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KAHLERIAN SPACE WITH BOCHNER CURVATURE TENSOR

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Abstract

In this paper, we have defined some important definitions. In the end, an important theorem on kahler space with Bochner Curvature Tensor has been established.

Key Words: Bochner Curvature tensor, Riemannian Curvature tensor, Ricci tensor, recurrene vector.

1. INTRODUCTION:

n (=2m) dimensional kahlerian space is a Riemannian space which admits a tensor field F_i^h and satisfying the following relations [1]

(1.1) $F_j^h F_h^i = -\delta_j^i$

(1.2) $F_{ij} = F_i^a g_{aj}$

(1.3) $F_{ij} = -F_{ji}$

(1.4) $\nabla_j F_{j.}^{i} = 0$

(1.5) Let $R^h_{ijk} = \partial_i \{^{\,h}_{j\ k}\} - \partial_j \{^{\,h}_{i\ k}\} + \{^{\,h}_{i\ l}\}\{^{\,l}_{j\ k}\} - \{^{\,h}_{j\ l}\}\{^{\,l}_{i\ k}\}$

$R = R_{jk} g^{jk}$ and $R_{jk} = R'_{ijk}$ are the Riemannian Curvature tensor, the scalar curvature and Ricci tensor respectively.

In the real coordinate system, the Bochner Curvature tensor may be defined as

(1.6) $K^h_{ijk} = R^h_{ijk} + \{1/(n+4)\}(R_{ik}\delta^h_j - R_{jk}\delta^h_i + g_{ik}R^h_j - g_{jk}R^h_i$
$+ S_{ik}F^h_j - S_{jk}F^h_i + F_{ik}S^h_j - F_{jk}F^h_i + 2S_{ij}F^h_k) - \{1/(n+2)(n+4)\}R(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j -$
$F_{jk}F^h_i + 2F_{ij}F^h_k$

Wherein,

$S_{ij} = F^a_iR_{aj}$

In the Kahlerian space, the Kahlerian Conharmonic Curvature tensor is defined as

(1.7) $T^h_{ijk} = R^h_{ijk} + \{1/(n+4)\}(R_{ik}\delta^h_j - R_{jk}\delta^h_i + g_{ik}R^h_j - g_{jk}R^h_i$
$+ S_{ik}F^h_j - S_{jk}F^h_i + F_{ik}S^h_j - F_{jk}S^h_i + 2S_{ij}F^h_k + 2F_{ij}S^h_k)$

Remark 1.1 :

It is to be noted that if a Kahlerian space is an Einstein one, then the Ricci tensor satisfies [2] [3]

(1.8) $R_{ij} = (1/n)Rg_{ij}$

(1.9) $\nabla_aR = 0$

Yields

(1.10) $\nabla_aR_{ji} = 0$

(1.11) $\nabla_aS_{ij} = 0$

(1.12) $S_{ij} = (1/n)RF_{ij}$

2. KAHLERIAN CONHARMONIC CURVATURE TENSOR :

If a Kahlerian space is an Einstein one, then the Bochner Curvature tensor becomes

(2.1) $U^h_{ijk} = R^h_{ijk} + \{1/n(n+2)\}R(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j - F_{jk}F^h_i + 2F_{ij}F^h_k)$

If a Kahlerian space is an Einstein one then the Kahlerian Conharmonic Curvature tensor reduces in the form

(2.2) $E^h_{ijk} = R^h_{ijk} + \{2/n(n+4)\}R(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j - F_{jk}F^h_i + 2F_{ij}F^h_k)$

By virtue of equations (2.1) and (2.2), we obtain

(2.3) $U^h_{ijk} = E^h_{ijk} - \{1/(n+2)(n+4)\}R(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j - F_{jk} F^h_i + 2F_{ij}F^h_k)$

In this regard, we have the following definitions:

Definition 2.1 :

A Kahler space is termed recurrent if it satisfies the relation

(2.4) $\nabla_aR^h_{ijk} = \lambda_aR^h_{ijk}$

for some non-zero recurrence vector λ_a.

Definition 2.2 :

A Kahlerian space is said to be Ricci recurrent if it satisfies the relation

(2.5) $\nabla_aR_{ij} = \lambda_aR_{ij}$

Contracting equation (2.5) with g^{ij} yields

(2.6) $\nabla_a R = \lambda_a R$

Definition 2.3 :

A Kahler space satisfying the relation

(2.7) $\nabla_a E^h_{ijk} = \lambda_a E^h_{ijk}$

Where λ_a is a non-zero recurrence vector, is termed Einstein kahlerian Conharmonic recurrent space.

Definition 2.4 :

A kahlerian space satisfying the relation

(2.8) $\nabla_a U^h_{ijk} = \lambda_a U^h_{ijk}$

Where λ_a is a non-zero recurrence vector, is called, an Einstein kahlerian space with recurrent Bochner Curvature tensor.

Theorem 2.1 :

Kahlerian Conharmonic Curvature tensor satisfies the following relation [4]

(2.9) $\nabla_a E^h_{ijk} + \nabla_i E^h_{jak} + \nabla_j E^h_{aik} = 0$

Theorem 2.2 :

In a kahlerian space, the Bochner Curvature tensor satisfies the following relation

(2.10) $\nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = 0$

Proof:

Inserting equation (2.8) in the L.H.S. of equation (2.10), we get

(2.11) $\nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \lambda_a U^h_{ijk} + \lambda_i U^h_{jak} + \lambda_j U^h_{aik}$

By virtue of equations (2.11) and (2.3), we obtain

(2.12) $\nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \lambda_a [E^h_{ijk} - \{R/(n+2)(n+4)\}(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j - F_{jk}F^h_i + 2F_{ij}F^h_k)] + \lambda_i [E^h_{jak} - \{R/(n+2)(n+4)\}(g_{ik}\delta^h_a - g_{ak}\delta^h_j + F_{jk}F^h_a - F_{ak}\delta^h_j + 2F_{ja}F^h_k)] + \lambda_j [E^h_{aik} - \{R/(n+2)(n+4)\}(g_{ak}\delta^h_i - g_{ik}\delta^h_a + F_{ak}F^h_i - F_{ik}F^h_a + 2F_{ai}F^h_k)]$

As a consequence of equations (2.12),(2.6) and (2.7), we get

(2.13) $\nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \nabla_a E^h_{ijk} - \{\nabla_a R/(n+2)(n+4)\}(g_{ik}\delta^h_j - g_{jk}\delta^h_i + F_{ik}F^h_j - F_{jk}F^h_i + 2F_{ij}F^h_k)] + \nabla_i E^h_{jak} - \{\nabla_i R/(n+2)(n+4)\}(g_{ik}\delta^h_a - g_{ak}\delta^h_j + F_{jk}F^h_a - F_{ak}\delta^h_j + 2F_{ja}F^h_k)] + \nabla_j E^h_{aik} - \{\nabla_j R/(n+2)(n+4)\}(g_{ak}\delta^h_i - g_{ik}\delta^h_a + F_{ak}F^h_i - F_{ik}F^h_a + 2F_{ai}F^h_k)]$

In view of equation (1.9) then equation (2.13) reduces in the form

(2.14) $\nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = \nabla_a E^h_{ijk} + \nabla_i E^h_{jak} + \nabla_j E^h_{aik}$

By virtue of equations (2.14) and (2.9), we obtain

$$\boxed{\nabla_a U^h_{ijk} + \nabla_i U^h_{jak} + \nabla_j U^h_{aik} = 0}$$

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