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One Approach to Semi-structured Time Series Forecasting

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Abstract

By specific example of the semi-structured time series there are considered known fuzzy forecasting models which differ in rules of fuzzification and/or defuzzification. In the context of this study this paper presents a new approach to defuzzification of outputs of fuzzy time series on the base of applying the fuzzy set point-estimation method. As compared with some well-known defuzzification rules proposed method improves the statistical quality of semi-structured time series forecasting.

Keywords: *semi-structured time series, fuzzy set, fuzzy relationship, defuzzification.*

1. Introduction

Many companies for years accumulate business information, hoping that in the future it will help them with complex analytical research of development tendencies of interesting their processes. Really, in certain cases the set of imperceptible (*ex facte*) "crude" data can become a source for additional, much more valuable information – data on regularities, tendencies or interdependence between any data, which can not be obtained on the basis of one concrete record.

One of ways of researches of the hidden regularities is the intelligent analysis of the time series extracted from storages of historical data. The concept of the intelligent analysis of data (Data Mining) defines the problems of search of functional and logical regularities in stored information, supports to create the models and rules, which explain the found anomalies and/or predict development of studied processes. However, the last researches showed that Data Mining is still at an early stage of the own development. Many companies are interested in this technology, but only some of them actively introduce similar projects.

Composed of Oracle Database time series forecasting is supported by Oracle OLAP FORECAST command and Oracle Data mining option, which applying standard mechanisms of forecasting handle with net historical data, i.e. the data presented in the form of usual numbers (Marcos Campos, 2013). However, in most cases these data are nevertheless poorly structured (semi-structured) or even unstructured, i.e. such about which it is known only their belonging to a certain type. Therefore, for obtaining more adequate results nothing like to represent them by interval, for example, as $x \in [x_{\min}, x_{\max}]$, or, even better, in the form of statements like "*x=close to* 7", i.e. in the form of terms of the linguistic variables described by fuzzy sets.

In the present article on a concrete example of the semi-structured data set known there are considered the fuzzy models of the time series which specify by one's rules of fuzzification and/or defuzzification. As is known, validity of the obtained predicts depends on how well these rules allow to describe adequately semi-structured data of the time series by fuzzy sets and respectively to interpret the obtained results in the traditional numerical manner. Thereupon, it is offered the point-estimation method of fuzzy predicts, which in comparison with the considered known rules of defuzzification allows improving quality of time series forecasting.

2. Problem definition

In the absence thereof adequate mathematical model the intellectual analysis of the time series allows to detect accurate information about the researched phenomenon in the past. Therefore, the object of our research will be time series (TS):

$$\{\tilde{A}(k)\}\ (k=1\div t), \tag{1}$$

where $\tilde{A}(k)$ is semi-structured data or, in our representation, the fuzzy set characterized by tuple:

$$\left\{x_j^k / \mu(x_j^k)\right\},\ \mu(x_j^k) \to [0,1],\ j = 1 \div J\ . \tag{2}$$

Our target is development of a defuzzification method for outputs of known fuzzy models of time series, which would allow improving results of forecasting in comparison with existing techniques. For this purpose as a basic it was chosen the time series of variation of "Marginality of sales" indicator reflecting dynamics of company profitability from the beginning of the 1988th year on the end of the 2nd quarter of the 2009th year (Table 1). It is reasonable to consider that the average historical data presented in Table 1 under objective and subjective reasons are not absolutely validity and, therefore, it is expedient to consider them as semi-structured, i.e. in the fuzzy interpretation. In practice it allows to refer more adequately to dynamics of the time series and to its prediction respectively.

Table 1. Semi-structured time series of "Marginality of Sales" indicator

Year, quarter	Indicator	Year, quarter	Indicator	Year, quarter	Indicator	Year, quarter	Indicator
1988, I	15.024	1993, IV	7.596	1999, III	13.186	2005, II	12.902
1988, II	13.514	1994, I	8.381	1999, IV	15.211	2005, III	13.606
1988, III	11.637	1994, II	7.216	2000, I	17.030	2005, IV	14.401
1988, IV	11.691	1994, III	6.540	2000, II	16.012	2006, I	15.803
1989, I	12.651	1994, IV	6.239	2000, III	16.202	2006, II	15.704
1989, II	13.973	1995, I	5.487	2000, IV	15.320	2006, III	15.297
1989, III	12.777	1995, II	5.759	2001, I	16.450	2006, IV	14.497
1989, IV	11.005	1995, III	5.993	2001, II	14.298	2007, I	14.598
1990, I	12.137	1995, IV	7.475	2001, III	13.495	2007, II	15.701
1990, II	13.096	1996, I	7.349	2001, IV	13.920	2007, III	14.773
1990, III	13.183	1996, II	7.303	2002, I	15.045	2007, IV	13.313
1990, IV	13.441	1996, III	7.119	2002, II	13.862	2008, I	14.403
1991, I	13.748	1996, IV	6.994	2002, III	13.188	2008, II	14.708
1991, II	14.091	1997, I	6.958	2002, IV	13.183	2008, III	16.432
1991, III	14.123	1997, II	7.596	2003, I	12.611	2008, IV	15.825
1991, IV	16.186	1997, III	8.088	2003, II	12.734	2009, I	14.911
1992, I	14.633	1997, IV	7.556	2003, III	12.937	2009, II	13.951
1992, II	12.848	1998, I	7.315	2003, IV	12.870	2009, III	14.197
1992, III	13.379	1998, II	7.893	2004, I	13.406	2009, IV	13.421
1992, IV	13.987	1998, III	8.859	2004, II	12.794	2010, I	12.619
1993, I	13.336	1998, IV	8.839	2004, III	13.100	2010, II	11.736
1993, II	13.071	1999, I	8.015	2004, IV	13.600		
1993, III	12.113	1999, II	12.096	2005, I	13.096		

Source: call reports of IT-Company Sinam Ltd (Baku, Azerbaijan), www.sinam.net

3. Fuzzy Time Series Forecasting

The problem of fuzzy time series (FTS) forecasting is actively discussed over a period of past two decades. Further exploration of this aspect may be found in many sources (e.g. Song and Chissom, 1993, 1994; Chen, 1996, 2002; Cheng et al., 2006). Theirs approaches to FTS forecasting provide consistent implementation of the following procedures:

1. Universe definition and its division into equal intervals;
2. Fuzzification of historical data;
3. Identifying the internal fuzzy relations and their localization in groups;
4. Definition of fuzzy predicts and its defuzzification.

Apply this procedure to the prediction of the time series, which characterizes the dynamics of change in the values of the "Marginality of Sales" indicator for the period the beginning of the 1988-th year on the end of the 2-nd quarter of the 2009th year (see Figure 1).

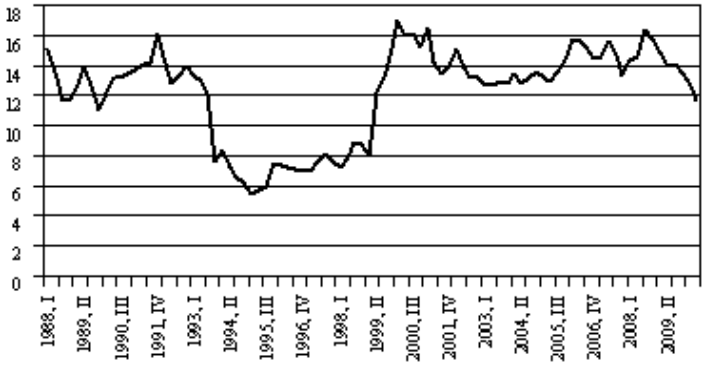

Figure 1. Time series of "Marginality of Sales" indicator

3.1. Fuzzy time series forecasting based on fuzzification of its data by given fuzzy criteria

Step 1. *Universe definition and its division into equal intervals*. One of ways of finding of universe as a covering of data range of the time series is described by Song and Chissom (1994), where the universe U is defined as $[D_{min}-D_1; D_{max}+D_2]$. Here D_{min} and D_{max} are respectively the minimum and maximum value of data of time series; D_1 and D_2 are the positive numbers chosen, as a rule, by the user. In our case: D_{min}=8.169 and D_{max}=17.031. Then choosing positive numbers as D_1=0.169 and D_2=0.069 we will obtain correspondent universe U=[8.0, 17.1], which one can divide into follow seven equal intervals: u_1=[8.0, 9.3], u_2=[9.3, 10.6], u_3=[10.6, 11.9], u_4=[11.9, 13.2], u_5=[13.2, 14.5], u_6=[14.5, 15.8], u_7=[15.8, 17.1].

Step 2. *Fuzzification of historical data.* Suppose that $\tilde{A}_1$, $\tilde{A}_2$, …, $\tilde{A}_k$ are fuzzy sets (assessment criteria) describing the terms of the linguistic variable "Marginality of Sales". Then on U these sets are generally defined as:

$$\tilde{A}_1=\mu_{11}/u_1+\mu_{12}/u_2+\ldots+\mu_{1m}/u_m,$$
$$\tilde{A}_2=\mu_{21}/u_1+\mu_{22}/u_2+\ldots+\mu_{2m}/u_m,$$
$$\ldots$$
$$\tilde{A}_k=\mu_{k1}/u_1+\mu_{k2}/u_2+\ldots+\mu_{km}/u_m,$$

where $\mu_{ij} \in [0,1]$ ($i=1 \div k$, $j=1 \div m$) denotes the grade of membership of crisp intervale u_j to fuzzy set $\tilde{A}_i$. Here, the most important thing is rightly to choose the parameters of membership functions that restore the fuzzy sets to describe the possible values (terms) of the linguistic variable "Marginality of Sales".

For a considered case as possible values of the linguistic variable "Marginality of Sales" choose the following terms:

LOW: $\tilde{A}_1=1/u_1+0.5/u_2+0/u_3+0/u_4+0/u_5+0/u_6+0/u_7$,

NOT HIGH: $\tilde{A}_2=0.5/u_1+1/u_2+0.5/u_3+0/u_4+0/u_5+0/u_6+0/u_7$,

HIGH: $\tilde{A}_3=0/u_1+0.5/u_2+1/u_3+0.5/u_4+0/u_5+0/u_6+0/u_7$,

MORE THEN HIGH: $\tilde{A}_4=0/u_1+0/u_2+0.5/u_3+1/u_4+0.5/u_5+0/u_6+0/u_7$,

CONSIDERABLY HIGH: $\tilde{A}_5=0/u_1+0/u_2+0/u_3+0.5/u_4+1/u_5+0.5/u_6+0/u_7$,

VERY HIGH: $\tilde{A}_6=0/u_1+0/u_2+0/u_3+0/u_4+0.5/u_5+1/u_6+0.5/u_7$,

TOO HIGH: $\tilde{A}_7=0/u_1+0/u_2+0/u_3+0/u_4+0/u_5+0.5/u_6+1/u_7$.

Fuzzy inputs of the desired model obtained by fuzzification time series data (see Figure 1) are presented in Table 2. Here, dominant factor is the maximum degree of belonging to the fuzzy set of the interval u_k ($k=1 \div 7$) corresponding to the current predicate – crisp historical data.

Table 2. Fuzzy inputs of FTS model

Year, quarter	Indicator	Interval	Fuzzy analog	Year, quarter	Indicator	Interval	Fuzzy analog	Year, quarter	Indicator	Interval	Fuzzy analog
1988, I	15.024	u_6	$\tilde{A}_6$	1995, III	8.712	u_1	$\tilde{A}_1$	2003, I	12.611	u_4	$\tilde{A}_4$
1988, II	13.514	u_5	$\tilde{A}_5$	1995, IV	11.012	u_3	$\tilde{A}_3$	2003, II	12.734	u_4	$\tilde{A}_4$
1988, III	11.637	u_3	$\tilde{A}_3$	1996, I	11.044	u_3	$\tilde{A}_3$	2003, III	12.937	u_5	$\tilde{A}_5$
1988, IV	11.691	u_3	$\tilde{A}_3$	1996, II	10.701	u_3	$\tilde{A}_3$	2003, IV	12.870	u_4	$\tilde{A}_4$
1989, I	12.651	u_4	$\tilde{A}_4$	1996, III	10.685	u_3	$\tilde{A}_3$	2004, I	13.406	u_4	$\tilde{A}_4$
1989, II	13.973	u_5	$\tilde{A}_5$	1996, IV	10.332	u_2	$\tilde{A}_2$	2004, II	12.794	u_5	$\tilde{A}_5$
1989, III	12.777	u_4	$\tilde{A}_4$	1997, I	10.911	u_3	$\tilde{A}_3$	2004, III	13.100	u_5	$\tilde{A}_5$
1989, IV	11.005	u_3	$\tilde{A}_3$	1997, II	12.111	u_4	$\tilde{A}_4$	2004, IV	13.600	u_7	$\tilde{A}_7$
1990, I	12.137	u_4	$\tilde{A}_4$	1997, III	12.183	u_4	$\tilde{A}_4$	2005, I	13.096	u_6	$\tilde{A}_6$
1990, II	13.096	u_4	$\tilde{A}_4$	1997, IV	12.085	u_4	$\tilde{A}_4$	2005, II	12.902	u_6	$\tilde{A}_6$
1990, III	13.183	u_4	$\tilde{A}_4$	1998, I	11.684	u_3	$\tilde{A}_3$	2005, III	13.606	u_5	$\tilde{A}_5$
1990, IV	13.441	u_5	$\tilde{A}_5$	1998, II	12.158	u_4	$\tilde{A}_4$	2005, IV	14.401	u_5	$\tilde{A}_5$
1991, I	13.748	u_5	$\tilde{A}_5$	1998, III	13.455	u_5	$\tilde{A}_5$	2006, I	15.803	u_7	$\tilde{A}_7$
1991, II	14.091	u_5	$\tilde{A}_5$	1998, IV	13.787	u_5	$\tilde{A}_5$	2006, II	15.704	u_6	$\tilde{A}_6$
1991, III	14.123	u_5	$\tilde{A}_5$	1999, I	12.570	u_4	$\tilde{A}_4$	2006, III	15.297	u_6	$\tilde{A}_6$
1991, IV	16.186	u_7	$\tilde{A}_7$	1999, II	12.096	u_4	$\tilde{A}_4$	2006, IV	14.497	u_5	$\tilde{A}_5$
1992, I	14.633	u_6	$\tilde{A}_6$	1999, III	13.186	u_4	$\tilde{A}_4$	2007, I	14.598	u_6	$\tilde{A}_6$
1992, II	12.848	u_4	$\tilde{A}_4$	1999, IV	15.211	u_6	$\tilde{A}_6$	2007, II	15.701	u_6	$\tilde{A}_6$
1992, III	13.379	u_5	$\tilde{A}_5$	2000, I	17.030	u_7	$\tilde{A}_7$	2007, III	14.773	u_6	$\tilde{A}_6$
1992, IV	13.987	u_5	$\tilde{A}_5$	2000, II	16.012	u_7	$\tilde{A}_7$	2007, IV	13.313	u_5	$\tilde{A}_5$
1993, I	13.336	u_5	$\tilde{A}_5$	2000, III	16.202	u_7	$\tilde{A}_7$	2008, I	14.403	u_5	$\tilde{A}_5$
1993, II	13.071	u_4	$\tilde{A}_4$	2000, IV	15.320	u_6	$\tilde{A}_6$	2008, II	14.708	u_6	$\tilde{A}_6$
1993, III	12.113	u_4	$\tilde{A}_4$	2001, I	16.450	u_7	$\tilde{A}_7$	2008, III	16.432	u_7	$\tilde{A}_7$
1993, IV	11.988	u_4	$\tilde{A}_4$	2001, II	14.298	u_5	$\tilde{A}_5$	2008, IV	15.825	u_7	$\tilde{A}_7$

1994, I	12.284	u_4	$\tilde{A}_4$	2001, III	13.495	u_4	$\tilde{A}_4$	2009, I	14.911	u_6	$\tilde{A}_6$
1994, II	11.761	u_3	$\tilde{A}_3$	2001, IV	13.920	u_4	$\tilde{A}_4$	2009, II	13.951	u_5	$\tilde{A}_5$
1994, III	9.620	u_2	$\tilde{A}_2$	2002, I	15.045	u_4	$\tilde{A}_4$	2009, III	14.197	u_5	$\tilde{A}_5$
1994, IV	9.595	u_2	$\tilde{A}_2$	2002, II	13.862	u_4	$\tilde{A}_4$	2009, IV	13.421	u_5	$\tilde{A}_5$
1995, I	8.169	u_1	$\tilde{A}_1$	2002, III	13.188	u_4	$\tilde{A}_4$	2010, I	12.619	u_4	$\tilde{A}_4$
1995, II	8.837	u_1	$\tilde{A}_1$	2002, IV	13.183	u_5	$\tilde{A}_5$	2010, II	11.736	u_3	$\tilde{A}_3$

Step 3. *Identifying the internal fuzzy relations and their localization in groups*. Fuzzy logic relationships are identified from the fuzzified historical data, i.e. as relationships between fuzzy descriptions of historical data. Obtained from Table 2 these relationships are shown in Table 3.

Table 3. Set of fuzzy relationships

$\tilde{A}_1 \to \tilde{A}_1$	$\tilde{A}_2 \to \tilde{A}_3$	$\tilde{A}_4 \to \tilde{A}_5$	$\tilde{A}_5 \to \tilde{A}_3$	$\tilde{A}_5 \to \tilde{A}_6$	$\tilde{A}_6 \to \tilde{A}_6$
$\tilde{A}_1 \to \tilde{A}_3$	$\tilde{A}_3 \to \tilde{A}_3$	$\tilde{A}_4 \to \tilde{A}_3$	$\tilde{A}_5 \to \tilde{A}_4$	$\tilde{A}_6 \to \tilde{A}_5$	$\tilde{A}_7 \to \tilde{A}_6$
$\tilde{A}_2 \to \tilde{A}_2$	$\tilde{A}_3 \to \tilde{A}_4$	$\tilde{A}_4 \to \tilde{A}_4$	$\tilde{A}_5 \to \tilde{A}_5$	$\tilde{A}_6 \to \tilde{A}_4$	$\tilde{A}_7 \to \tilde{A}_7$
$\tilde{A}_2 \to \tilde{A}_1$	$\tilde{A}_3 \to \tilde{A}_2$	$\tilde{A}_4 \to \tilde{A}_6$	$\tilde{A}_5 \to \tilde{A}_7$	$\tilde{A}_6 \to \tilde{A}_7$	$\tilde{A}_7 \to \tilde{A}_5$

Presented in the Table 3 fuzzy relationships are grouped by following principle: if the time series variable $F(t\text{-}1)$ is fuzzified as $\tilde{A}_i$ and $F(t)$ is fuzzified as $\tilde{A}_j$, then $\tilde{A}_i$ is related to $\tilde{A}_j$ ($\tilde{A}_i \to \tilde{A}_j$). If $\tilde{A}_i$ is related to other fuzzy set too, for example, to $\tilde{A}_k$, then relative to $\tilde{A}_i$ it is formed local group of the first order: $\tilde{A}_i \to \tilde{A}_j, \tilde{A}_k$. From the data in Table 3, we get the following first order relationships (Table 4).

Table 4. First order fuzzy relationships groups

Group	Relationships	Group	Relationships	Group	Relationships	Group	Relationships
Group 1:	$\tilde{A}_1 \to \tilde{A}_1, \tilde{A}_3$	Group 3:	$\tilde{A}_3 \to \tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	Group 5:	$\tilde{A}_5 \to \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	Group 7:	$\tilde{A}_7 \to \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$
Group 2:	$\tilde{A}_2 \to \tilde{A}_1, \tilde{A}_2, \tilde{A}_3$	Group 4:	$\tilde{A}_4 \to \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	Group 6:	$\tilde{A}_6 \to \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$		

Second order fuzzy relationships between semi-structured data, described by fuzzy assessment criterion, are grouped in Table 5.

Table 5. Second order fuzzy relationships groups

Group	Relationships	Group	Relationships	Group	Relationships	Group	Relationships
Group 1:	$\tilde{A}_1, \tilde{A}_1 \to \tilde{A}_1,$ $\tilde{A}_1, \tilde{A}_1 \to \tilde{A}_3$	Group 6:	$\tilde{A}_3, \tilde{A}_4 \to \tilde{A}_4,$ $\tilde{A}_3, \tilde{A}_4 \to \tilde{A}_5$	Group 11:	$\tilde{A}_5, \tilde{A}_4 \to \tilde{A}_3,$ $\tilde{A}_5, \tilde{A}_4 \to \tilde{A}_4,$	Group 17:	$\tilde{A}_6, \tilde{A}_4 \to \tilde{A}_5$
Group 2:	$\tilde{A}_1, \tilde{A}_3 \to \tilde{A}_3$	Group 6:	$\tilde{A}_3, \tilde{A}_2 \to \tilde{A}_2,$ $\tilde{A}_3, \tilde{A}_2 \to \tilde{A}_3$	Group 12:	$\tilde{A}_5, \tilde{A}_5 \to \tilde{A}_5,$ $\tilde{A}_5, \tilde{A}_5 \to \tilde{A}_7,$ $\tilde{A}_5, \tilde{A}_5 \to \tilde{A}_4,$ $\tilde{A}_5, \tilde{A}_5 \to \tilde{A}_6$	Group 18:	$\tilde{A}_6, \tilde{A}_7 \to \tilde{A}_7,$ $\tilde{A}_6, \tilde{A}_7 \to \tilde{A}_5$
Group 3:	$\tilde{A}_2, \tilde{A}_2 \to \tilde{A}_1$	Group 7:	$\tilde{A}_4, \tilde{A}_5 \to \tilde{A}_4,$ $\tilde{A}_4, \tilde{A}_5 \to \tilde{A}_5$	Group 13:	$\tilde{A}_5, \tilde{A}_3 \to \tilde{A}_3,$	Group 19:	$\tilde{A}_6, \tilde{A}_6 \to \tilde{A}_5,$ $\tilde{A}_6, \tilde{A}_6 \to \tilde{A}_6$
Group 4:	$\tilde{A}_2, \tilde{A}_1 \to \tilde{A}_1$	Group 8:	$\tilde{A}_4, \tilde{A}_3 \to \tilde{A}_4,$ $\tilde{A}_4, \tilde{A}_3 \to \tilde{A}_2$	Group 14:	$\tilde{A}_5, \tilde{A}_7 \to \tilde{A}_6$	Group 20:	$\tilde{A}_7, \tilde{A}_5 \to \tilde{A}_4$
Group 5:	$\tilde{A}_2, \tilde{A}_3 \to \tilde{A}_4$	Group 9:	$\tilde{A}_4, \tilde{A}_4 \to \tilde{A}_4,$ $\tilde{A}_4, \tilde{A}_4 \to \tilde{A}_3,$ $\tilde{A}_4, \tilde{A}_4 \to \tilde{A}_6,$ $\tilde{A}_4, \tilde{A}_4 \to \tilde{A}_5$	Group 15:	$\tilde{A}_5, \tilde{A}_6 \to \tilde{A}_6$	Group 21:	$\tilde{A}_7, \tilde{A}_7 \to \tilde{A}_7,$ $\tilde{A}_7, \tilde{A}_7 \to \tilde{A}_6$
Group 6:	$\tilde{A}_3, \tilde{A}_3 \to \tilde{A}_4,$ $\tilde{A}_3, \tilde{A}_3 \to \tilde{A}_3,$ $\tilde{A}_3, \tilde{A}_3 \to \tilde{A}_2$	Group 10:	$\tilde{A}_4, \tilde{A}_6 \to \tilde{A}_7$	Group 16:	$\tilde{A}_6, \tilde{A}_5 \to \tilde{A}_3,$ $\tilde{A}_6, \tilde{A}_5 \to \tilde{A}_5,$ $\tilde{A}_6, \tilde{A}_5 \to \tilde{A}_6$	Group 22:	$\tilde{A}_7, \tilde{A}_6 \to \tilde{A}_4,$ $\tilde{A}_7, \tilde{A}_6 \to \tilde{A}_7,$ $\tilde{A}_7, \tilde{A}_6 \to \tilde{A}_6,$ $\tilde{A}_7, \tilde{A}_6 \to \tilde{A}_5$

Step 4. *definition of fuzzy predicts and its defuzzification*. To determine the fuzzy predicts and their defuzzification there are used extensively two models:

- Chen's model (1996) based on the use of simple arithmetic operations;
- Analytical Song-Chissom model (1993).

According to Chen's model if at the current moment (i-th year) semi-structured data x_i is described as the fuzzy set $\tilde{A}_j$, which composed of group is related to only one fuzzy set $\tilde{A}_k$, ($\tilde{A}_j \to \tilde{A}_k$), then the predict for the next (i+1)-th year will be the fuzzy set $\tilde{A}_k$,. Otherwise, i.e. when there is a group of multiform relationships, for example $\tilde{A}_j \to \tilde{A}_k{,}1, \tilde{A}_k{,}2, \ldots, \tilde{A}_k{,}\text{p}$, then just this bunch in the complex will be fuzzy prediction for the (i+1)-th year.

Song-Chissom approach assumes finding of prediction from recurrence equation $\tilde{A}_t=\tilde{A}_{t-1}\circ R$, where «∘» denotes *max-min* operator; R is a fuzzy relation determined as $R=\bigcup_{i=1}^{k} R_i$, where $R_i = A_s^T \times A_q$ is defined for all relationships $\tilde{A}_s\rightarrow\tilde{A}_q$, and "∪" denotes the union operator.

3.1.1. Chen's model

Assume that $\tilde{A}_i$ is the fuzzy analog of semi-structured data x_{t-1} for (t-1)-th year. Then defuzzified prediction for next year is determined as consistent with the following rules:

1. if there is a unique relationship in the localized around $\tilde{A}_i$ group, for example $\tilde{A}_i\rightarrow\tilde{A}_k$, where $\tilde{A}_k$ includes interval u_k with the greatest degree of membership, then the middle point of the u_k will be the predicted value;
2. if $\tilde{A}_i$ has no any relationships, i.e. $\tilde{A}_i\rightarrow\varnothing$, and $\tilde{A}_i$ with the highest degree of membership includes u_i, then the middle point of the u_k will be the predicted value;
3. if there is a multi-valued relationship in the localized around $\tilde{A}_i$ group, for example $\tilde{A}_i\rightarrow\tilde{A}_1,\tilde{A}_2,\ldots,\tilde{A}_n$, where $\tilde{A}_1,\tilde{A}_2,\ldots,\tilde{A}_n$ with highest degree of membership include appropriate intervals $u_1,u_2,\ldots,u_n$, then predicted value is calculated as $x_t=(m_1+m_2+\ldots+m_n)/n$, where $m_1,m_2,\ldots,m_n$ are the midpoints respectively of $u_1,u_2,\ldots,u_n$.

Applying these rules to defuzzification of outputs of FTS model (fuzzy predictions) (see Table 2) under fuzzy relations of the 1st and 2nd order, we get the following prediction results (Tables 6, 7).

Table 6. Defuzzified outputs of Chen's model under fuzzy relations of the 1st order

Year, quarter	Indicator	Prediction	Fuzzy relationships group	Middle point of intervals
1988, I	15.024		$\tilde{A}_6\rightarrow\tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
1988, II	13.514	14.500	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1988, III	11.637	13.850	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1988, IV	11.691	11.250	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1989, I	12.651	11.250	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1989, II	13.973	13.200	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1989, III	12.777	13.850	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1989, IV	11.005	13.200	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1990, I	12.137	11.250	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1990, II	13.096	13.200	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1990, III	13.183	13.200	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1990, IV	13.441	13.200	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1991, I	13.748	13.850	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1991, II	14.091	13.850	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1991, III	14.123	13.850	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1991, IV	16.186	13.850	$\tilde{A}_7\rightarrow\tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
1992, I	14.633	15.150	$\tilde{A}_6\rightarrow\tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
1992, II	12.848	14.500	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1992, III	13.379	13.200	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1992, IV	13.987	13.850	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1993, I	13.336	13.850	$\tilde{A}_5\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1993, II	13.071	13.850	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1993, III	12.113	13.200	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1993, IV	11.988	13.200	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1994, I	12.284	13.200	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1994, II	11.761	13.200	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1994, III	9.620	11.250	$\tilde{A}_2\rightarrow\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$	8.650, 9.950, 11.250
1994, IV	9.595	9.950	$\tilde{A}_2\rightarrow\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$	8.650, 9.950, 11.250
1995, I	8.169	9.950	$\tilde{A}_1\rightarrow\tilde{A}_1, \tilde{A}_3$	8.650, 11.250
1995, II	8.837	9.950	$\tilde{A}_1\rightarrow\tilde{A}_1, \tilde{A}_3$	8.650, 11.250
1995, III	8.712	9.950	$\tilde{A}_1\rightarrow\tilde{A}_1, \tilde{A}_3$	8.650, 11.250
1995, IV	11.012	9.950	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1996, I	11.044	11.250	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1996, II	10.701	11.250	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1996, III	10.685	11.250	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1996, IV	10.332	11.250	$\tilde{A}_2\rightarrow\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$	8.650, 9.950, 11.250
1997, I	10.911	9.950	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1997, II	12.111	11.250	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1997, III	12.183	13.200	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1997, IV	12.085	13.200	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1998, I	11.684	13.200	$\tilde{A}_3\rightarrow\tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550
1998, II	12.158	11.250	$\tilde{A}_4\rightarrow\tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150

Year, quarter	Indicator	Prediction	Fuzzy relationships group	Middle point of intervals
1998, III	13.455	13.200	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1998, IV	13.787	13.850	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
1999, I	12.570	13.850	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1999, II	12.096	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1999, III	13.186	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
1999, IV	15.211	13.200	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2000, I	17.030	14.500	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2000, II	16.012	15.150	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2000, III	16.202	15.150	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2000, IV	15.320	15.150	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2001, I	16.450	14.500	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2001, II	14.298	15.150	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2001, III	13.495	13.850	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2001, IV	13.920	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2002, I	15.045	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2002, II	13.862	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2002, III	13.188	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2002, IV	13.183	13.200	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2003, I	12.611	13.850	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2003, II	12.734	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2003, III	12.937	13.200	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2003, IV	12.870	13.850	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2004, I	13.406	13.200	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2004, II	12.794	13.200	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2004, III	13.100	13.850	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2004, IV	13.600	13.850	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2005, I	13.096	15.150	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2005, II	12.902	14.500	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2005, III	13.606	14.500	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2005, IV	14.401	13.850	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2006, I	15.803	13.850	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2006, II	15.704	15.150	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2006, III	15.297	14.500	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2006, IV	14.497	14.500	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2007, I	14.598	13.850	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2007, II	15.701	14.500	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2007, III	14.773	14.500	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2007, IV	13.313	14.500	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2008, I	14.403	13.850	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2008, II	14.708	13.850	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2008, III	16.432	14.500	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2008, IV	15.825	15.150	$\tilde{A}_7 \rightarrow \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	13.850, 15.150, 16.450
2009, I	14.911	15.150	$\tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	12.550, 13.850, 15.150, 16.450
2009, II	13.951	14.500	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2009, III	14.197	13.850	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2009, IV	13.421	13.850	$\tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6, \tilde{A}_7$	11.250, 12.550, 13.850, 15.150, 16.450
2010, I	12.619	13.850	$\tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2010, II	11.736	13.200	$\tilde{A}_3 \rightarrow \tilde{A}_2, \tilde{A}_3, \tilde{A}_4$	9.950, 11.250, 12.550

Table 7. Defuzzified outputs of Chen's model under fuzzy relations of the 2nd order

Year, quarter	Indicator	Prediction	Fuzzy relationships group	Middle point of intervals
1988, I	15.024			
1988, II	13.514		$\tilde{A}_6, \tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_5, \tilde{A}_6$	11.250, 13.850, 15.150
1988, III	11.637	13.417	$\tilde{A}_5, \tilde{A}_3 \rightarrow \tilde{A}_3$	11.250
1988, IV	11.691	11.250	$\tilde{A}_3, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_2$	12.550, 11.250, 9.950
1989, I	12.651	11.250	$\tilde{A}_3, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
1989, II	13.973	13.200	$\tilde{A}_4, \tilde{A}_5 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
1989, III	12.777	13.200	$\tilde{A}_5, \tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4$	11.250, 12.550
1989, IV	11.005	11.900	$\tilde{A}_4, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_2$	9.950, 12.550
1990, I	12.137	11.250	$\tilde{A}_3, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850

Year, quarter	Indicator	Prediction	Fuzzy relationships group	Middle point of intervals
1990, II	13.096	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1990, III	13.183	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1990, IV	13.441	13.200	$\tilde{A}_4, \tilde{A}_5 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
1991, I	13.748	13.200	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	12.550, 13.850, 15.150, 16.450
1991, II	14.091	14.500	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	12.550, 13.850, 15.150, 16.450
1991, III	14.123	14.500	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	12.550, 13.850, 15.150, 16.450
1991, IV	16.186	14.500	$\tilde{A}_5, \tilde{A}_7 \rightarrow \tilde{A}_6$	15.150
1992, I	14.633	15.150	$\tilde{A}_7, \tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_7, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1992, II	12.848	14.500	$\tilde{A}_6, \tilde{A}_4 \rightarrow \tilde{A}_5$	13.850
1992, III	13.379	13.850	$\tilde{A}_4, \tilde{A}_5 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
1992, IV	13.987	13.200	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	12.550, 13.850, 15.150, 16.450
1993, I	13.336	14.500	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	12.550, 13.850, 15.150, 16.450
1993, II	13.071	14.500	$\tilde{A}_5, \tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4$	11.250, 12.550
1993, III	12.113	11.900	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1993, IV	11.988	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1994, I	12.284	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1994, II	11.761	13.200	$\tilde{A}_4, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_2$	9.950, 12.550
1994, III	9.620	13.200	$\tilde{A}_3, \tilde{A}_2 \rightarrow \tilde{A}_2, \tilde{A}_3$	9.950, 11.250
1994, IV	9.595	10.600	$\tilde{A}_2, \tilde{A}_2 \rightarrow \tilde{A}_1$	8.650
1995, I	8.169	8.650	$\tilde{A}_2, \tilde{A}_1 \rightarrow \tilde{A}_1$	8.650
1995, II	8.837	8.650	$\tilde{A}_1, \tilde{A}_1 \rightarrow \tilde{A}_1, \tilde{A}_3$	8.650, 11.250
1995, III	8.712	9.950	$\tilde{A}_1, \tilde{A}_1 \rightarrow \tilde{A}_1, \tilde{A}_3$	8.650, 11.250
1995, IV	11.012	9.950	$\tilde{A}_1, \tilde{A}_3 \rightarrow \tilde{A}_3$	11.250
1996, I	11.044	11.250	$\tilde{A}_3, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_2$	12.550, 11.250, 9.950
1996, II	10.701	11.250	$\tilde{A}_3, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_2$	12.550, 11.250, 9.950
1996, III	10.685	11.250	$\tilde{A}_3, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_2$	12.550, 11.250, 9.950
1996, IV	10.332	11.250	$\tilde{A}_3, \tilde{A}_2 \rightarrow \tilde{A}_2, \tilde{A}_3$	9.950, 11.250
1997, I	10.911	10.600	$\tilde{A}_2, \tilde{A}_3 \rightarrow \tilde{A}_4$	12.550
1997, II	12.111	12.550	$\tilde{A}_3, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
1997, III	12.183	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1997, IV	12.085	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1998, I	11.684	13.200	$\tilde{A}_4, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_2$	9.950, 12.550
1998, II	12.158	11.250	$\tilde{A}_3, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
1998, III	13.455	13.200	$\tilde{A}_4, \tilde{A}_5 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
1998, IV	13.787	13.200	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	12.550, 13.850, 15.150, 16.450
1999, I	12.570	14.500	$\tilde{A}_5, \tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4$	11.250, 12.550
1999, II	12.096	11.900	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1999, III	13.186	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
1999, IV	15.211	13.200	$\tilde{A}_4, \tilde{A}_6 \rightarrow \tilde{A}_7$	16.450
2000, I	17.030	16.450	$\tilde{A}_6, \tilde{A}_7 \rightarrow \tilde{A}_7, \tilde{A}_5$	13.850, 16.450
2000, II	16.012	15.150	$\tilde{A}_7, \tilde{A}_7 \rightarrow \tilde{A}_7, \tilde{A}_6$	15.150, 16.450
2000, III	16.202	15.800	$\tilde{A}_7, \tilde{A}_7 \rightarrow \tilde{A}_7, \tilde{A}_6$	15.150, 16.450
2000, IV	15.320	15.800	$\tilde{A}_7, \tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_7, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2001, I	16.450	14.500	$\tilde{A}_6, \tilde{A}_7 \rightarrow \tilde{A}_7, \tilde{A}_5$	13.850, 16.450
2001, II	14.298	15.150	$\tilde{A}_7, \tilde{A}_5 \rightarrow \tilde{A}_4$	12.550
2001, III	13.495	12.550	$\tilde{A}_5, \tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4$	11.250, 12.550
2001, IV	13.920	11.900	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2002, I	15.045	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2002, II	13.862	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2002, III	13.188	13.200	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2002, IV	13.183	13.200	$\tilde{A}_4, \tilde{A}_5 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
2003, I	12.611	13.200	$\tilde{A}_5, \tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4$	11.250, 12.550
2003, II	12.734	11.900	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2003, III	12.937	13.200	$\tilde{A}_4, \tilde{A}_5 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
2003, IV	12.870	13.200	$\tilde{A}_5, \tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4$	11.250, 12.550
2004, I	13.406	11.900	$\tilde{A}_4, \tilde{A}_4 \rightarrow \tilde{A}_4, \tilde{A}_3, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2004, II	12.794	13.200	$\tilde{A}_4, \tilde{A}_5 \rightarrow \tilde{A}_4, \tilde{A}_5$	12.550, 13.850
2004, III	13.100	13.200	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2004, IV	13.600	14.500	$\tilde{A}_5, \tilde{A}_7 \rightarrow \tilde{A}_6$	15.150

Year, quarter	Indicator	Prediction	Fuzzy relationships group	Middle point of intervals
2005, I	13.096	15.150	$\tilde{A}_7, \tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_7, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2005, II	12.902	14.500	$\tilde{A}_6, \tilde{A}_6 \rightarrow \tilde{A}_5,$	13.850
2005, III	13.606	13.850	$\tilde{A}_6, \tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_5, \tilde{A}_6$	11.250, 13.850, 15.150
2005, IV	14.401	13.417	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2006, I	15.803	14.500	$\tilde{A}_5, \tilde{A}_7 \rightarrow \tilde{A}_6$	15.150
2006, II	15.704	15.150	$\tilde{A}_7, \tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_7, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2006, III	15.297	14.500	$\tilde{A}_6, \tilde{A}_6 \rightarrow \tilde{A}_5, \tilde{A}_6$	13.850, 15.150
2006, IV	14.497	14.500	$\tilde{A}_6, \tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_5, \tilde{A}_6$	11.250, 13.850, 15.150
2007, I	14.598	13.417	$\tilde{A}_5, \tilde{A}_6 \rightarrow \tilde{A}_6$	15.150
2007, II	15.701	15.150	$\tilde{A}_6, \tilde{A}_6 \rightarrow \tilde{A}_5, \tilde{A}_6$	13.850, 15.150
2007, III	14.773	14.500	$\tilde{A}_6, \tilde{A}_6 \rightarrow \tilde{A}_5, \tilde{A}_6$	13.850, 15.150
2007, IV	13.313	14.500	$\tilde{A}_6, \tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_5, \tilde{A}_6$	11.250, 13.850, 15.150
2008, I	14.403	13.417	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2008, II	14.708	14.500	$\tilde{A}_5, \tilde{A}_6 \rightarrow \tilde{A}_6$	15.150
2008, III	16.432	15.150	$\tilde{A}_6, \tilde{A}_7 \rightarrow \tilde{A}_7, \tilde{A}_5$	13.850, 16.450
2008, IV	15.825	15.150	$\tilde{A}_7, \tilde{A}_7 \rightarrow \tilde{A}_7, \tilde{A}_6$	15.150, 16.450
2009, I	14.911	15.800	$\tilde{A}_7, \tilde{A}_6 \rightarrow \tilde{A}_4, \tilde{A}_7, \tilde{A}_6, \tilde{A}_5$	11.250, 12.550, 13.850, 15.150
2009, II	13.951	14.500	$\tilde{A}_6, \tilde{A}_5 \rightarrow \tilde{A}_3, \tilde{A}_5, \tilde{A}_6$	11.250, 13.850, 15.150
2009, III	14.197	13.417	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2009, IV	13.421	14.500	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_5, \tilde{A}_7, \tilde{A}_4, \tilde{A}_6$	11.250, 12.550, 13.850, 15.150
2010, I	12.619	14.500	$\tilde{A}_5, \tilde{A}_4 \rightarrow \tilde{A}_3, \tilde{A}_4$	11.250, 12.550
2010, II	11.736	11.900	$\tilde{A}_4, \tilde{A}_3 \rightarrow \tilde{A}_4, \tilde{A}_2$	9.950, 12.550

3.1.2. Song-Chissom model

Generalizing (time-invariant) fuzzy relation R is create on the based of the union $R = \bigcup_{i=1}^{24} R_i$ of identified 24 internal relationships (fuzzy relations) R_i: $\tilde{A}_s \rightarrow \tilde{A}_q$, where "$\cup$" is the union operator, which is realized by Poulsen (2009) as *max* operation. In essence, the relationship R_i: $\tilde{A}_s \rightarrow \tilde{A}_q$ is a fuzzy implicative rule of the form "If ..., then ...", for which is used various implication operations. In our notation it is chosen implication E. Mamdani:

$$\mu_R(w,u) = \min(\mu_{\tilde{A}_i}(w), \mu_{\tilde{A}_k}(u)), \tag{3}$$

where R is the fuzzy subset on $\tilde{A}_i \times \tilde{A}_k$ representing 7×7-dimensional matrix – fuzzy relation; $w \in \tilde{A}_i$ and $u \in \tilde{A}_k$. Applying the implication (3) one can represent the identified fuzzy relations (see Table 3) in the form of the following matrix:

$$R(\tilde{A}_1 \rightarrow \tilde{A}_1) = \begin{array}{c|ccccccc} & 1 & 0{,}5 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0{,}5 & 0 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \quad R(\tilde{A}_1 \rightarrow \tilde{A}_3) = \begin{array}{c|ccccccc} & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 \\ 1 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 \\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_2 \rightarrow \tilde{A}_2) = \begin{array}{c|ccccccc} & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 \\ 1 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \quad R(\tilde{A}_2 \rightarrow \tilde{A}_1) = \begin{array}{c|ccccccc} & 1 & 0{,}5 & 0 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0{,}5 & 0 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_2 \rightarrow \tilde{A}_3) = \begin{array}{c|ccccccc} & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 \\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 \\ 1 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 \\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \quad R(\tilde{A}_3 \rightarrow \tilde{A}_3) = \begin{array}{c|ccccccc} & 0 & 0{,}5 & 1 & 0{,}5 & 0{,}0 & 0{,}0 & 0{,}0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 \\ 1 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 \\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_3 \rightarrow \tilde{A}_4) = \begin{array}{c|ccccccc} & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}0 & 0{,}0 & 0 & 0 \\ 0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 \\ 1 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 \\ 1 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \quad R(\tilde{A}_3 \rightarrow \tilde{A}_2) = \begin{array}{c|ccccccc} & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 \\ 1 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_4\to\tilde{A}_5)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0\\ 1 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0\\ 0{,}5 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_4\to\tilde{A}_3)=\begin{array}{c|ccccccc} & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0\\ 1 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0\\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_4\to\tilde{A}_4)=\begin{array}{c|ccccccc} & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0\\ 1{,}0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0\\ 0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_4\to\tilde{A}_6)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5\\ 1 & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_5\to\tilde{A}_3)=\begin{array}{c|ccccccc} & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0\\ 1 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0\\ 0{,}5 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_5\to\tilde{A}_4)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0\\ 1 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0\\ 0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_5\to\tilde{A}_5)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0{,}5 & 1{,}0 & 0{,}5 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0\\ 1 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0\\ 0{,}5 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_5\to\tilde{A}_7)=\begin{array}{c|ccccccc} & 0{,}0 & 0{,}0 & 0{,}0 & 0{,}0 & 0{,}0 & 0{,}5 & 1{,}0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5\\ 1 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 1\\ 0{,}5 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_5\to\tilde{A}_6)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5\\ 1 & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$R(\tilde{A}_6\to\tilde{A}_5)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0\\ 1 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0\\ 0{,}5 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 \end{array}$$

$$R(\tilde{A}_6\to\tilde{A}_4)=\begin{array}{c|ccccccc} & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0\\ 1 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 & 0\\ 0{,}5 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0 & 0 \end{array}$$

$$R(\tilde{A}_6\to\tilde{A}_7)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5\\ 1 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 1\\ 0{,}5 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 \end{array}$$

$$R(\tilde{A}_6\to\tilde{A}_6)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5\\ 1 & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 \end{array}$$

$$R(\tilde{A}_7\to\tilde{A}_6)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5\\ 1 & 0 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 \end{array}$$

$$R(\tilde{A}_7\to\tilde{A}_7)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 0{,}5\\ 1 & 0 & 0 & 0 & 0 & 0 & 0{,}5 & 1 \end{array}$$

$$R(\tilde{A}_7\to\tilde{A}_5)=\begin{array}{c|ccccccc} & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0{,}5 & 0 & 0 & 0 & 0{,}5 & 0{,}5 & 0{,}5 & 0\\ 1 & 0 & 0 & 0 & 0{,}5 & 1 & 0{,}5 & 0 \end{array}$$

Then the time-invariant fuzzy relation will be as following:

$$R=\begin{Vmatrix} 1 & 0{,}5 & 1 & 0{,}5 & 0 & 0 & 0\\ 1 & 1 & 1 & 0{,}5 & 0{,}5 & 0 & 0\\ 0{,}5 & 1 & 1 & 1 & 0{,}5 & 0{,}5 & 0{,}5\\ 0{,}5 & 0{,}5 & 1 & 1 & 1 & 1 & 0{,}5\\ 0 & 0{,}5 & 1 & 1 & 1 & 1 & 1\\ 0 & 0{,}5 & 0{,}5 & 1 & 1 & 1 & 1\\ 0 & 0{,}0 & 0{,}5 & 0{,}5 & 1 & 1 & 1 \end{Vmatrix}$$

So, according to the Song-Chisson approach (1993) time series forecasting is carried out by following recurrent equality:

$$\tilde{A}_t = \tilde{A}_{t-1} \circ \tilde{R}, \quad (4)$$

where $\tilde{A}_{t-1}$ is the fuzzy analogue of the actual value of "Marginality of Sales" in (t-1) th year; $\tilde{A}_t$ is its fuzzy forecast for the next t-th year; $\tilde{R}$ is the generalizing time-constant fuzzy relation; «∘» denotes a composition of two fuzzy sets, which is formally defined by Poulsen (2009) as:

$$[\tilde{P} \circ \tilde{Q}](x, y) = \max_{z \in Z} \min\{\mu_{\tilde{P}}(x, z), \mu_{\tilde{Q}}(z, y)\}, \quad (5)$$

where $x \in X$ and $y \in Y$ are elements from correspondent universe X and Y. In matrix form this rule is formulated as:

$$[r_{ij}] = [p_{ik}] \circ [q_{kj}] = \max \min\{p_{ik}, q_{kj}\}.$$

Particularly, according to (4)-(5) fuzzy forecast for second quarter of 1988 year is defined as:

$$\tilde{A}_{1988\text{II}} = \tilde{A}_{1988\text{I}} \circ \tilde{R} = \tilde{A}_6 \circ \tilde{R} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0{,}5 \;\; 1 \;\; 0{,}5] \circ \begin{bmatrix} 1 & 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0.5 & 0.5 & 0 & 0 \\ 0.5 & 1 & 1 & 1 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 & 1 & 1 & 1 & 0.5 \\ 0 & 0.5 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.5 & 0.5 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0.5 & 0.5 & 1 & 1 & 1 \end{bmatrix} =$$

$$= [0 \;\; 0.5 \;\; 0.5 \;\; 1 \;\; 1 \;\; 1 \;\; 1],$$

where, for example, the first component c_1 of the desired row vector (or fuzzy output) is determined from the equation:

c_1=max{min(a_1,r_{11}); min(a_2,r_{21}); min(a_3,r_{31}); min(a_4,r_{41}); min(a_5,r_{51}); min(a_6,r_{61}); min(a_7,r_{71})}= =max{min(1, 0.5); min(0.5, 0.5); min(0, 0.5); min(0, 0.5); min(0, 0.5); min(0, 0);min(0, 0)}=0.5.

Thus, the result of the composition $\tilde{A}_{1988,\text{I}} \circ \tilde{R}$ is a fuzzy forecast $\tilde{A}_{1988,\text{II}}$, i.e. the fuzzy set with support vector (u_1,u_2,...,u_7):

$$\tilde{A}_{1988\text{II}} = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{0.5}{u_3} + \frac{1}{u_4} + \frac{1}{u_5} + \frac{1}{u_6} + \frac{1}{u_7}.$$

In this case, the last four component of the support vector (u_1,u_2,...,u_7) with a maximum degree of membership are included in the fuzzy set $\tilde{A}_{1988,\text{II}}$. According to the Chen's 3rd rule defuzzification numerical analogue of this set, i.e. the crisp prediction for second quarter of 1988 year will be:

$$\frac{m(u_4) + m(u_5) + m(u_6) + m(u_7)}{4} = \frac{\frac{11.9+13.2}{2} + \frac{13.2+14.5}{2} + \frac{14.5+15.8}{2} + \frac{15.8+17.1}{2}}{2} = 14.5.$$

This prediction is identical to the corresponding prediction obtained using Chen's model (see Table 6). Moreover, the application of Chen's defuzzification roles to the rest of fuzzy inference of compositional rule (4)-(5) in most cases also generate absolutely similar predictions showed in Table 6 (see Table 8).

Table 8. Compositional rule based semi-structured time series forecasting

Year, quarter	Actual data	Intervals							Mean values	Prediction
		u_1	u_2	u_3	u_4	u_5	u_6	u_7		
1988, I	0.522									
1988, II	0.425	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
1988, III	0.425	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1988, IV	0.477	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1989, I	0.828	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1989, II	0.616	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1989, III	0.367	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1989, IV	0.431	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1990, I	0.281	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1990, II	0.465	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1990, III	0.269	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1990, IV	0.578	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1991, I	0.566	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1991, II	14.091	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850

1991, III	14.123	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1991, IV	16.186	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
1992, I	14.633	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
1992, II	12.848	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1992, III	13.379	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1992, IV	13.987	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1993, I	13.336	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1993, II	13.071	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1993, III	12.113	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1993, IV	11.988	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1994, I	12.284	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1994, II	11.761	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1994, III	9.620	1	1	1	0.5	0.5	0.5	0.5	8.650, 9.950, 11.250	9.950
1994, IV	9.595	1	1	1	0.5	0.5	0.5	0.5	8.650, 9.950, 11.250	9.950
1995, I	8.169	1	0.5	1	0.5	0.5	0	0	8.650, 11.250	9.950
1995, II	8.837	1	0.5	1	0.5	0.5	0	0	8.650, 11.250	9.950
1995, III	8.712	1	0.5	1	0.5	0.5	0	0	8.650, 11.250	9.950
1995, IV	11.012	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1996, I	11.044	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1996, II	10.701	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1996, III	10.685	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1996, IV	10.332	1	1	1	0.5	0.5	0.5	0.5	8.650, 9.950, 11.250	9.950
1997, I	10.911	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1997, II	12.111	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1997, III	12.183	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1997, IV	12.085	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1998, I	11.684	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250
1998, II	12.158	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1998, III	13.455	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1998, IV	13.787	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
1999, I	12.570	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1999, II	12.096	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1999, III	13.186	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
1999, IV	15.211	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2000, I	17.030	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2000, II	16.012	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2000, III	16.202	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2000, IV	15.320	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2001, I	16.450	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2001, II	14.298	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2001, III	13.495	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2001,	13.920	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200

IV										
2002, I	15.045	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2002, II	13.862	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2002, III	13.188	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2002, IV	13.183	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2003, I	12.611	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2003, II	12.734	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2003, III	12.937	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2003, IV	12.870	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2004, I	13.406	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2004, II	12.794	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2004, III	13.100	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2004, IV	13.600	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2005, I	13.096	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2005, II	12.902	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2005, III	13.606	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2005, IV	14.401	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2006, I	15.803	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2006, II	15.704	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2006, III	15.297	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2006, IV	14.497	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2007, I	14.598	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2007, II	15.701	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2007, III	14.773	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2007, IV	13.313	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2008, I	14.403	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2008, II	14.708	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2008, III	16.432	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2008, IV	15.825	0	0.5	0.5	0.5	1	1	1	13.850, 15.150, 16.450	15.150
2009, I	14.911	0	0.5	0.5	1	1	1	1	12.550, 13.850, 15.150, 16.450	14.500
2009, II	13.951	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2009, III	14.197	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2009, IV	13.421	0.5	0.5	1	1	1	1	1	11.250, 12.550, 13.850, 15.150, 16.450	13.850
2010, I	12.619	0.5	0.5	1	1	1	1	0.5	11.250, 12.550, 13.850, 15.150	13.200
2010, II	11.736	0.5	1	1	1	0.5	0.5	0.5	9.950, 11.250, 12.550	11.250

In fact, Song-Chissom approach assumes more weighted rules of defuzzification, which are focused on the presence of the maximal component of fuzzy outputs of model represented by support vector (u_1, u_2, ..., u_7). In particular, predicted fuzzy set can be to have single and/or more maximal component that may be disposed sequentially or separately, which, in turn, is essential for defuzzification of fuzzy predictions. Subject to these considerations one can formulate defuzzification rules for fuzzy outputs as follows:

1. If fuzzy output (vector) has only one maximal component, then desired crisp prediction is mean value of the interval corresponding to this component. For example, only second component of fuzzy set $\tilde{A} = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0.5}{u_4} + \frac{0.5}{u_5} + \frac{0.5}{u_6} + \frac{0.5}{u_7}$ with value 1 is the maximum. Therefore, defuzzified (crisp) prediction for this period will be midpoint of interval u_2=[9.3; 10.6], notably: 9.95=(9.3+10.6)/2.

2. If fuzzy output has consistently two or more maximal component, then desired crisp prediction is mean value of the corresponding concatenated interval. For example, for the fuzzy output $\tilde{A}_{1988II} = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{0.5}{u_3} + \frac{1}{u_4} + \frac{1}{u_5} + \frac{1}{u_6} + \frac{1}{u_7}$ midpoint of combining intervals u_4, u_5, u_6, and u_7 is defuzzified prediction. In this case it is the number 14.5.
3. If the maximal components of the fuzzy output determine the minimum and maximum basic fuzzy sets accordingly, then selection of defuzzified prediction will depend on the values of the indicator for the preceding period. If in the previous period the fuzzy analogue of value of indicator closer to the base minimal set, then defuzzified prediction selected as midpoint of interval, which includes this minimal set with the maximum degree of membership. Otherwise, it is selected midpoint of interval, which includes this maximal set with the maximum degree of membership.
4. In other cases, as the prediction it is selected midpoint of all intervals corresponding to the maximal components of the fuzzy output.

Listed rules are just part of the all rules considered by Kumar et al. (2010). Such sampling is sufficient, because it covers all cases of the distribution of maximal components of outputs of Song-Chisson model applied to the given time series.

3.2.2. Paulsen's algorithm

Paulsen's approach to the fuzzy modeling and forecasting of semi-structured time series provides the following steps (2009).

Step 1: *definition of the universe of time series data*. To determine the width of universe *U* for time series data, as a rule, it is used the following standard indicators: average distance (*AD*) between two consecutive numbers of the series and corresponding standard deviation (σ), which are respectively calculated as:

$$AD = \frac{1}{t-1}\sum_{k=1}^{t-1}\left|x_k - x_{k+1}\right|, \tag{6}$$

$$\sigma_{AD} = \sqrt{\frac{1}{t}\sum_{k=1}^{t}\left(\bar{x}_k - AD\right)^2}, \tag{7}$$

where $\bar{x}_k = \left|x_k - x_{k+1}\right|$.

U is determined as interval $[x_{\min}\text{-}AD_R, x_{\max}\text{+}AD_R]$, where AD_R is adjusted value of *AD*; $x_{\min}$ is minimal value and $x_{\max}$ is maximal value of time series data. Particularly, for our time series, where t=91, we have: $x_{\min}$=8.169, $x_{\max}$=17.031. To find lower bound (*LB*) and upper bound (*UB*) of *U* at the beginning it is necessary to calculate correspondent values of indicators *AD* and σ. In our case, these are *AD*=0.7751 and σ_{AD}=0.5923. Further, it is chosen minimal value of deviations in the consecutive data of time series, which satisfy the conditions:

$$0.7751 - 0.5923 \le \bar{x}_k \le 0.7751 + 0.5923 \quad (k = 1 \div t-1).$$

In our case this is $\bar{x}_{50} = 0.19$. Then lower and upper bounds of *U* are determined accordingly as:

$$LB = 8.169 - 0.19 = 7.979,$$
$$UB = 17.031 + 0.19 = 17.221.$$

Thus, desired covering will be interval *U*=[7.979; 17.221], width of which is calculated as difference between lower and upper bounds:

$$D = UB - LB = 17.221 - 7.979 = 9.242.$$

Finally, the number of component intervals of *U* is calculated by formula:

$$n = \frac{D - \bar{x}_{50}}{2 \cdot \bar{x}_{50}} = \frac{9.242 - 0.19}{2 \cdot 0.19} = 23.8211 \approx 24.$$

Step 2: *creation the fuzzy subsets of universe U*. To create the fuzzy subsets of *U* it used trapezoidal membership function in the form:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2; \\ 1, a_2 \le x \le a_3; \\ \dfrac{a_4 - x}{a_4 - a_3}, a_3 \le x \le a_4; \\ 0, otherwise. \end{cases} \tag{8}$$

To find the values of parameters a_j (j=1÷4) for each trapezoidal membership function it used keypoints of universe partitioning to 24 intervals. Particular, first and second membership functions look as shown in Figure 2.

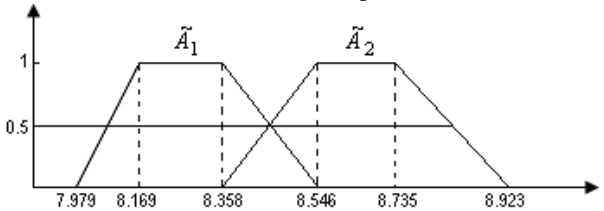

Figure 2. Trapezoidal membership functions

Constructed in a similar manner fuzzy sets are presented in Table 9.

Table 9. Fuzzy sets describing semi-structured data of time series

Fuzzy subset	Parameters of membership function				Fuzzy subset	Parameters of membership function			
	a_1	a_2	a_3	a_4		a_1	a_2	a_3	a_4
$\tilde{A}_1$	7.979	8.169	8.358	8.546	$\tilde{A}_{13}$	12.506	12.694	12.883	13.072
$\tilde{A}_2$	8.358	8.546	8.735	8.923	$\tilde{A}_{14}$	12.883	13.072	13.260	13.449
$\tilde{A}_3$	8.735	8.923	9.112	9.300	$\tilde{A}_{15}$	13.260	13.449	13.637	13.826
$\tilde{A}_4$	9.112	9.300	9.489	9.677	$\tilde{A}_{16}$	13.637	13.826	14.014	14.203
$\tilde{A}_5$	9.489	9.677	9.866	10.055	$\tilde{A}_{17}$	14.014	14.203	14.391	14.580
$\tilde{A}_6$	9.866	10.055	10.243	10.432	$\tilde{A}_{18}$	14.391	14.580	14.769	14.957
$\tilde{A}_7$	10.243	10.432	10.620	10.809	$\tilde{A}_{19}$	14.769	14.957	15.146	15.334
$\tilde{A}_8$	10.620	10.809	10.997	11.186	$\tilde{A}_{20}$	15.146	15.334	15.523	15.711
$\tilde{A}_9$	10.997	11.186	11.375	11.563	$\tilde{A}_{21}$	15.523	15.711	15.900	16.089
$\tilde{A}_{10}$	11.375	11.563	11.752	11.940	$\tilde{A}_{22}$	15.900	16.089	16.277	16.466
$\tilde{A}_{11}$	11.752	11.940	12.129	12.317	$\tilde{A}_{23}$	16.277	16.466	16.654	16.843
$\tilde{A}_{12}$	12.129	12.317	12.506	12.694	$\tilde{A}_{24}$	16.654	16.843	17.031	17.221

Step 3: *fuzzification of time series data*. In process of fuzzification of time series for each historical data it is chosen analogous fuzzy set, that trapezoidal membership function of its data in comparison with the others would have the greatest value. Results of time series fuzzification are showed in Table 10.

Table 10. Fuzzification of time series data

Year, quarter	Indicator	Fuzzy analog	Year, quarter	Indicator	Fuzzy analog	Year, quarter	Indicator	Fuzzy analog
1988, I	15.024	$\tilde{A}_{19}$	1995, III	8.712	$\tilde{A}_2$	2003, I	12.611	$\tilde{A}_{13}$
1988, II	13.514	$\tilde{A}_{15}$	1995, IV	11.012	$\tilde{A}_8$	2003, II	12.734	$\tilde{A}_{13}$
1988, III	11.637	$\tilde{A}_{10}$	1996, I	11.044	$\tilde{A}_8$	2003, III	12.937	$\tilde{A}_{13}$
1988, IV	11.691	$\tilde{A}_{10}$	1996, II	10.701	$\tilde{A}_7$	2003, IV	12.870	$\tilde{A}_{13}$
1989, I	12.651	$\tilde{A}_{13}$	1996, III	10.685	$\tilde{A}_7$	2004, I	13.406	$\tilde{A}_{15}$
1989, II	13.973	$\tilde{A}_{16}$	1996, IV	10.332	$\tilde{A}_6$	2004, II	12.794	$\tilde{A}_{13}$
1989, III	12.777	$\tilde{A}_{13}$	1997, I	10.911	$\tilde{A}_8$	2004, III	13.100	$\tilde{A}_{14}$
1989, IV	11.005	$\tilde{A}_8$	1997, II	12.111	$\tilde{A}_{11}$	2004, IV	13.600	$\tilde{A}_{15}$
1990, I	12.137	$\tilde{A}_{11}$	1997, III	12.183	$\tilde{A}_{11}$	2005, I	13.096	$\tilde{A}_{14}$
1990, II	13.096	$\tilde{A}_{14}$	1997, IV	12.085	$\tilde{A}_{11}$	2005, II	12.902	$\tilde{A}_{13}$
1990, III	13.183	$\tilde{A}_{14}$	1998, I	11.684	$\tilde{A}_{10}$	2005, III	13.606	$\tilde{A}_{15}$
1990, IV	13.441	$\tilde{A}_{15}$	1998, II	12.158	$\tilde{A}_{11}$	2005, IV	14.401	$\tilde{A}_{17}$
1991, I	13.748	$\tilde{A}_{16}$	1998, III	13.455	$\tilde{A}_{15}$	2006, I	15.803	$\tilde{A}_{21}$
1991, II	14.091	$\tilde{A}_{16}$	1998, IV	13.787	$\tilde{A}_{16}$	2006, II	15.704	$\tilde{A}_{21}$
1991, III	14.123	$\tilde{A}_{17}$	1999, I	12.570	$\tilde{A}_{12}$	2006, III	15.297	$\tilde{A}_{20}$
1991, IV	16.186	$\tilde{A}_{20}$	1999, II	12.096	$\tilde{A}_{11}$	2006, IV	14.497	$\tilde{A}_{18}$
1992, I	14.633	$\tilde{A}_{18}$	1999, III	13.186	$\tilde{A}_{14}$	2007, I	14.598	$\tilde{A}_{18}$
1992, II	12.848	$\tilde{A}_{13}$	1999, IV	15.211	$\tilde{A}_{19}$	2007, II	15.701	$\tilde{A}_{21}$
1992, III	13.379	$\tilde{A}_{15}$	2000, I	17.030	$\tilde{A}_{24}$	2007, III	14.773	$\tilde{A}_{18}$
1992, IV	13.987	$\tilde{A}_{16}$	2000, II	16.012	$\tilde{A}_{22}$	2007, IV	13.313	$\tilde{A}_{14}$
1993, I	13.336	$\tilde{A}_{14}$	2000, III	16.202	$\tilde{A}_{22}$	2008, I	14.403	$\tilde{A}_{17}$
1993, II	13.071	$\tilde{A}_{14}$	2000, IV	15.320	$\tilde{A}_{20}$	2008, II	14.708	$\tilde{A}_{18}$
1993, III	12.113	$\tilde{A}_{11}$	2001, I	16.450	$\tilde{A}_{23}$	2008, III	16.432	$\tilde{A}_{23}$
1993, IV	11.988	$\tilde{A}_{11}$	2001, II	14.298	$\tilde{A}_{17}$	2008, IV	15.825	$\tilde{A}_{21}$
1994, I	12.284	$\tilde{A}_{12}$	2001, III	13.495	$\tilde{A}_{15}$	2009, I	14.911	$\tilde{A}_{19}$
1994, II	11.761	$\tilde{A}_{10}$	2001, IV	13.920	$\tilde{A}_{16}$	2009, II	13.951	$\tilde{A}_{16}$
1994, III	9.620	$\tilde{A}_5$	2002, I	15.045	$\tilde{A}_{19}$	2009, III	14.197	$\tilde{A}_{17}$
1994, IV	9.595	$\tilde{A}_5$	2002, II	13.862	$\tilde{A}_{16}$	2009, IV	13.421	$\tilde{A}_{15}$
1995, I	8.169	$\tilde{A}_1$	2002, III	13.188	$\tilde{A}_{14}$	2010, I	12.619	$\tilde{A}_{13}$
1995, II	8.837	$\tilde{A}_3$	2002, IV	13.183	$\tilde{A}_{14}$	2010, II	11.736	$\tilde{A}_{10}$

Step 4: *Identifying the internal fuzzy relations and their localization in groups*. On the base of determined fuzzy analogues of historical data (Table 10) we get existing internal fuzzy relationships of first and second orders.

Table 11. Fuzzy relationships of first order

$\tilde{A}_1\rightarrow\tilde{A}_3$	$\tilde{A}_{10}\rightarrow\tilde{A}_{13}$	$\tilde{A}_{13}\rightarrow\tilde{A}_{15}$	$\tilde{A}_{15}\rightarrow\tilde{A}_{13}$	$\tilde{A}_{17}\rightarrow\tilde{A}_{18}$	$\tilde{A}_{21}\rightarrow\tilde{A}_{20}$
$\tilde{A}_2\rightarrow\tilde{A}_8$	$\tilde{A}_{10}\rightarrow\tilde{A}_5$	$\tilde{A}_{13}\rightarrow\tilde{A}_{13}$	$\tilde{A}_{15}\rightarrow\tilde{A}_{14}$	$\tilde{A}_{18}\rightarrow\tilde{A}_{13}$	$\tilde{A}_{21}\rightarrow\tilde{A}_{18}$
$\tilde{A}_3\rightarrow\tilde{A}_2$	$\tilde{A}_{10}\rightarrow\tilde{A}_{11}$	$\tilde{A}_{13}\rightarrow\tilde{A}_{14}$	$\tilde{A}_{15}\rightarrow\tilde{A}_{17}$	$\tilde{A}_{18}\rightarrow\tilde{A}_{18}$	$\tilde{A}_{21}\rightarrow\tilde{A}_{19}$
$\tilde{A}_5\rightarrow\tilde{A}_5$	$\tilde{A}_{11}\rightarrow\tilde{A}_{14}$	$\tilde{A}_{13}\rightarrow\tilde{A}_{10}$	$\tilde{A}_{16}\rightarrow\tilde{A}_{13}$	$\tilde{A}_{18}\rightarrow\tilde{A}_{21}$	$\tilde{A}_{22}\rightarrow\tilde{A}_{22}$
$\tilde{A}_5\rightarrow\tilde{A}_1$	$\tilde{A}_{11}\rightarrow\tilde{A}_{11}$	$\tilde{A}_{14}\rightarrow\tilde{A}_{14}$	$\tilde{A}_{16}\rightarrow\tilde{A}_{16}$	$\tilde{A}_{18}\rightarrow\tilde{A}_{14}$	$\tilde{A}_{22}\rightarrow\tilde{A}_{20}$
$\tilde{A}_6\rightarrow\tilde{A}_8$	$\tilde{A}_{11}\rightarrow\tilde{A}_{12}$	$\tilde{A}_{14}\rightarrow\tilde{A}_{15}$	$\tilde{A}_{16}\rightarrow\tilde{A}_{17}$	$\tilde{A}_{18}\rightarrow\tilde{A}_{23}$	$\tilde{A}_{23}\rightarrow\tilde{A}_{17}$
$\tilde{A}_7\rightarrow\tilde{A}_7$	$\tilde{A}_{11}\rightarrow\tilde{A}_{10}$	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}$	$\tilde{A}_{16}\rightarrow\tilde{A}_{14}$	$\tilde{A}_{19}\rightarrow\tilde{A}_{15}$	$\tilde{A}_{23}\rightarrow\tilde{A}_{21}$
$\tilde{A}_7\rightarrow\tilde{A}_6$	$\tilde{A}_{11}\rightarrow\tilde{A}_{15}$	$\tilde{A}_{14}\rightarrow\tilde{A}_{19}$	$\tilde{A}_{16}\rightarrow\tilde{A}_{12}$	$\tilde{A}_{19}\rightarrow\tilde{A}_{24}$	$\tilde{A}_{24}\rightarrow\tilde{A}_{22}$
$\tilde{A}_8\rightarrow\tilde{A}_{11}$	$\tilde{A}_{12}\rightarrow\tilde{A}_{10}$	$\tilde{A}_{14}\rightarrow\tilde{A}_{13}$	$\tilde{A}_{16}\rightarrow\tilde{A}_{19}$	$\tilde{A}_{19}\rightarrow\tilde{A}_{16}$	

$\tilde{A}_8 \rightarrow \tilde{A}_8$	$\tilde{A}_{12} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{14} \rightarrow \tilde{A}_{17}$	$\tilde{A}_{17} \rightarrow \tilde{A}_{20}$	$\tilde{A}_{20} \rightarrow \tilde{A}_{18}$	
$\tilde{A}_8 \rightarrow \tilde{A}_7$	$\tilde{A}_{13} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{15} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{17} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{20} \rightarrow \tilde{A}_{23}$	
$\tilde{A}_{10} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{13} \rightarrow \tilde{A}_8$	$\tilde{A}_{15} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{17} \rightarrow \tilde{A}_{21}$	$\tilde{A}_{21} \rightarrow \tilde{A}_{21}$	

Table 12. Fuzzy relationships of second order

$\tilde{A}_1, \tilde{A}_3 \rightarrow \tilde{A}_2$	$\tilde{A}_{10}, \tilde{A}_{11} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{13}, \tilde{A}_{13} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{17}, \tilde{A}_{20} \rightarrow \tilde{A}_{18}$	$\tilde{A}_{20}, \tilde{A}_{18} \rightarrow \tilde{A}_{18}$
$\tilde{A}_2, \tilde{A}_8 \rightarrow \tilde{A}_8$	$\tilde{A}_{11}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{13}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{12}$	$\tilde{A}_{17}, \tilde{A}_{15} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{20}, \tilde{A}_{23} \rightarrow \tilde{A}_{17}$
$\tilde{A}_3, \tilde{A}_2 \rightarrow \tilde{A}_8$	$\tilde{A}_{11}, \tilde{A}_{14} \rightarrow \tilde{A}_{19}$	$\tilde{A}_{13}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{19}$	$\tilde{A}_{17}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{21}, \tilde{A}_{21} \rightarrow \tilde{A}_{20}$
$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_1$	$\tilde{A}_{11}, \tilde{A}_{11} \rightarrow \tilde{A}_{12}$	$\tilde{A}_{13}, \tilde{A}_{14} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{17}, \tilde{A}_{21} \rightarrow \tilde{A}_{21}$	$\tilde{A}_{21}, \tilde{A}_{20} \rightarrow \tilde{A}_{18}$
$\tilde{A}_5, \tilde{A}_1 \rightarrow \tilde{A}_3$	$\tilde{A}_{11}, \tilde{A}_{11} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{14}, \tilde{A}_{14} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{15}, \tilde{A}_{13} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{17}, \tilde{A}_{18} \rightarrow \tilde{A}_{23}$	$\tilde{A}_{21}, \tilde{A}_{18} \rightarrow \tilde{A}_{14}$
$\tilde{A}_6, \tilde{A}_8 \rightarrow \tilde{A}_{11}$	$\tilde{A}_{11}, \tilde{A}_{11} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{14}, \tilde{A}_{14} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{15}, \tilde{A}_{13} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{18}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{21}, \tilde{A}_{19} \rightarrow \tilde{A}_{16}$
$\tilde{A}_7, \tilde{A}_7 \rightarrow \tilde{A}_6$	$\tilde{A}_{11}, \tilde{A}_{10} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{14}, \tilde{A}_{14} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{15}, \tilde{A}_{14} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{18}, \tilde{A}_{18} \rightarrow \tilde{A}_{21}$	$\tilde{A}_{22}, \tilde{A}_{22} \rightarrow \tilde{A}_{20}$
$\tilde{A}_7, \tilde{A}_6 \rightarrow \tilde{A}_8$	$\tilde{A}_{11}, \tilde{A}_{12} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{14}, \tilde{A}_{15} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{15}, \tilde{A}_{17} \rightarrow \tilde{A}_{21}$	$\tilde{A}_{18}, \tilde{A}_{21} \rightarrow \tilde{A}_{18}$	$\tilde{A}_{22}, \tilde{A}_{20} \rightarrow \tilde{A}_{23}$
$\tilde{A}_8, \tilde{A}_{11} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{11}, \tilde{A}_{15} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{14}, \tilde{A}_{15} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{16}, \tilde{A}_{13} \rightarrow \tilde{A}_8$	$\tilde{A}_{18}, \tilde{A}_{14} \rightarrow \tilde{A}_{17}$	$\tilde{A}_{23}, \tilde{A}_{17} \rightarrow \tilde{A}_{15}$
$\tilde{A}_8, \tilde{A}_{11} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{12}, \tilde{A}_{10} \rightarrow \tilde{A}_5$	$\tilde{A}_{14}, \tilde{A}_{11} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{16}, \tilde{A}_{16} \rightarrow \tilde{A}_{17}$	$\tilde{A}_{18}, \tilde{A}_{23} \rightarrow \tilde{A}_{21}$	$\tilde{A}_{23}, \tilde{A}_{21} \rightarrow \tilde{A}_{19}$
$\tilde{A}_8, \tilde{A}_8 \rightarrow \tilde{A}_7$	$\tilde{A}_{12}, \tilde{A}_{11} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{14}, \tilde{A}_{19} \rightarrow \tilde{A}_{24}$	$\tilde{A}_{16}, \tilde{A}_{17} \rightarrow \tilde{A}_{20}$	$\tilde{A}_{19}, \tilde{A}_{15} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{24}, \tilde{A}_{22} \rightarrow \tilde{A}_{22}$
$\tilde{A}_8, \tilde{A}_7 \rightarrow \tilde{A}_7$	$\tilde{A}_{13}, \tilde{A}_{16} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{14}, \tilde{A}_{13} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{16}, \tilde{A}_{17} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{19}, \tilde{A}_{24} \rightarrow \tilde{A}_{22}$	
$\tilde{A}_{10}, \tilde{A}_{10} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{13}, \tilde{A}_8 \rightarrow \tilde{A}_{11}$	$\tilde{A}_{14}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{16}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{19}, \tilde{A}_{16} \rightarrow \tilde{A}_{14}$	
$\tilde{A}_{10}, \tilde{A}_{13} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{13}, \tilde{A}_{15} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{14}, \tilde{A}_{17} \rightarrow \tilde{A}_{18}$	$\tilde{A}_{16}, \tilde{A}_{12} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{19}, \tilde{A}_{16} \rightarrow \tilde{A}_{17}$	
$\tilde{A}_{10}, \tilde{A}_5 \rightarrow \tilde{A}_5$	$\tilde{A}_{13}, \tilde{A}_{15} \rightarrow \tilde{A}_{17}$	$\tilde{A}_{15}, \tilde{A}_{10} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{16}, \tilde{A}_{19} \rightarrow \tilde{A}_{16}$	$\tilde{A}_{20}, \tilde{A}_{18} \rightarrow \tilde{A}_{13}$	

As in the previous case, the identified fuzzy relationships are grouped by previously mentioned principle. Groups of fuzzy relationships of the first and second order are presented in Tables 13, 14.

Table 13. First order fuzzy relationships groups

Group 1:	$\tilde{A}_1 \rightarrow \tilde{A}_3$	Group 9:	$\tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	Group 17:	$\tilde{A}_{19} \rightarrow \tilde{A}_{15}, \tilde{A}_{16}, \tilde{A}_{24}$
Group 2:	$\tilde{A}_2 \rightarrow \tilde{A}_8$	Group 10:	$\tilde{A}_{12} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}$	Group 18:	$\tilde{A}_{20} \rightarrow \tilde{A}_{18}, \tilde{A}_{23}$
Group 3:	$\tilde{A}_3 \rightarrow \tilde{A}_2$	Group 11:	$\tilde{A}_{13} \rightarrow \tilde{A}_8, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	Group 19:	$\tilde{A}_{21} \rightarrow \tilde{A}_{18}, \tilde{A}_{19}, \tilde{A}_{20}, \tilde{A}_{21}$
Group 4:	$\tilde{A}_5 \rightarrow \tilde{A}_1, \tilde{A}_5$	Group 12:	$\tilde{A}_{14} \rightarrow \tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	Group 20:	$\tilde{A}_{22} \rightarrow \tilde{A}_{20}, \tilde{A}_{22}$
Group 5:	$\tilde{A}_6 \rightarrow \tilde{A}_8$	Group 13:	$\tilde{A}_{15} \rightarrow \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	Group 21:	$\tilde{A}_{23} \rightarrow \tilde{A}_{17}, \tilde{A}_{21}$
Group 6:	$\tilde{A}_7 \rightarrow \tilde{A}_6, \tilde{A}_7$	Group 14:	$\tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	Group 22:	$\tilde{A}_{24} \rightarrow \tilde{A}_{22}$
Group 7:	$\tilde{A}_8 \rightarrow \tilde{A}_7, \tilde{A}_8, \tilde{A}_{11}$	Group 15:	$\tilde{A}_{17} \rightarrow \tilde{A}_{15}, \tilde{A}_{18}, \tilde{A}_{20}, \tilde{A}_{21}$		
Group 8:	$\tilde{A}_{10} \rightarrow \tilde{A}_5, \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{13}$	Group 16:	$\tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{18}, \tilde{A}_{21}, \tilde{A}_{23}$		

Table 14. Second order fuzzy relationships groups

Group 1:	$\tilde{A}_1, \tilde{A}_3 \rightarrow \tilde{A}_2$	Group 24:	$\tilde{A}_{13}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}, \tilde{A}_{13}$	Group 47:	$\tilde{A}_{17}, \tilde{A}_{20} \rightarrow \tilde{A}_{18}$
Group 2:	$\tilde{A}_2, \tilde{A}_8 \rightarrow \tilde{A}_8$	Group 25:	$\tilde{A}_{13}, \tilde{A}_{14} \rightarrow \tilde{A}_{15}$	Group 48:	$\tilde{A}_{17}, \tilde{A}_{21} \rightarrow \tilde{A}_{21}$
Group 3:	$\tilde{A}_3, \tilde{A}_2 \rightarrow \tilde{A}_8$	Group 26:	$\tilde{A}_{13}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}, \tilde{A}_{16}, \tilde{A}_{17}$	Group 49:	$\tilde{A}_{18}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}$
Group 4:	$\tilde{A}_5, \tilde{A}_1 \rightarrow \tilde{A}_3$	Group 27:	$\tilde{A}_{13}, \tilde{A}_{16} \rightarrow \tilde{A}_{13}$	Group 50:	$\tilde{A}_{18}, \tilde{A}_{14} \rightarrow \tilde{A}_{17}$
Group 5:	$\tilde{A}_5, \tilde{A}_5 \rightarrow \tilde{A}_1$	Group 28:	$\tilde{A}_{14}, \tilde{A}_{11} \rightarrow \tilde{A}_{11}$	Group 51:	$\tilde{A}_{18}, \tilde{A}_{18} \rightarrow \tilde{A}_{21}$
Group 6:	$\tilde{A}_6, \tilde{A}_8 \rightarrow \tilde{A}_{11}$	Group 29:	$\tilde{A}_{14}, \tilde{A}_{13} \rightarrow \tilde{A}_{13}, \tilde{A}_{15}$	Group 52:	$\tilde{A}_{18}, \tilde{A}_{21} \rightarrow \tilde{A}_{18}$
Group 7:	$\tilde{A}_7, \tilde{A}_6 \rightarrow \tilde{A}_8$	Group 30:	$\tilde{A}_{14}, \tilde{A}_{14} \rightarrow \tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{15}$	Group 53:	$\tilde{A}_{18}, \tilde{A}_{23} \rightarrow \tilde{A}_{21}$
Group 8:	$\tilde{A}_7, \tilde{A}_7 \rightarrow \tilde{A}_6$	Group 31:	$\tilde{A}_{14}, \tilde{A}_{15} \rightarrow \tilde{A}_{14}, \tilde{A}_{16}$	Group 54:	$\tilde{A}_{19}, \tilde{A}_{15} \rightarrow \tilde{A}_{10}$
Group 9:	$\tilde{A}_8, \tilde{A}_7 \rightarrow \tilde{A}_7$	Group 32:	$\tilde{A}_{14}, \tilde{A}_{17} \rightarrow \tilde{A}_{18}$	Group 55:	$\tilde{A}_{19}, \tilde{A}_{16} \rightarrow \tilde{A}_{14}, \tilde{A}_{17}$
Group 10:	$\tilde{A}_8, \tilde{A}_8 \rightarrow \tilde{A}_7$	Group 33:	$\tilde{A}_{14}, \tilde{A}_{19} \rightarrow \tilde{A}_{24}$	Group 56:	$\tilde{A}_{19}, \tilde{A}_{24} \rightarrow \tilde{A}_{22}$
Group 11:	$\tilde{A}_8, \tilde{A}_{11} \rightarrow \tilde{A}_{11}, \tilde{A}_{14}$	Group 34:	$\tilde{A}_{15}, \tilde{A}_{10} \rightarrow \tilde{A}_{10}$	Group 57:	$\tilde{A}_{20}, \tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{18}$
Group 12:	$\tilde{A}_{10}, \tilde{A}_5 \rightarrow \tilde{A}_5$	Group 35:	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{19}$	Group 58:	$\tilde{A}_{20}, \tilde{A}_{23} \rightarrow \tilde{A}_{17}$
Group 13:	$\tilde{A}_{10}, \tilde{A}_{10} \rightarrow \tilde{A}_{13}$	Group 36:	$\tilde{A}_{15}, \tilde{A}_{13} \rightarrow \tilde{A}_{10}, \tilde{A}_{14}$	Group 59:	$\tilde{A}_{21}, \tilde{A}_{18} \rightarrow \tilde{A}_{14}$
Group 14:	$\tilde{A}_{10}, \tilde{A}_{11} \rightarrow \tilde{A}_{15}$	Group 37:	$\tilde{A}_{15}, \tilde{A}_{14} \rightarrow \tilde{A}_{13}$	Group 60:	$\tilde{A}_{21}, \tilde{A}_{19} \rightarrow \tilde{A}_{16}$
Group 15:	$\tilde{A}_{10}, \tilde{A}_{13} \rightarrow \tilde{A}_{16}$	Group 38:	$\tilde{A}_{15}, \tilde{A}_{17} \rightarrow \tilde{A}_{21}$	Group 61:	$\tilde{A}_{21}, \tilde{A}_{20} \rightarrow \tilde{A}_{18}$
Group 16:	$\tilde{A}_{11}, \tilde{A}_{10} \rightarrow \tilde{A}_{11}$	Group 39:	$\tilde{A}_{16}, \tilde{A}_{12} \rightarrow \tilde{A}_{11}$	Group 62:	$\tilde{A}_{21}, \tilde{A}_{21} \rightarrow \tilde{A}_{20}$
Group 17:	$\tilde{A}_{11}, \tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}$	Group 40:	$\tilde{A}_{16}, \tilde{A}_{13} \rightarrow \tilde{A}_8$	Group 63:	$\tilde{A}_{22}, \tilde{A}_{20} \rightarrow \tilde{A}_{23}$
Group 18:	$\tilde{A}_{11}, \tilde{A}_{12} \rightarrow \tilde{A}_{10}$	Group 41:	$\tilde{A}_{16}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}$	Group 64:	$\tilde{A}_{22}, \tilde{A}_{22} \rightarrow \tilde{A}_{20}$
Group 19:	$\tilde{A}_{11}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}, \tilde{A}_{19}$	Group 42:	$\tilde{A}_{16}, \tilde{A}_{16} \rightarrow \tilde{A}_{17}$	Group 65:	$\tilde{A}_{23}, \tilde{A}_{17} \rightarrow \tilde{A}_{15}$
Group 20:	$\tilde{A}_{11}, \tilde{A}_{15} \rightarrow \tilde{A}_{16}$	Group 43:	$\tilde{A}_{16}, \tilde{A}_{17} \rightarrow \tilde{A}_{15}, \tilde{A}_{20}$	Group 66:	$\tilde{A}_{23}, \tilde{A}_{21} \rightarrow \tilde{A}_{19}$
Group 21:	$\tilde{A}_{12}, \tilde{A}_{10} \rightarrow \tilde{A}_5$	Group 44:	$\tilde{A}_{16}, \tilde{A}_{19} \rightarrow \tilde{A}_{16}$	Group 67:	$\tilde{A}_{24}, \tilde{A}_{22} \rightarrow \tilde{A}_{22}$
Group 22:	$\tilde{A}_{12}, \tilde{A}_{11} \rightarrow \tilde{A}_{14}$	Group 45:	$\tilde{A}_{17}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}, \tilde{A}_{16}$		
Group 23:	$\tilde{A}_{13}, \tilde{A}_8 \rightarrow \tilde{A}_{11}$	Group 46:	$\tilde{A}_{17}, \tilde{A}_{18} \rightarrow \tilde{A}_{23}$		

Step 5: *defuzzification of model outputs.* To defuzzify the forecasted outputs it is applied Chen's rules. Its results for fuzzy relationships of first and second orders are showed in Table 17.

4. Defuzzification of outputs of Poulsen's model by point-estimation method

Defuzzification of fuzzy outputs is a key step of time series forecasting prosess. It affects greatly the accuracy of the prediction in the ordinary numbers. The above Chen's rules being a fairly trivial allow obtaining the relatively crude

numerical estimation of fuzzy predictions. Defuzzification rules of fuzzy predictions obtained by application of the recurrence composite conclusion (4)-(5) [4] are more adequacy than the Chen's rules. However, they mainly use the average values (midpoints) of the constituent intervals of the universe partition.

In this section it is proposed to use point-estimation method of fuzzy predictions. The essence of this method is as follows.

Suppose that a fuzzy subset $\tilde{A}_t$ of the universe U ($\tilde{A}_t \subset U$) is the fuzzy predictions obtained by the application of one of the above models. As a rule, this set consolidates by combining two or more elementary fuzzy sets from the list of sets that describe the historical data of the given time series. For example, according to the Paulsen's algorithm the fuzzy output for second quarter of 1988 year (1988, II) is a group of fuzzy relationships of the first order: $\tilde{A}_{19} \to \tilde{A}_{15}$, $\tilde{A}_{16}$, $\tilde{A}_{24}$. In the notation of fuzzy inference mechanism, this implies following implicative rule:

"if predicate is $\tilde{A}_{19}$, then prediction will be $\tilde{A}_{15}$ or $\tilde{A}_{16}$ or $\tilde{A}_{24}$".

Taking into account existence of OR operator in the right part of this rule, common membership function is defined as $\mu_{\tilde{A}_{1916}}(u) = \mu_{\tilde{A}_{15} \cup \tilde{A}_{16} \cup \tilde{A}_{24}}(u) = \max\{\mu_{\tilde{A}_{15}}(u), \mu_{\tilde{A}_{16}}(u), \mu_{\tilde{A}_{24}}(u)\}$. Here, as membership function one can use trapezoidal function in the form:

$$\mu_{\tilde{A}_k}(x) = \begin{cases} \dfrac{x - a_{1k}}{a_{2k} - a_{1k}}, \ a_{1k} \le x \le a_{2k}; \\ 1, \ a_{2k} \le x \le a_{3k}; \\ \dfrac{a_{4k} - x}{a_{4k} - a_{3k}}, \ a_{3k} \le x \le a_{4k}; \\ 0, \ otherwise, \end{cases} \tag{9}$$

which produces k-th fuzzy analog $\tilde{A}_k$ (k=1÷24) correspondent semi-structured data of time series.

For point-estimation of fuzzy prediction it is necessary to determine α-level sets ($\alpha \in [0; 1]$) in the form $A_\alpha = \{i | \mu_{\tilde{A}}(i) \ge \alpha, i \in I\}$, where I is a finite aggregate of numbers from $U_{\min}$ to $U_{\max}$, which form the arithmetical progression.

Further, for each level set it is determined correspondent cardinal number $M(A_\alpha)$ by formula:

$$M(A_\alpha) = \sum_{j=1}^{n} \frac{i_j}{n}, \ i \in C_\alpha. \tag{10}$$

Finnaly, point-estimation of fuzzy set $\tilde{A}_t$ is calculated from equality:

$$F(\tilde{A}) = \frac{1}{\alpha_{\max}} \int_0^{\alpha_{\max}} M(A_\alpha) d\alpha, \tag{11}$$

where $\alpha_{\max}$ is maximal value on $\tilde{A}_t$.

Apply formulated point-estimation method to fuzzy outputs of Poulsen's model, which in most cases are offered the union of several elementary fuzzy sets from the list $\{\tilde{A}_k\}$ (k=1÷24). For the construction of these sets as the support vector one can choose a suitable set of numbers from the universe U=[7.979; 17.221]. Let this be a set of 51-th numbers varying from 7.979 to 17.221 by step of 0.185:

C={7.9790, 8.164, 8.349, 8.534, 8.718, 8.903, 9.088, 9.273, 9.458, 9.643, 9.827, 10.012, 10.197, 10.382, 10.567, 10.752, 10.936, 11.121, 11.306, 11.491, 11.676, 11.861, 12.045, 12.230, 12.415, 12.600, 12.785, 12.970, 13.155, 13.339, 13.524, 13.709, 13.894, 14.079, 14.264, 14.448, 14.633, 14.818, 15.003, 15.188, 15.373, 15.557, 15.742, 15.927, 16.112, 16.297, 16.482, 16.666, 16.851, 17.036, 17.221}.

As an example, we choose a fuzzy output of Poulsen's model for second quarter of 1988 year ($\tilde{A}_{1988,II}$), which is the union of fuzzy sets $\tilde{A}_{15}$, $\tilde{A}_{16}$ and $\tilde{A}_{24}$ (see Table 12). Restoring these sets with the appropriate trapezoidal membership functions of the form (9), on the basis of the support vector C we obtain the following interpretation of a fuzzy set $\tilde{A}_{1988,II}$:

$$\begin{aligned} \tilde{A}_{1988II} = {} & \frac{0}{7.979} + \frac{0}{8.164} + \frac{0}{8.349} + \frac{0}{8.534} + \frac{0}{8.718} + \frac{0}{8.903} + \frac{0}{9.088} + \frac{0}{9.273} + \frac{0}{9.458} + \frac{0}{9.643} + \\ & + \frac{0}{9.827} + \frac{0}{10.012} + \frac{0}{10.197} + \frac{0}{10.382} + \frac{0}{10.567} + \frac{0}{10.752} + \frac{0}{10.936} + \frac{0}{11.121} + \frac{0}{11.306} + \\ & + \frac{0}{11.491} + \frac{0}{11.676} + \frac{0}{11.861} + \frac{0}{12.045} + \frac{0}{12.230} + \frac{0}{12.415} + \frac{0}{12.600} + \frac{0}{12.785} + \frac{0}{12.970} + \\ & + \frac{0}{13.155} + \frac{0.420}{13.339} + \frac{1}{13.524} + \frac{0.619}{13.709} + \frac{1}{13.894} + \frac{0.658}{14.079} + \frac{0}{14.264} + \frac{0}{14.448} + \frac{0}{14.633} + \\ & + \frac{0}{14.818} + \frac{0}{15.003} + \frac{0}{15.188} + \frac{0}{15.373} + \frac{0}{15.557} + \frac{0}{15.742} + \frac{0}{15.927} + \frac{0}{16.112} + \frac{0}{16.297} + \\ & + \frac{0}{16.482} + \frac{0.066}{16.666} + \frac{1}{16.851} + \frac{0.973}{17.036} + \frac{0}{17.221}. \end{aligned}$$

Level set A_α and correspondent cardinal number $M(A_\alpha)$ are determined as following:

- for $0<\alpha<0.066$, $d\alpha$=0.066, A_α={13.339, 13.524, 13.709, 13.894, 14.079, 16.666, 16.851, 17.036}; $M(A_\alpha)$=14.888;
- for $0.066<\alpha<0.420$, $d\alpha$=0.354, A_α={13.339, 13.524, 13.709, 13.894, 14.079, 16.851, 17.036}; $M(A_\alpha)$=14.633;
- for $0.420<\alpha<0.619$, $d\alpha$=0.199, A_α={13.524, 13.709, 13.894, 14.079, 16.851, 17.036}; $M(A_\alpha)$=14.849;
- for $0.619<\alpha<0.658$, $d\alpha$=0.039, A_α={13.524, 13.894, 14.079, 16.851, 17.036}; $M(A_\alpha)$=15.077;
- for $0.658<\alpha<0.973$, $d\alpha$=0.315, A_α={13.524,13.894,16.851,17.036}; $M(A_\alpha)$=15.326;
- for $0.973<\alpha<1$, $d\alpha$=0.027, A_α={13.524, 13.894, 16.851}; $M(A_\alpha)$=14.757.

Then in accordance with (11) point-estimation of fuzzy prediction $\tilde{A}_{1988,II}$ will be:

$$F(\tilde{A}_{1988II}) = \frac{1}{1}\int_0^1 M(A_\alpha)d\alpha = \frac{1}{1}(0.066\cdot 14.888 + 0.3539\cdot 14.633 +$$

$$+ 0.199\cdot 14.849 + 0.039\cdot 15.077 + 0.315\cdot 15.326 + 0.027\cdot 14.757) = 14.932.$$

Thus, using the procedure of the point-estimation of fuzzy sets to the output of Poulsun's model induced by relationships of first and second orders we get the correspondent target predictions (see Tables 15, 16).

Table 15. Point-estimation of outputs of Poulsen's model induced by relationships of 1st order

Year, quarter	Actual data	Fuzzy relationships group of first order	Point-estimation of outputs	Year, quarter	Actual data	Fuzzy relationships group of first order	Point-estimation of outputs
1988, I	15.024	$\tilde{A}_{19}\rightarrow\tilde{A}_{15}, \tilde{A}_{16}, \tilde{A}_{24}$		1999, II	12.096	$\tilde{A}_{11}\rightarrow\tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	11.845
1988, II	13.514	$\tilde{A}_{15}\rightarrow\tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	14.932	1999, III	13.186	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	12.573
1988, III	11.637	$\tilde{A}_{10}\rightarrow\tilde{A}_{5}, \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{13}$	13.158	1999, IV	15.211	$\tilde{A}_{19}\rightarrow\tilde{A}_{15}, \tilde{A}_{16}, \tilde{A}_{24}$	13.523
1988, IV	11.691	$\tilde{A}_{10}\rightarrow\tilde{A}_{5}, \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{13}$	11.509	2000, I	17.030	$\tilde{A}_{24}\rightarrow\tilde{A}_{22}$	14.932
1989, I	12.651	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	11.509	2000, II	16.012	$\tilde{A}_{22}\rightarrow\tilde{A}_{20}, \tilde{A}_{22}$	16.181
1989, II	13.973	$\tilde{A}_{16}\rightarrow\tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	12.601	2000, III	16.202	$\tilde{A}_{22}\rightarrow\tilde{A}_{20}, \tilde{A}_{22}$	15.809
1989, III	12.777	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	13.671	2000, IV	15.320	$\tilde{A}_{20}\rightarrow\tilde{A}_{18}, \tilde{A}_{23}$	15.809
1989, IV	11.005	$\tilde{A}_{8}\rightarrow\tilde{A}_{7}, \tilde{A}_{8}, \tilde{A}_{11}$	12.601	2001, I	16.450	$\tilde{A}_{23}\rightarrow\tilde{A}_{17}, \tilde{A}_{21}$	15.640
1990, I	12.137	$\tilde{A}_{11}\rightarrow\tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	11.162	2001, II	14.298	$\tilde{A}_{17}\rightarrow\tilde{A}_{15}, \tilde{A}_{18}, \tilde{A}_{20}, \tilde{A}_{21}$	15.067
1990, II	13.096	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	12.573	2001, III	13.495	$\tilde{A}_{15}\rightarrow\tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	14.867
1990, III	13.183	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	13.523	2001, IV	13.920	$\tilde{A}_{16}\rightarrow\tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	13.158
1990, IV	13.441	$\tilde{A}_{15}\rightarrow\tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	13.523	2002, I	15.045	$\tilde{A}_{19}\rightarrow\tilde{A}_{15}, \tilde{A}_{16}, \tilde{A}_{24}$	13.671
1991, I	13.748	$\tilde{A}_{16}\rightarrow\tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	13.158	2002, II	13.862	$\tilde{A}_{16}\rightarrow\tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	14.932
1991, II	14.091	$\tilde{A}_{16}\rightarrow\tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	13.671	2002, III	13.188	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	13.671
1991, III	14.123	$\tilde{A}_{17}\rightarrow\tilde{A}_{15}, \tilde{A}_{18}, \tilde{A}_{20}, \tilde{A}_{21}$	13.671	2002, IV	13.183	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	13.523
1991, IV	16.186	$\tilde{A}_{20}\rightarrow\tilde{A}_{18}, \tilde{A}_{23}$	14.867	2003, I	12.611	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	13.523
1992, I	14.633	$\tilde{A}_{18}\rightarrow\tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{18}, \tilde{A}_{21}, \tilde{A}_{23}$	15.640	2003, II	12.734	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	12.601
1992, II	12.848	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	14.715	2003, III	12.937	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	12.601
1992, III	13.379	$\tilde{A}_{15}\rightarrow\tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	12.601	2003, IV	12.870	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	12.601
1992, IV	13.987	$\tilde{A}_{16}\rightarrow\tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	13.158	2004, I	13.406	$\tilde{A}_{15}\rightarrow\tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	12.601
1993, I	13.336	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	13.671	2004, II	12.794	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	13.158
1993, II	13.071	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	13.523	2004, III	13.100	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	12.601
1993, III	12.113	$\tilde{A}_{11}\rightarrow\tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	13.523	2004, IV	13.600	$\tilde{A}_{15}\rightarrow\tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	13.523
1993, IV	11.988	$\tilde{A}_{11}\rightarrow\tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	12.573	2005, I	13.096	$\tilde{A}_{14}\rightarrow\tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	13.158
1994, I	12.284	$\tilde{A}_{12}\rightarrow\tilde{A}_{10}, \tilde{A}_{11}$	12.573	2005, II	12.902	$\tilde{A}_{13}\rightarrow\tilde{A}_{8}, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	13.523
1994, II	11.761	$\tilde{A}_{10}\rightarrow\tilde{A}_{5}, \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{13}$	11.845	2005, III	13.606	$\tilde{A}_{15}\rightarrow\tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	12.601

1994, III	9.620	$\tilde{A}_5 \rightarrow \tilde{A}_1, \tilde{A}_5$	11.509	2005, IV	14.401	$\tilde{A}_{17} \rightarrow \tilde{A}_{15}, \tilde{A}_{18}, \tilde{A}_{20}, \tilde{A}_{21}$	13.158
1994, IV	9.595	$\tilde{A}_5 \rightarrow \tilde{A}_1, \tilde{A}_5$	9.003	2006, I	15.803	$\tilde{A}_{21} \rightarrow \tilde{A}_{18}, \tilde{A}_{19}, \tilde{A}_{20}, \tilde{A}_{21}$	14.867
1995, I	8.169	$\tilde{A}_1 \rightarrow \tilde{A}_3$	9.003	2006, II	15.704	$\tilde{A}_{21} \rightarrow \tilde{A}_{18}, \tilde{A}_{19}, \tilde{A}_{20}, \tilde{A}_{21}$	15.247
1995, II	8.837	$\tilde{A}_3 \rightarrow \tilde{A}_2$	9.019	2006, III	15.297	$\tilde{A}_{20} \rightarrow \tilde{A}_{18}, \tilde{A}_{23}$	15.247
1995, III	8.712	$\tilde{A}_2 \rightarrow \tilde{A}_8$	8.642	2006, IV	14.497	$\tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{18}, \tilde{A}_{21}, \tilde{A}_{23}$	15.640
1995, IV	11.012	$\tilde{A}_8 \rightarrow \tilde{A}_7, \tilde{A}_8, \tilde{A}_{11}$	10.904	2007, I	14.598	$\tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{18}, \tilde{A}_{21}, \tilde{A}_{23}$	14.715
1996, I	11.044	$\tilde{A}_8 \rightarrow \tilde{A}_7, \tilde{A}_8, \tilde{A}_{11}$	11.162	2007, II	15.701	$\tilde{A}_{21} \rightarrow \tilde{A}_{18}, \tilde{A}_{19}, \tilde{A}_{20}, \tilde{A}_{21}$	14.715
1996, II	10.701	$\tilde{A}_7 \rightarrow \tilde{A}_6, \tilde{A}_7$	11.162	2007, III	14.773	$\tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{18}, \tilde{A}_{21}, \tilde{A}_{23}$	15.247
1996, III	10.685	$\tilde{A}_7 \rightarrow \tilde{A}_6, \tilde{A}_7$	10.337	2007, IV	13.313	$\tilde{A}_{14} \rightarrow \tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{17}, \tilde{A}_{19}$	14.715
1996, IV	10.332	$\tilde{A}_6 \rightarrow \tilde{A}_8$	10.337	2008, I	14.403	$\tilde{A}_{17} \rightarrow \tilde{A}_{15}, \tilde{A}_{18}, \tilde{A}_{20}, \tilde{A}_{21}$	13.523
1997, I	10.911	$\tilde{A}_8 \rightarrow \tilde{A}_7, \tilde{A}_8, \tilde{A}_{11}$	10.904	2008, II	14.708	$\tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{18}, \tilde{A}_{21}, \tilde{A}_{23}$	14.867
1997, II	12.111	$\tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	11.162	2008, III	16.432	$\tilde{A}_{23} \rightarrow \tilde{A}_{17}, \tilde{A}_{21}$	14.715
1997, III	12.183	$\tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	12.573	2008, IV	15.825	$\tilde{A}_{21} \rightarrow \tilde{A}_{18}, \tilde{A}_{19}, \tilde{A}_{20}, \tilde{A}_{21}$	15.067
1997, IV	12.085	$\tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	12.573	2009, I	14.911	$\tilde{A}_{19} \rightarrow \tilde{A}_{15}, \tilde{A}_{16}, \tilde{A}_{24}$	15.247
1998, I	11.684	$\tilde{A}_{10} \rightarrow \tilde{A}_5, \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{13}$	12.573	2009, II	13.951	$\tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	14.932
1998, II	12.158	$\tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{15}$	11.509	2009, III	14.197	$\tilde{A}_{17} \rightarrow \tilde{A}_{15}, \tilde{A}_{18}, \tilde{A}_{20}, \tilde{A}_{21}$	13.671
1998, III	13.455	$\tilde{A}_{15} \rightarrow \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	12.573	2009, IV	13.421	$\tilde{A}_{15} \rightarrow \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}$	14.867
1998, IV	13.787	$\tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{17}, \tilde{A}_{19}$	13.158	2010, I	12.619	$\tilde{A}_{13} \rightarrow \tilde{A}_8, \tilde{A}_{10}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{15}, \tilde{A}_{16}$	13.158
1999, I	12.570	$\tilde{A}_{12} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}$	13.671	2010, II	11.736	$\tilde{A}_{10} \rightarrow \tilde{A}_5, \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{13}$	12.601

Table 16. Point-estimation of outputs of Poulsen's model induced by relationships of 2^d order

Year, quarter	Actual data	Fuzzy relationships group of second order	Point-estimation of outputs	Year, quarter	Actual data	Fuzzy relationships group of second order	Point-estimation of outputs
1988, I	15.024			1999, II	12.096	$\tilde{A}_{12}, \tilde{A}_{11} \rightarrow \tilde{A}_{14}$	12.035
1988, II	13.514	$\tilde{A}_{19}, \tilde{A}_{15} \rightarrow \tilde{A}_{10}$		1999, III	13.186	$\tilde{A}_{11}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}, \tilde{A}_{19}$	13.166
1988, III	11.637	$\tilde{A}_{15}, \tilde{A}_{10} \rightarrow \tilde{A}_{10}$	11.658	1999, IV	15.211	$\tilde{A}_{14}, \tilde{A}_{19} \rightarrow \tilde{A}_{24}$	14.134
1988, IV	11.691	$\tilde{A}_{10}, \tilde{A}_{10} \rightarrow \tilde{A}_{13}$	11.658	2000, I	17.030	$\tilde{A}_{19}, \tilde{A}_{24} \rightarrow \tilde{A}_{22}$	17.035
1989, I	12.651	$\tilde{A}_{10}, \tilde{A}_{13} \rightarrow \tilde{A}_{16}$	12.789	2000, II	16.012	$\tilde{A}_{24}, \tilde{A}_{22} \rightarrow \tilde{A}_{22}$	16.181
1989, II	13.973	$\tilde{A}_{13}, \tilde{A}_{16} \rightarrow \tilde{A}_{13}$	13.919	2000, III	16.202	$\tilde{A}_{22}, \tilde{A}_{22} \rightarrow \tilde{A}_{20}$	16.181
1989, III	12.777	$\tilde{A}_{16}, \tilde{A}_{13} \rightarrow \tilde{A}_8$	12.789	2000, IV	15.320	$\tilde{A}_{22}, \tilde{A}_{20} \rightarrow \tilde{A}_{23}$	15.428
1989, IV	11.005	$\tilde{A}_{13}, \tilde{A}_8 \rightarrow \tilde{A}_{11}$	10.904	2001, I	16.450	$\tilde{A}_{20}, \tilde{A}_{23} \rightarrow \tilde{A}_{17}$	16.558
1990, I	12.137	$\tilde{A}_8, \tilde{A}_{11} \rightarrow \tilde{A}_{11}, \tilde{A}_{14}$	12.035	2001, II	14.298	$\tilde{A}_{23}, \tilde{A}_{17} \rightarrow \tilde{A}_{15}$	14.296
1990, II	13.096	$\tilde{A}_{11}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}, \tilde{A}_{19}$	12.600	2001, III	13.495	$\tilde{A}_{17}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}, \tilde{A}_{16}$	13.543
1990, III	13.183	$\tilde{A}_{14}, \tilde{A}_{14} \rightarrow \tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{15}$	14.134	2001, IV	13.920	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{19}$	13.354

1990, IV	13.441	$\tilde{A}_{14}, \tilde{A}_{15} \rightarrow \tilde{A}_{14}, \tilde{A}_{16}$	12.793	2002, I	15.045	$\tilde{A}_{16}, \tilde{A}_{19} \rightarrow \tilde{A}_{16}$	13.667
1991, I	13.748	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{19}$	13.547	2002, II	13.862	$\tilde{A}_{19}, \tilde{A}_{16} \rightarrow \tilde{A}_{14}, \tilde{A}_{17}$	13.919
1991, II	14.091	$\tilde{A}_{16}, \tilde{A}_{16} \rightarrow \tilde{A}_{17}$	13.667	2002, III	13.188	$\tilde{A}_{16}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}$	13.741
1991, III	14.123	$\tilde{A}_{16}, \tilde{A}_{17} \rightarrow \tilde{A}_{15}, \tilde{A}_{20}$	14.296	2002, IV	13.183	$\tilde{A}_{14}, \tilde{A}_{14} \rightarrow \tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{15}$	13.166
1991, IV	16.186	$\tilde{A}_{17}, \tilde{A}_{20} \rightarrow \tilde{A}_{18}$	14.511	2003, I	12.611	$\tilde{A}_{14}, \tilde{A}_{13} \rightarrow \tilde{A}_{13}, \tilde{A}_{15}$	12.793
1992, I	14.633	$\tilde{A}_{20}, \tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{18}$	14.673	2003, II	12.734	$\tilde{A}_{13}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}, \tilde{A}_{13}$	13.170
1992, II	12.848	$\tilde{A}_{18}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}$	13.757	2003, III	12.937	$\tilde{A}_{13}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}, \tilde{A}_{13}$	13.170
1992, III	13.379	$\tilde{A}_{13}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}, \tilde{A}_{16}, \tilde{A}_{17}$	13.543	2003, IV	12.870	$\tilde{A}_{13}, \tilde{A}_{13} \rightarrow \tilde{A}_{15}, \tilde{A}_{13}$	13.170
1992, IV	13.987	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{19}$	13.659	2004, I	13.406	$\tilde{A}_{13}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}, \tilde{A}_{16}, \tilde{A}_{17}$	13.170
1993, I	13.336	$\tilde{A}_{16}, \tilde{A}_{14} \rightarrow \tilde{A}_{14}$	13.667	2004, II	12.794	$\tilde{A}_{15}, \tilde{A}_{13} \rightarrow \tilde{A}_{10}, \tilde{A}_{14}$	13.659
1993, II	13.071	$\tilde{A}_{14}, \tilde{A}_{14} \rightarrow \tilde{A}_{11}, \tilde{A}_{13}, \tilde{A}_{15}$	13.166	2004, III	13.100	$\tilde{A}_{13}, \tilde{A}_{14} \rightarrow \tilde{A}_{15}$	12.408
1993, III	12.113	$\tilde{A}_{14}, \tilde{A}_{11} \rightarrow \tilde{A}_{11}$	12.793	2004, IV	13.600	$\tilde{A}_{14}, \tilde{A}_{15} \rightarrow \tilde{A}_{14}, \tilde{A}_{16}$	13.543
1993, IV	11.988	$\tilde{A}_{11}, \tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}$	12.035	2005, I	13.096	$\tilde{A}_{15}, \tilde{A}_{14} \rightarrow \tilde{A}_{13}$	13.547
1994, I	12.284	$\tilde{A}_{11}, \tilde{A}_{12} \rightarrow \tilde{A}_{10}$	12.031	2005, II	12.902	$\tilde{A}_{14}, \tilde{A}_{13} \rightarrow \tilde{A}_{13}, \tilde{A}_{15}$	12.789
1994, II	11.761	$\tilde{A}_{12}, \tilde{A}_{10} \rightarrow \tilde{A}_{5}$	11.658	2005, III	13.606	$\tilde{A}_{13}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}, \tilde{A}_{16}, \tilde{A}_{17}$	13.170
1994, III	9.620	$\tilde{A}_{10}, \tilde{A}_{5} \rightarrow \tilde{A}_{5}$	9.773	2005, IV	14.401	$\tilde{A}_{15}, \tilde{A}_{17} \rightarrow \tilde{A}_{21}$	13.659
1994, IV	9.595	$\tilde{A}_{5}, \tilde{A}_{5} \rightarrow \tilde{A}_{1}$	9.773	2006, I	15.803	$\tilde{A}_{17}, \tilde{A}_{21} \rightarrow \tilde{A}_{21}$	15.805
1995, I	8.169	$\tilde{A}_{5}, \tilde{A}_{1} \rightarrow \tilde{A}_{3}$	8.265	2006, II	15.704	$\tilde{A}_{21}, \tilde{A}_{21} \rightarrow \tilde{A}_{20}$	15.805
1995, II	8.837	$\tilde{A}_{1}, \tilde{A}_{3} \rightarrow \tilde{A}_{2}$	9.019	2006, III	15.297	$\tilde{A}_{21}, \tilde{A}_{20} \rightarrow \tilde{A}_{18}$	15.428
1995, III	8.712	$\tilde{A}_{3}, \tilde{A}_{2} \rightarrow \tilde{A}_{8}$	8.642	2006, IV	14.497	$\tilde{A}_{20}, \tilde{A}_{18} \rightarrow \tilde{A}_{13}, \tilde{A}_{18}$	14.673
1995, IV	11.012	$\tilde{A}_{2}, \tilde{A}_{8} \rightarrow \tilde{A}_{8}$	10.904	2007, I	14.598	$\tilde{A}_{18}, \tilde{A}_{18} \rightarrow \tilde{A}_{21}$	13.757
1996, I	11.044	$\tilde{A}_{8}, \tilde{A}_{8} \rightarrow \tilde{A}_{7}$	10.904	2007, II	15.701	$\tilde{A}_{18}, \tilde{A}_{21} \rightarrow \tilde{A}_{18}$	15.805
1996, II	10.701	$\tilde{A}_{8}, \tilde{A}_{7} \rightarrow \tilde{A}_{7}$	10.527	2007, III	14.773	$\tilde{A}_{21}, \tilde{A}_{18} \rightarrow \tilde{A}_{14}$	14.673
1996, III	10.685	$\tilde{A}_{7}, \tilde{A}_{7} \rightarrow \tilde{A}_{6}$	10.527	2007, IV	13.313	$\tilde{A}_{18}, \tilde{A}_{14} \rightarrow \tilde{A}_{17}$	13.166
1996, IV	10.332	$\tilde{A}_{7}, \tilde{A}_{6} \rightarrow \tilde{A}_{8}$	10.150	2008, I	14.403	$\tilde{A}_{14}, \tilde{A}_{17} \rightarrow \tilde{A}_{18}$	14.296
1997, I	10.911	$\tilde{A}_{6}, \tilde{A}_{8} \rightarrow \tilde{A}_{11}$	10.904	2008, II	14.708	$\tilde{A}_{17}, \tilde{A}_{18} \rightarrow \tilde{A}_{23}$	14.673
1997, II	12.111	$\tilde{A}_{8}, \tilde{A}_{11} \rightarrow \tilde{A}_{11}, \tilde{A}_{14}$	12.035	2008, III	16.432	$\tilde{A}_{18}, \tilde{A}_{23} \rightarrow \tilde{A}_{21}$	16.558
1997, III	12.183	$\tilde{A}_{11}, \tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}$	12.601	2008, IV	15.825	$\tilde{A}_{23}, \tilde{A}_{21} \rightarrow \tilde{A}_{19}$	15.805
1997, IV	12.085	$\tilde{A}_{11}, \tilde{A}_{11} \rightarrow \tilde{A}_{10}, \tilde{A}_{11}, \tilde{A}_{12}$	12.031	2009, I	14.911	$\tilde{A}_{21}, \tilde{A}_{19} \rightarrow \tilde{A}_{16}$	15.051
1998, I	11.684	$\tilde{A}_{11}, \tilde{A}_{10} \rightarrow \tilde{A}_{11}$	12.031	2009, II	13.951	$\tilde{A}_{19}, \tilde{A}_{16} \rightarrow \tilde{A}_{14}, \tilde{A}_{17}$	13.919
1998, II	12.158	$\tilde{A}_{10}, \tilde{A}_{11} \rightarrow \tilde{A}_{15}$	12.035	2009, III	14.197	$\tilde{A}_{16}, \tilde{A}_{17} \rightarrow \tilde{A}_{15}, \tilde{A}_{20}$	13.741
1998, III	13.455	$\tilde{A}_{11}, \tilde{A}_{15} \rightarrow \tilde{A}_{16}$	13.543	2009, IV	13.421	$\tilde{A}_{17}, \tilde{A}_{15} \rightarrow \tilde{A}_{13}, \tilde{A}_{16}$	14.511
1998,	13.78	$\tilde{A}_{15}, \tilde{A}_{16} \rightarrow \tilde{A}_{12}, \tilde{A}_{14}, \tilde{A}_{16}, \tilde{A}_{19}$	13.919	2010, I	12.61	$\tilde{A}_{15}, \tilde{A}_{13} \rightarrow \tilde{A}_{10}, \tilde{A}_{14}$	13.355

IV	7				9		
1999, I	12.570	$\tilde{A}_{16}, \tilde{A}_{12} \rightarrow \tilde{A}_{11}$	13.667	2010, II	11.736	$\tilde{A}_{15}, \tilde{A}_{13} \rightarrow \tilde{A}_{10}, \tilde{A}_{14}$	12.408

5. Comparison of forecasting results

To compare the considered approaches to semi-structured time series forecasting we use the following statistical evaluation criteria (see Table 17): Mean Absolute Percentage Error (*MAPE*) and Mean Squared Error (*MSE*), which calculated as:

$$MAPE = \frac{1}{n}\sum_{j=1}^{n}\frac{\left|forecast_j - actual_j\right|}{actual_j} \times 100 \quad (12)$$

$$MSE = \frac{1}{n}\sum_{j=1}^{n}\left(forecast_j - actual_j\right)^2 . \quad (13)$$

Table 17. Comparison of forecasting results

Year, quarter	Indicator	Chen's model under relationships of:		Song-Chissom's model	Poulsen's model under relationships of:		Defuzzification of outputs of Poulsen's model by point-estimation method under relationships of:	
		first order	second order		first order	second order	first order	second order
1988, I	15.024							
1988, II	13.514	14.500		14.500	14.800		14.932	
1988, III	11.637	13.850	13.417	13.850	13.166	11.657	13.158	11.658
1988, IV	11.691	11.250	11.250	11.250	11.563	11.657	11.509	11.658
1989, I	12.651	11.250	11.250	11.250	11.563	12.789	11.509	12.789
1989, II	13.973	13.200	13.200	13.200	12.663	13.920	12.601	13.919
1989, III	12.777	13.850	13.200	13.850	13.606	12.789	13.671	12.789
1989, IV	11.005	13.200	11.900	13.200	12.663	10.903	12.601	10.904
1990, I	12.137	11.250	11.250	11.250	11.155	12.034	11.162	12.035
1990, II	13.096	13.200	13.200	13.200	12.562	12.600	12.573	12.600
1990, III	13.183	13.200	13.200	13.200	13.480	14.109	13.523	14.134
1990, IV	13.441	13.200	13.200	13.200	13.480	12.789	13.523	12.793
1991, I	13.748	13.850	13.200	13.850	13.166	13.543	13.158	13.547
1991, II	14.091	13.850	14.500	13.850	13.606	13.637	13.671	13.667
1991, III	14.123	13.850	14.500	13.850	13.606	14.297	13.671	14.296
1991, IV	16.186	13.850	14.500	15.150	14.863	14.486	14.867	14.511
1992, I	14.633	15.150	15.150	14.500	15.617	14.674	15.64	14.673
1992, II	12.848	14.500	14.500	13.200	14.599	13.732	14.715	13.757
1992, III	13.379	13.200	13.850	13.850	12.663	13.543	12.601	13.543
1992, IV	13.987	13.850	13.200	13.850	13.166	13.669	13.158	13.659
1993, I	13.336	13.850	14.500	13.850	13.606	13.637	13.671	13.667
1993, II	13.071	13.850	14.500	13.200	13.480	13.166	13.523	13.166
1993, III	12.113	13.200	11.900	13.200	13.480	12.789	13.523	12.793
1993, IV	11.988	13.200	13.200	13.200	12.562	12.034	12.573	12.035
1994, I	12.284	13.200	13.200	13.200	12.562	12.034	12.573	12.031
1994, II	11.761	13.200	13.200	11.250	11.846	11.657	11.845	11.658
1994, III	9.620	11.250	13.200	9.950	11.563	9.772	11.509	9.773
1994, IV	9.595	9.950	10.600	9.950	9.018	9.772	9.003	9.773
1995, I	8.169	9.950	8.650	9.950	9.018	8.263	9.003	8.265
1995, II	8.837	9.950	8.650	9.950	9.018	9.018	9.019	9.019
1995, III	8.712	9.950	9.950	9.950	8.640	8.640	8.642	8.642
1995, IV	11.012	9.950	9.950	11.250	10.903	10.903	10.904	10.904
1996, I	11.044	11.250	11.250	11.250	11.155	10.903	11.162	10.904
1996, II	10.701	11.250	11.250	11.250	11.155	10.526	11.162	10.527
1996, III	10.685	11.250	11.250	11.250	10.337	10.526	10.337	10.527
1996, IV	10.332	11.250	11.250	9.950	10.337	10.149	10.337	10.150
1997, I	10.911	9.950	10.600	11.250	10.903	10.903	10.904	10.904
1997, II	12.111	11.250	12.550	13.200	11.155	12.034	11.162	12.035
1997, III	12.183	13.200	13.200	13.200	12.562	12.600	12.573	12.601
1997, IV	12.085	13.200	13.200	13.200	12.562	12.034	12.573	12.031
1998, I	11.684	13.200	13.200	11.250	12.562	12.034	12.573	12.031
1998, II	12.158	11.250	11.250	13.200	11.563	12.034	11.509	12.035
1998, III	13.455	13.200	13.200	13.850	12.562	13.543	12.573	13.543
1998, IV	13.787	13.850	13.200	13.850	13.166	13.920	13.158	13.919

1999, I	12.570	13.850	14.500	13.200	13.606	13.637	13.671	13.667
1999, II	12.096	13.200	11.900	13.200	11.846	12.034	11.845	12.035
1999, III	13.186	13.200	13.200	13.200	12.562	13.166	12.573	13.166
1999, IV	15.211	13.200	13.200	14.500	13.480	14.109	13.523	14.134
2000, I	17.030	14.500	16.450	15.150	14.800	16.937	14.932	17.035
2000, II	16.012	15.150	15.150	15.150	16.183	16.183	16.181	16.181
2000, III	16.202	15.150	15.800	15.150	15.806	16.183	15.809	16.181
2000, IV	15.320	15.150	15.800	14.500	15.806	15.429	15.809	15.428
2001, I	16.450	14.500	14.500	15.150	15.617	16.560	15.64	16.558
2001, II	14.298	15.150	15.150	13.850	15.051	14.297	15.067	14.296
2001, III	13.495	13.850	12.550	13.200	14.863	13.543	14.867	13.543
2001, IV	13.920	13.200	11.900	13.200	13.166	13.355	13.158	13.354
2002, I	15.045	13.200	13.200	13.200	13.606	13.637	13.671	13.667
2002, II	13.862	13.200	13.200	13.200	14.800	13.920	14.932	13.919
2002, III	13.188	13.200	13.200	13.200	13.606	13.732	13.671	13.741
2002, IV	13.183	13.200	13.200	13.850	13.480	13.166	13.523	13.166
2003, I	12.611	13.850	13.200	13.200	13.480	12.789	13.523	12.793
2003, II	12.734	13.200	11.900	13.200	12.663	13.166	12.601	13.170
2003, III	12.937	13.200	13.200	13.850	12.663	13.166	12.601	13.170
2003, IV	12.870	13.850	13.200	13.200	12.663	13.166	12.601	13.170
2004, I	13.406	13.200	11.900	13.200	12.663	13.166	12.601	13.170
2004, II	12.794	13.200	13.200	13.850	13.166	13.669	13.158	13.659
2004, III	13.100	13.850	13.200	13.850	12.663	12.412	12.601	12.408
2004, IV	13.600	13.850	14.500	15.150	13.480	13.543	13.523	13.543
2005, I	13.096	15.150	15.150	14.500	13.166	13.355	13.158	13.547
2005, II	12.902	14.500	14.500	14.500	13.480	12.789	13.523	12.789
2005, III	13.606	14.500	13.850	13.850	12.663	13.166	12.601	13.170
2005, IV	14.401	13.850	13.417	13.850	13.166	13.669	13.158	13.659
2006, I	15.803	13.850	14.500	15.150	14.863	15.806	14.867	15.805
2006, II	15.704	15.150	15.150	14.500	15.240	15.806	15.247	15.805
2006, III	15.297	14.500	14.500	14.500	15.240	15.429	15.247	15.428
2006, IV	14.497	14.500	14.500	13.850	15.617	14.674	15.64	14.673
2007, I	14.598	13.850	13.417	14.500	14.599	13.732	14.715	13.757
2007, II	15.701	14.500	15.150	14.500	14.599	15.806	14.715	15.805
2007, III	14.773	14.500	14.500	14.500	15.240	14.674	15.247	14.673
2007, IV	13.313	14.500	14.500	13.850	14.599	13.166	14.715	13.166
2008, I	14.403	13.850	13.417	13.850	13.480	14.297	13.523	14.296
2008, II	14.708	13.850	14.500	14.500	14.863	14.674	14.867	14.673
2008, III	16.432	14.500	15.150	15.150	14.599	16.560	14.715	16.558
2008, IV	15.825	15.150	15.150	15.150	15.051	15.806	15.067	15.805
2009, I	14.911	15.150	15.800	14.500	15.240	15.051	15.247	15.051
2009, II	13.951	14.500	14.500	13.850	14.800	13.920	14.932	13.919
2009, III	14.197	13.850	13.417	13.850	13.606	13.732	13.671	13.741
2009, IV	13.421	13.850	14.500	13.850	14.863	14.486	14.867	14.511
2010, I	12.619	13.850	14.500	13.200	13.166	13.355	13.158	13.355
2010, II	11.736	13.200	11.900	11.250	12.663	12.412	12.601	12.408
MAPE		**6.8372**	**6.5198**	**5.5188**	**5.3357**	**2.1630**	**5.4333**	**2.1755**
MSE		**1.1517**	**1.0954**	**0.7513**	**0.7515**	**0.1977**	**0.7610**	**0.1985**

6. Conclusions

Comparison of forecasting results obtained by point-estimation method with the results obtained by known forecasting methods showed that defuzzification method of outputs of fuzzy TSM have a right to exist. In the illustrated variant of the application of point-estimation method the outputs of fuzzy TSM described by the fuzzy set on support vector, which includes 50 components of the specified universe. Further experiments showed that an increase of number of the support vector components (for example, up to 100 units and more) significantly improves the prediction quality.

Discussed fuzzy models of the semi-structured time series are an integral part of the rapidly developing of Data Intelligence Analysis Theory (Data Mining). By application of the fuzzy analysis methods it is possible to describe the semi-structured data of the time series; to detect and above all to formalize internal multi ordinal relationships between data. This field of Data Mining still will find its further development. However, the results already obtained in the form of fuzzy methodology of semi-structured time series forecasting can be adapted to integrate into the existing software of Data Mining Information Technology, for example, in Oracle Data Mining. In particular, it will significantly enrich used in Oracle Data Mining limited set of standard functions.

References

Chen, S.M. (1996). Forecasting enrollments based on fuzzy time series. *Fuzzy Sets and Systems*, 81, pp. 311–319.

Chen, S.M. (2002). Forecasting enrollments based on high-order fuzzy time series. *Cybernetics and Systems*, 33, pp. 1–16.

Cheng, C.H., Chang, J.R. & Yen, C.A. (2006). Entropy-based and trapezoid fuzzification fuzzy time series approaches for forecasting IT project cost. *Technological Forecasting & Social Change*, 73, pp. 524–542.

Kumar, N. et al. (2010). Fuzzy time series forecasting of wheat production. *International Journal on Computer Science and Engineering*, 2(3), pp. 635-640.

Marcos Campos, M. (2013). Personal Blog "Oracle Data Mining and Analytics". Retrieved from http://oracledmt.blogspot.com/

Poulsen, J.R. (2009). *Fuzzy Time Series Forecasting – Developing a new forecasting model based on high order fuzzy time series*. AAUE, Semester: CIS 4.

Song, Q. & Chissom, B.S. (1993, a). Forecasting enrollments with fuzzy time series – part I. *Fuzzy Sets and Systems*, 54, pp.1–9.

Song, Q. & Chissom, B.S. (1993,b). Fuzzy time series and its models. *Fuzzy Sets and Systems*, 54, pp. 269–277.

Song, Q. & Chissom, B.S. (1994). Forecasting enrollments with fuzzy time series – part II. *Fuzzy Sets and Systems*, 62, pp. 1–8.