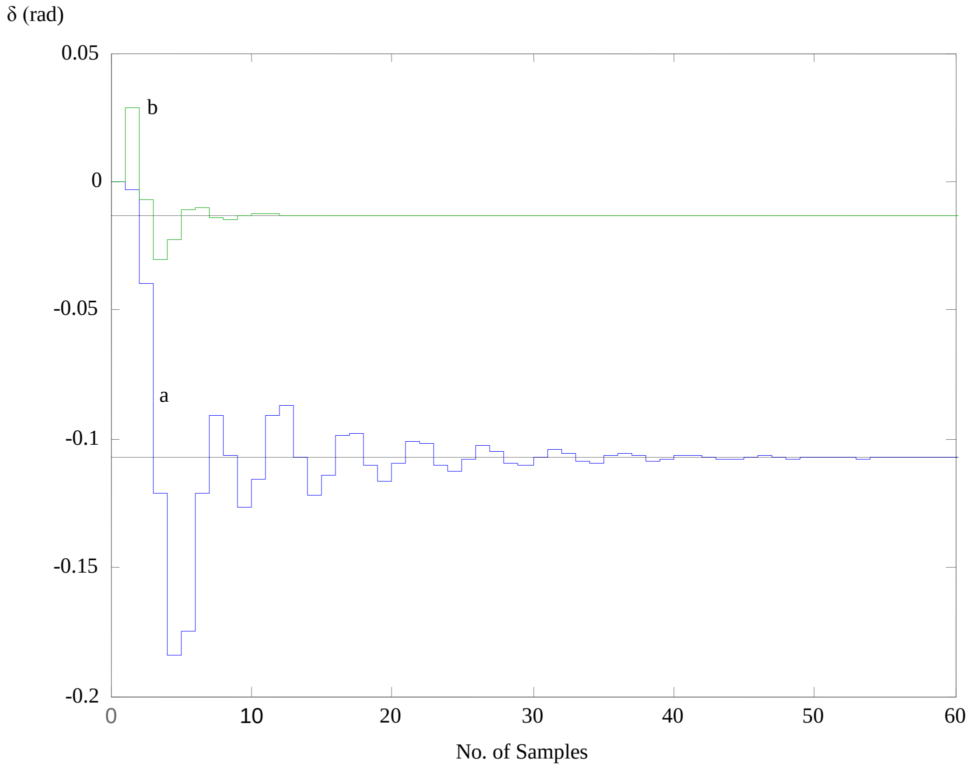
The magnitude of the eigenvalues of the discrete original open-loop and of the designed closed-loop power system model are shown in Table 1.

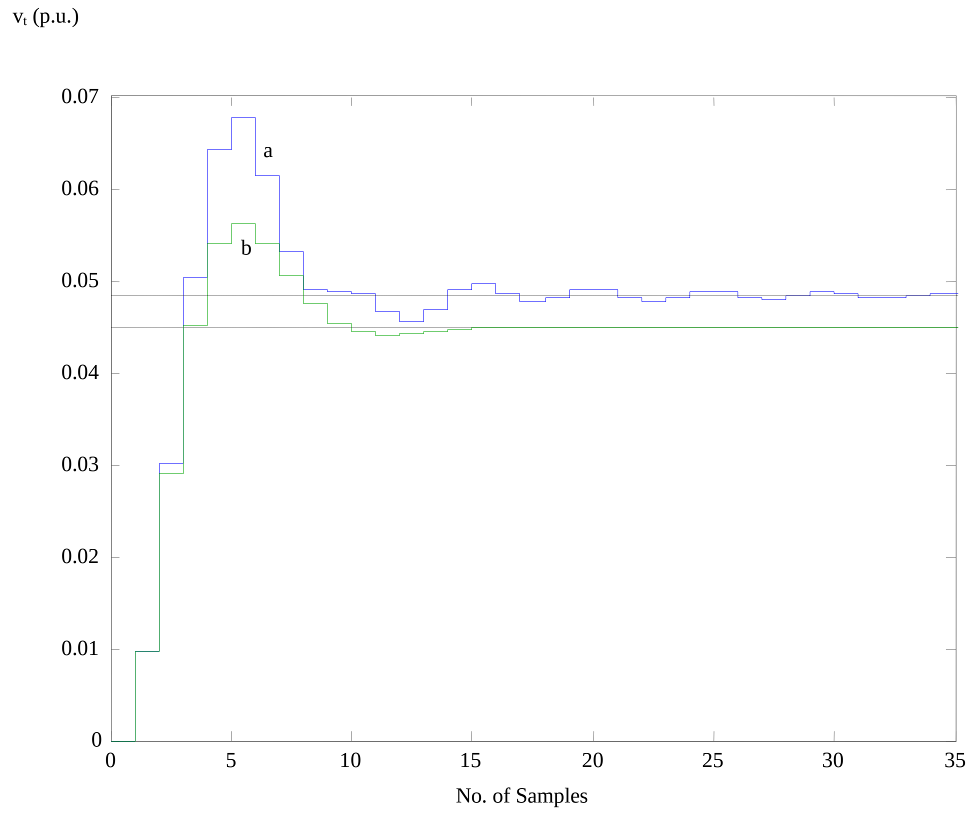
Table 1: Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.

Original open-loop power system model	$\lvert\lambda\rvert$	0.9087 0.9087 0.6985 0.6985 0.9608 0.4263 0.0211 0.0076 0.0005 0.0005
Designed closed-loop power system model	$\lvert\hat{\lambda}\rvert$	0.6358 0.6358 0.6657 0.6657 0.4395 0.9608 0.0211 0.0005 0.0005 0.0076

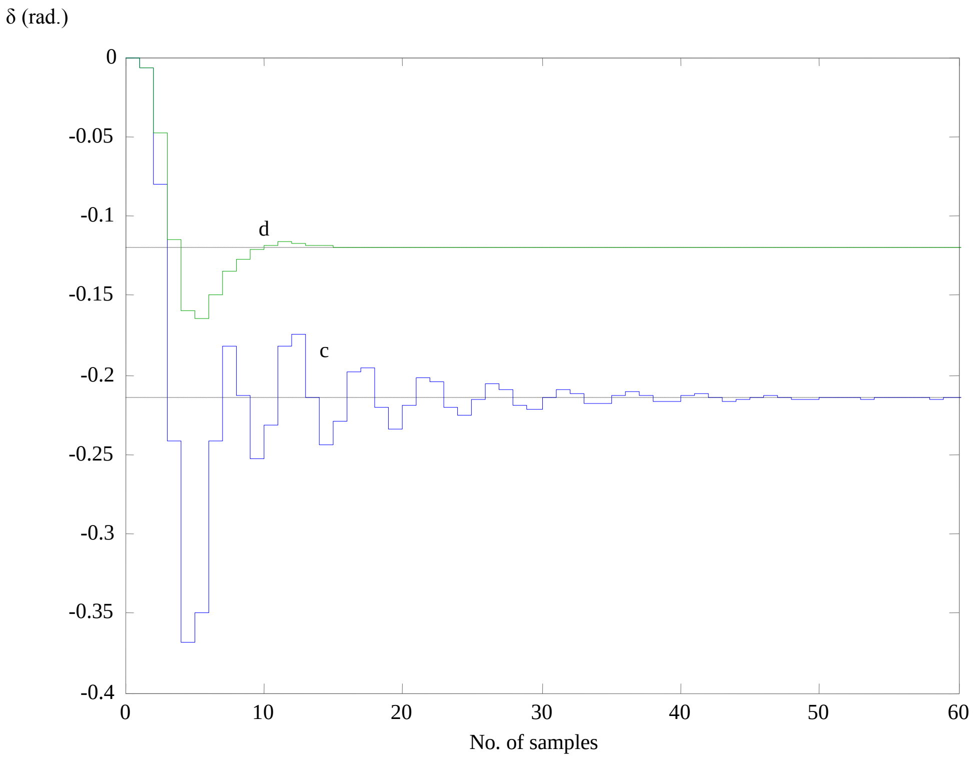
The solution results of the discrete system models (i.e. eigenvalues, eigenvectors, responses of system variables etc.) for zero initial conditions were obtained using a special software program (based on the theory of & 2 and runs in MATLAB program (environment).

The time responses of the variables δ, v_t, corresponding to the linear discrete open-loop and closed-loop system models, are shown in figure 3





v_t (p.u.)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
5
10
15
20
25
30
35
a
b
No. of Samples



δ (rad.)
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
0
10
20
30
40
50
60
d
c
No. of samples

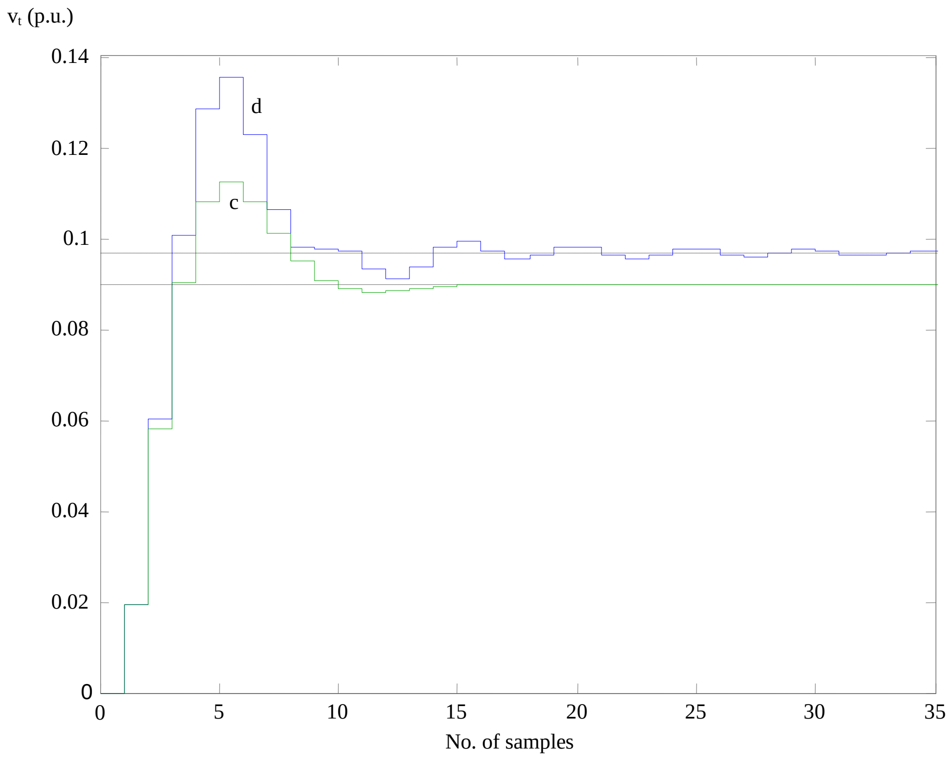


Fig.3. Responses of δ, v_t, outputs subject to unit step input change where (a) and (b) refer to discrete open loop and close loop to input change ΔVref.=0,05 p.u., and ΔTm=0.0 and (c) and (d) refer to discrete open loop and close loop to input change ΔVref.=0,10 p.u.,, and ΔTm=0.0.

It is to be noted that the multirate output sampling controllers are based on samplings with different period in every output variable of the system under control.

The results of figure 3 demonstrate clearly that the application of the OMCM leads to the design of a very efficient two-point multirate controller with distinct robustness characteristics, i.e. to a linear discrete closed-loop system model with superior dynamic stability characteristics. The motivation for designing and using OMCM controllers has to do with the fact that they many be implemented directly using digital computers, which malies them very useful in practical applications.

4. Conclusions

An efficient optimal multirate control method was presented in concise form. In turn based on it an appropriate software program was used to obtain a linear discrete open-loop system model from an original continuous linearized open-loop system model of a practical power system at a given operating point. In addition, through the same program, a robust OMCM controller was designed, which yielded a discrete closed-loop system model with superior dynamic stability characteristics by comparison to the associated ones of the discrete open-loop system model. The clear simplicity of the OMCM used makes an appropriate and reliable tool for the design of such implementable controllers.

Appendix A (Numerical values of system parameters and operating point)

Synchronous machine: 3-phase, 160 MVA, pf=0.094, xd=1.7, xq=1.6, $x_d' = 0.245\,p.u.$; $\tau_{do}' = 5.9$, H=5.4 s; ωR =314 rad. s-1.

Type-1 exciter: KA=50, KE= -0.17, SE = 0.95, KF = 0.04, KR = 1, Ko =1; τA = 0,05, τE = 0,95, τF = 1, τR = 0.05, τo = 10 p.u., τ1 = τ3 = 0.440, τ2 = τ4 =0,092 s.

External system: Re = 0.02, Xe = 0.40 p.u., (on 160 MVA base).

Operating point: Po=1, Qo=0.5, EFDo=2.5128, Eqo=0.9986, vto=1, Tmo=1 p.u.; δo=1.1966 rad.; K1=1.1330, K2=1.3295, K3=0.3072, K4=1.8235, K5=-0.0433, K6=0.4777.

Appendix B (Numerical values of matrices A, B and C of the original 10th-order system)

$$\mathbf{A} = \begin{bmatrix} -0.5517 & 0 & -0.3091 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1695 \\ -0.0410 & 0 & -0.0350 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 314.1593 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9.5540 & 0 & -0.8660 & -20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0.0421 & -0.0328 \\ -0.1962 & 10.8696 & -0.1672 & 0 & 0 & -10.8696 & 0 & 0 & 0 & 0 \\ -0.9386 & 51.9849 & -0.7999 & 0 & 0 & -41.1153 & -10.8696 & 0 & 0 & 0 \\ -0.9386 & 51.9849 & -0.7999 & 0 & 0 & -41.1143 & -10.8696 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & -1000 & -1000 & 0 & 0 & 1000 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0526 & -0.8211 \end{bmatrix}$$

$$\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0.0926 & 0 & 0 & 0 & 0.4428 & 2.1179 & 2.1179 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4777 & 0 & -0.0433 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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