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Research Article

On Three-Dimensional Saigo-Maeda Operator of Weyl Type with Three-Dimensional H-Transforms

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ABSTRACT

In this paper we have made an attempt to establish a triple integral relation between weyl type three dimensional Saigo-Maeda operator of fractional integration and the three dimensional H-transform. Some special cases have also been established.

Keywords: Laplace transform, generalized fractional integral operator, H-transform, Appell function F_3.MSC2010: 26A33, 33C65, 44A10.

INTRODUCTION

Saxena, Gupta and Kumbhat [16] studied two dimensional Weyl fractional calculus. Nishimoto and Saxena [8] gave the generalization of results given by Arora, Raina and Koul [1], Saxena, Gupta and Kumbhat [16], and Raina and Kiryakova [12] by proving the theorems associated with two dimensional G-transforms. Saxena, Ram and Goree [14] proved a theorem on two dimensional generalized H-transforms involving Weyl type two dimensional Saigo operators. Saxena, Ram and Suthar [13] studied the generalized fractional integration operators associated with the Appell Function F_3 as kernel, introduced recently by Saigo and Maeda [10]. Chaurasia and Srivastava [19] established a theorem on two dimensional $\bar{H}$-transforms involving polynomials of general class with Weyl type two dimensional Saigo operators. Chaurasia and Jain [18] evaluated certain triple integral relations, and derived a theorem on three dimensional $\bar{H}$-transforms associated with the three dimensional Saigo operators of Weyl type.

In this paper we drive a relationship between H-transforms and the Saigo-Maeda operators of Weyl type of three dimensions. The results obtained here provide extension of Saigo results, Saxena and Ram [9], Saxena and Ram [12] and Chaurasia and Jain [18].

DEFINITIONS: The Fox's H-function [2, 3, 4] is in the form of Mellin-Barnes type [7] integral in following way

$$H_{p,q}^{m,n}[z] = H_{p,q}^{m,n}\left[z \left| \begin{matrix} (a_1, A_1),\ldots,(a_p, A_p) \\ (b_1, B_1),\ldots,(b_q, B_q) \end{matrix} \right. \right] = \frac{1}{2\pi i}\int_L h(s)\, z^s\, ds \qquad (2.1)$$

Where
$$h(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - B_j s)\prod_{j=1}^{n}\Gamma(1 - a_j + A_j s)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j + B_j s)\prod_{j=n+1}^{p}\Gamma(a_j - A_j s)} \qquad (2.2)$$

Where $0 \le n \le p$, $0 \le m \le q$ are non-negative integers; A_j and B_j are positive; a_j ($j=1,\ldots,p$) and b_j ($j=1,\ldots, q$) are real or complex, such that $A_j (b_h + \nu) \neq B_h (a_j - \lambda - 1)$, ($\nu, \lambda = 0,1, \ldots$; $h = 1,2, \ldots , m$; $j = 1,2, \ldots, n$), L is a suitable contour separating the simple poles of integrand $h(s)$ in (2.2).

Now we define generalized fractional calculus operators as follows: Let $\alpha, \alpha', \beta, \beta', \gamma \in C$ and $x > 0$ then the fractional calculus operators in generalized form of arbitrary order involving Appell function F_3, due to Saigo and Maeda [10] in the kernel are defined by the following equations:

$$^{\alpha,\alpha',\beta,\beta',\gamma}_{0,} f = \frac{(x)}{(\ )}\int(\ \text{-t})\ t\ \ F(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{t}{x},1-\frac{x}{t})f(t)\,dt;\mathrm{Re}(\gamma)>0 \qquad (2.3)$$

$$= \left(\frac{d}{dx}\right)^k I_{0,x}^{\alpha,\alpha',\beta+k,\beta',\gamma+k} f; \quad \mathrm{Re}(\gamma) \le 0, \quad k = [-\mathrm{Re}(\gamma)] + 1 \qquad (2.4)$$

$$I_{x,\infty}^{\alpha,\alpha',\beta,\beta',\gamma} f = \frac{(x)^{-\alpha'}}{\Gamma(\gamma)}\int_x^\infty (\text{t-}x)^{\gamma-1} t^{-\alpha} F_3(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t},1-\frac{t}{x})f(t)\,dt;\mathrm{Re}(\gamma)>0 \qquad (2.5)$$

$$= \left(-\frac{d}{dx}\right)^k I_{x,\infty}^{\alpha,\alpha',\beta,\beta'+k,\gamma+k} f;\mathrm{Re}(\gamma) \le 0, \quad k = [-\mathrm{Re}(\gamma)] + 1 \qquad (2.6)$$

The Saigo-Maeda operators are extensions of the Saigo operators defined as follows:

$$I_{0,x}^{\alpha,\beta,\eta} f = \frac{(x)^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x\text{-t})^{\alpha-1}\, {}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x})f(t)\,dt;\mathrm{Re}(\alpha)>0 \qquad (2.7)$$

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$$=\left(\frac{d}{dx}\right)^{k} I_{0,x}^{\alpha+k,\beta-k,\eta-k} f;\mathrm{Re}(\alpha)\le 0,\quad k=[-\mathrm{Re}(\alpha)]+1 \qquad (2.8)$$

$$I_{x,\infty}^{\alpha,\beta,\eta} f=\frac{1}{\Gamma(\alpha)}\int_{x}^{\infty}(\text{t-}x)^{\alpha-1} t^{-\alpha-\beta}\,{}_2F_1(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t})f(t)\,dt;\mathrm{Re}(\alpha)>0 \qquad (2.9)$$

$$=\left(-\frac{d}{dx}\right)^{k} I_{x,\infty}^{\alpha+k,\beta-k,\eta} f;\mathrm{Re}(\alpha)\le 0,\quad k=[-\mathrm{Re}(\alpha)]+1 \qquad (2.10)$$

Following Chaurasia and Jain [18] we denote by u_1 the class of functions *f(x)* on R_+ which are infinitely partialy differentiable with any order behaving as $O(|x|^{-\xi})$ when $x\to\infty$ for all ξ. Similarly by u_2, we denote the class of functions *f(x,y)* on the $R_+ * R_+$, which are infinitely partialy differentiable with any order behaving as $O(|x|^{-\xi_1}|y|^{-\xi_2})$ when $x\to\infty$, $y\to\infty$ for all ξ_i (i = 1,2).

On the same lines we denote by u_3, the class of functions *f(x,y,z)* defined on the $R_+ * R_+ * R_+$, which are infinitely partialy differentiable with any order behaving as $O(|x|^{-\xi_1}|y|^{-\xi_2}|z|^{-\xi_3})$ when $x\to\infty$, $y\to\infty$, $z\to\infty$ for all ξ_i (i = 1,2,3).

The Saigo-Maeda operator of Weyl type three dimensional fractional integration of order $Re(\alpha_i)>0$, $Re(\gamma_i)>0$ (i = 1,2,3) is defined in the class u_3 by

$$I_{x,\infty}^{\alpha_1,\alpha_1',\beta_1,\beta_1',\gamma_1} I_{x,\infty}^{\alpha_2,\alpha_2',\beta_2,\beta_2',\gamma_2} I_{x,\infty}^{\alpha_3,\alpha_3',\beta_3,\beta_3',\gamma_3}[f(x,y,z)]=\frac{x^{\alpha_1-\gamma_1}y^{\alpha_2-\gamma_2}z^{\alpha_3-\gamma_3}}{\Gamma(\gamma_1)\Gamma(\gamma_2)\Gamma(\gamma_3)}\int_x^\infty\int_y^\infty\int_z^\infty(u-x)^{\gamma_1-1}$$

$$(v-y)^{\gamma_2-1}(w-z)^{\gamma_3-1}u^{-\alpha_1}v^{-\alpha_2}w^{-\alpha_3}F_3(\alpha_1,\alpha_1',\beta_1,\beta_1',;\gamma_1;1-\frac{x}{u},1-\frac{u}{x}) \qquad (2.11)$$

$$F_3(\alpha_2,\alpha_2',\beta_2,\beta_2';\gamma_2;1-\frac{y}{v},1-\frac{v}{y})F_3(\alpha_3,\alpha_3',\beta_3,\beta_3';\gamma_3;1-\frac{z}{w},1-\frac{w}{z})f(u,v,w)\,du\,dv\,dw$$

THREE DIMENSIONAL LAPLACE TRANSFORM AND H-TRANSFORM: The Laplace transform *h(p,q,r)* of a function $f(x,y,z)\in u_3$ is written as [13] : let $Re(p)>0,Re(q)>0,Re(r)>0$

$$h(p,q,r)=L\left[f(x,y,z);p,q,r\right]=\int_0^\infty\int_0^\infty\int_0^\infty e^{-px-qy-rz}f(x,y,z)\,dx\,dy\,dz \qquad (3.1)$$

Similarly, the Laplace transform of

$$f\left[a\sqrt{x^2-b^2}\,H(x-b),c\sqrt{y^2-d^2}\,H(y-d),e\sqrt{z^2-f^2}\,H(z-f)\right]$$

is defined by *F(x,y,z)*, H(t) denotes Heaviside's unit step function.

By the three dimensional H-transform $\varphi(p,q,r)$ of a function *F(x,y,z)*, we mean the following repeated integral involving three different H-functions:

$$\phi(p,q,r)=\int_b^\infty\int_d^\infty\int_f^\infty(px)^{\rho-1}(qy)^{\sigma-1}(rz)^{\mu-1}H_{P_1,Q_1}^{M_1,N_1}\left[(px)^{k_1}\left|\begin{matrix}(a_{P_1},A_{P_1})\\(b_{Q_1},B_{Q_1})\end{matrix}\right.\right]$$

$$H_{P_2,Q_2}^{M_2,N_2}\left[(qy)^{k_2}\left|\begin{matrix}(c_{P_2},C_{P_2})\\(d_{Q_2},D_{Q_2})\end{matrix}\right.\right]H_{P_3,Q_3}^{M_3,3}\left[(rz)^{k_3}\left|\begin{matrix}(e_{P_3},E_{P_3})\\(f_{Q_3},F_{Q_3})\end{matrix}\right.\right]F(x,y,z)\,dxdydz \qquad \mathbf{(3.3)}$$

Here we assume that $b>0,d>0,f>0,k_1>0,k_2>0,k_3>0$; $\varphi(p,q,r)$ exists and belongs to u_3. **The generality of the H-function, the equation (3.3) provides a generalization of a number of integral transforms like the three-dimensional Laplace, Stieltjes, Hankel, Whittaker and G-transforms.**

RELATION BETWEEN THREE-DIMENSIONAL H-TRANSFORMS IN TERMS OF THREE-DIMENSIONAL SAIGO-MAEDA OPERATORS OF WEYL TYPE

In this section we evaluate three-dimensional H-transform $\varphi_1(p,q,r)$ of *F(x,y,z)* which will be needed for proof of the theorem considered in this section. We have

$$\phi_1(p,q,r)=H_{P_1+3,Q_1+3;P_2+3,Q_2+3;P_3+3,Q_3+3}^{M_1+3,N_1;\,M_2+3,N_2;\,M_3+3,N_3}[F(x,y,z;\rho,\sigma,\mu;p,q,r)=\int_b^\infty\int_d^\infty\int_f^\infty(px)^{\rho-1}(qy)^{\sigma-1}$$

$$(rz)^{\mu-1}H_{P_1+3,Q_1+3}^{M_1+3,N_1}\left[(px)^{k_1}\left|\begin{matrix}(a_{P_1},A_{P_1})\,(1-\rho,k_1)\,(1+\alpha_1-\beta_1-\rho,k_1)\,(1+\alpha_1+\alpha_1'+\beta_1'-\gamma_1-\rho,k_1)\\(1-\rho-\beta_1,k_1)\,(1+\alpha_1+\beta_1'-\gamma_1-\rho,k_1)\,(1+\alpha_1+\alpha_1'-\gamma_1-\rho,k_1)\,(b_{Q_1},B_{Q_1})\end{matrix}\right.\right]$$

$$H_{P_2+3,Q_2+3}^{M_2+3,N_2}\left[(qy)^{k_2}\left|\begin{matrix}(c_{P_2},C_{P_2})\,(1-\sigma,k_2)\,(1+\alpha_2-\beta_2-\sigma,k_2)\,(1+\alpha_2+\alpha_2'+\beta_2'-\gamma_2-\sigma,k_2)\\(1-\sigma-\beta_2,k_2)\,(1+\alpha_2+\beta_2'-\gamma_2-\sigma,k_2)\,(1+\alpha_2+\alpha_2'-\gamma_2-\sigma,k_2)\,(d_{Q_2},D_{Q_2})\end{matrix}\right.\right]$$

$$H_{P_3+3,Q_3+3}^{M_3+3,N_3}\left[(rz)^{k_3}\left|\begin{matrix}(e_{P_3},E_{P_3})\,(1-\mu,k_3)\,(1+\alpha_3-\beta_3-\mu,k_3)\,(1+\alpha_3+\alpha_3'+\beta_3'-\gamma_3-\mu,k_3)\\(1-\mu-\beta_3,k_3)\,(1+\alpha_3+\beta_3'-\gamma_3-\mu,k_3)\,(1+\alpha_3+\alpha_3'-\gamma_3-\mu,k_3)\,(f_{Q_3},F_{Q_3})\end{matrix}\right.\right]$$

$$.F(x,y,z)\,dx\,dy\,dz \qquad (4.1)$$

Where it is assumed that $\varphi_1(p,q,r)$ exists and belongs to u_3, the parameters $k_1>0$, $k_2>0$, $k_3>0$ and other conditions on the additional parameters $\alpha_1,\alpha_1',\beta_1,\beta_1',\gamma_1,\alpha_2,\alpha_2',\beta_2,\beta_2',\gamma_2\alpha_3,\alpha_3',\beta_3,\beta_3',\gamma_3$, corresponding to those integral involved exists.

Theorem 4.1: Let $\varphi(p,q,r)$ be given by (3.3) then for $Re(\gamma_1)>0,Re(\gamma_2)>0,Re(\gamma_3)>0,d>0,d>0,f>0,k_1>0,\ k_2>0,\ k_3>0$ there holds the formula

$$I_{p,\infty}^{\alpha_1,\alpha_1',\beta_1,\beta_1',\gamma_1} I_{q,\infty}^{\alpha_2,\alpha_2',\beta_2,\beta_2',\gamma_2} I_{r,\infty}^{\alpha_3,\alpha_3',\beta_3,\beta_3',\gamma_3}[\varphi(p,q,r)]=\varphi_1(p,q,r) \qquad (4.2)$$

Where $\varphi_1(p,q,r)$ is given by (4.1).

Proof: Let $Re(\gamma_1)>0$, $Re(\gamma_2)>0,Re(\gamma_3)>0$, then in view of (3.3), we find that

$$I_{p,\infty}^{\alpha_1,\alpha_1',\beta_1,\beta_1',\gamma_1} I_{q,\infty}^{\alpha_2,\alpha_2',\beta_2,\beta_2',\gamma_2} I_{r,\infty}^{\alpha_3,\alpha_3',\beta_3,\beta_3',\gamma_3}[\phi(p,q,r]=\frac{p^{\alpha_1-\gamma_1}q^{\alpha_2-\gamma_2}r^{\alpha_3-\gamma_3}}{\Gamma(\gamma_1)\Gamma(\gamma_2)\Gamma(\gamma_3)}$$

$$\int_p^\infty\int_q^\infty\int_r^\infty(u-p)^{\gamma_1-1}(v-q)^{\gamma_2-1}(w-r)^{\gamma_3-1}p^{-\alpha_1}q^{-\alpha_2}r^{-\alpha_3}F_3(\alpha_1,\alpha_1',\beta_1,\beta_1',;\gamma_1;1-\frac{p}{u},1-\frac{u}{p})$$

$$F_3(\alpha_2,\alpha_2',\beta_2,\beta_2';\gamma_2;1-\frac{q}{v},1-\frac{v}{q})F_3(\alpha_3,\alpha_3',\beta_3,\beta_3';\gamma_3;1-\frac{r}{w},1-\frac{w}{r})$$

$$\varphi(u,v,w)\,du\,dv\,dw$$

$$=\frac{p^{\alpha_1-\gamma_1}q^{\alpha_2-\gamma_2}r^{\alpha_3-\gamma_3}}{\Gamma(\gamma_1)\Gamma(\gamma_2)\Gamma(\gamma_3)}\int_p^\infty\int_q^\infty\int_r^\infty(u-p)^{\gamma_1-1}(v-q)^{\gamma_2-1}(w-r)^{\gamma_3-1}p^{-\alpha_1}q^{-\alpha_2}r^{-\alpha_3}$$

$$F_3(\alpha_1,\alpha_1',\beta_1,\beta_1',;\gamma_1;1-\frac{p}{u},1-\frac{u}{p}F_3(\alpha_2,\alpha_2',\beta_2,\beta_2';\gamma_2;1-\frac{q}{v},1-\frac{v}{q})$$

$$F_3(\alpha_3,\alpha_3',\beta_3,\beta_3';\gamma_3;1-\frac{r}{w},1-\frac{w}{r})\left\{\int_b^\infty\int_d^\infty\int_f^\infty(ux)^{\rho-1}(vy)^{\sigma-1}(wz)^{\mu-1}\right.$$

$$H_{P_1,Q_1}^{M_1,N_1}\left[(ux)^{k_1}\left|\begin{matrix}(a_{P_1},A_{P_1})\\(b_{Q_1},B_{Q_1})\end{matrix}\right.\right]H_{P_2,Q_2}^{M_2,N_2}\left[(vy)^{k_2}\left|\begin{matrix}(c_{P_2},C_{P_2})\\(d_{Q_2},D_{Q_2})\end{matrix}\right.\right]$$

$$\left.H_{P_3,Q_3}^{M_3,3}\left[(wz)^{k_3}\left|\begin{matrix}(e_{P_3},E_{P_3})\\(f_{Q_3},F_{Q_3})\end{matrix}\right.\right]F(x,y,z)\,dxdydz\right\}dudvdw \qquad (4.3)$$

On interchanging the order of integration and evaluating the *u*, *v*, *w*-integrals, we get

$$\int_x^\infty(u-p)^{\gamma-1}u^{\rho-\alpha-1}F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{p}{u},1-\frac{u}{p}\right)H_{P,Q}^{M,N}\left[(au)^k\left|\begin{matrix}(a_P,A_P)\\(b_Q,B_Q)\end{matrix}\right.\right]du=\quad(\gamma)x^{\rho+\gamma-\alpha-1}\times$$

$$H_{P+3,Q+3}^{M+3,N}\left[(au)^k\left|\begin{matrix}(a_P,A_P),(1-\rho,k),(1+\alpha-\beta-\rho,k)1+\alpha+\alpha'+\beta'-\gamma-\rho,k)\\(1-\rho-\beta,k),(1+\alpha+\beta'-\gamma-\rho,k),(1+\alpha+\alpha'-\gamma-\rho,k),(b_Q,B_Q)\end{matrix}\right.\right];Re(\gamma)>0\ldots(4.4)$$

Equation (4.4) can be established by means of the following formula [18]

$$\int_0^\infty x^{\rho-1}(1-x)^{\gamma-1}F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-x,1-\frac{1}{x}\right)dx=\left[\begin{matrix}\gamma,\ \rho+\alpha',\ \rho+\beta',\ \rho+\gamma-\alpha-\beta\\ \rho+\alpha'+\beta',\rho+\gamma-\alpha,\rho+\gamma-\beta\end{matrix}\right] \qquad (4.5)$$

Where $Re(\gamma)>0, Re(\rho)>0, Re(\gamma+\rho-\alpha-\beta)>0$ and $\Gamma\begin{bmatrix} a,b,c,\ldots \\ d,e,f,\ldots \end{bmatrix}$ represents the ratio of the product of several gamma functions i.e. $\frac{\Gamma(a)\Gamma(b)\Gamma(c)\ldots}{\Gamma(d)\Gamma(e)\Gamma(f)\ldots} = \Gamma\begin{bmatrix} a,b,c,\ldots \\ d,e,f,\ldots \end{bmatrix}$

The left hand side of (4.3) becomes

$$H^{M_1+3,N_1;\ M_2+3,N_2;\ M_3+3,N_3}_{P_1+3,Q_1+3;P_2+3,Q_2+3;P_3+3,Q_3+3}[F(x,y,z;\rho,\sigma,\mu;p,q,r)$$

$$=\varphi_1(p,q,r)$$

Which is required right hand side of the equation (4.2).

As far as the three-dimensional Weyl type Saigo-Maeda operators $I^{\alpha_1,\alpha_1',\beta_1,\beta_1',\gamma_1}_{x,\infty} I^{\alpha_2,\alpha_2',\beta_2,\beta_2',\gamma_2}_{y,\infty} I^{\alpha_3,\alpha_3',\beta_3,\beta_3',\gamma_3}_{z,\infty}$ preserve the class u_3, it follows that $\varphi_1(p,q,r)$ also belongs to u_3.

SPECIAL CASES: By putting $\alpha_1' = \alpha_2' = \alpha_3' = 0$, in theorem 4.1 and use the identity $\left(I^{\alpha+\beta,0,-\eta,\beta',\alpha}_{x,\infty} f\right)(x) = \left(I^{\alpha,\beta,\eta,}_{x,\infty} f\right)(x)$. Theorem reduced in to the form of two dimensional and one dimensional analogue.

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