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# Hierarchical synchronization in complex dynamical networks with multipartite graph structure

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# Abstract

This paper investigates the coupling schemes and corresponding criteria for hierarchical synchronization in multipartite graph networks consisting of multi-layers non-identical chaotic oscillators. The global asymptotically stable criteria for linearly or adaptively coupled networks are derived respectively to guarantee each layer of oscillators synchronize to the same behavior. The numerical simulations show that one can selectively implement certain layer synchrony while other layers not by means of taking part of adaptive coupling or increasing the corresponding linearly coupling strength.

Keywords: Complex dynamical network, hierarchical synchronization, multipartite graph.

## **1. Introduction**

Complex networks have been studied extensively in various disciplines, such as social, biological, mathematical, and engineering sciences and so on[1]. Besides the properties of "small-world"[2] and "scale-free"[3], the cluster (or hierarchical) structure is a common property in real-world complex networks. Many real networks are composed of several clusters (or multi-layers) within which the nodes have some common properties in a cluster different from other clusters[4-7]. The multipartite graph network is a special cluster network with multi-layers structure, there are connections of nodes between different layers but without any connection within a layer. For example, in social tagging systems, the relationships among "users - object - label" constitute a triple-graph network (see Fig.1 for the schematic diagram).

Because of its widespread use in variety contexts such as biology, chemistry, ecology, sociology, and technology, synchronization of complex dynamical networks has been extensively investigated. There is a remarkable synchronous phenomenon, cluster (or group) synchronization, has been observed in real dynamics of complex networks[8-13]. Recently, some progress in cluster synchronization of complex dynamical networks have been reported[14-17]. In Ref.[14], Belykh et al. studied the cluster synchronization for conditional clusters and unconditional clusters in an oscillator network with given configuration, they proposed a graph theoretical approach and obtained the criteria for the existence and stability of cluster synchronization. In Ref.[15], authors presented a linear feedback control strategy to achieve the cluster synchronization for a network with identical oscillators. In Ref.[16], authors investigated the cluster synchronization in a dynamical network consisting of two groups of non-identical oscillators, and upper bounds of input strength for the synchrony of each cluster are derived under the "same-input" condition. In Ref.[17], we proposed a mathematical model of a complex dynamical network consisting of two groups of different oscillators, and presented corresponding criteria of cluster synchronization based on linear coupling and adaptive control schemes, respectively. In this paper, we further investigate the cluster synchronization of a complex dynamical network. Different from previous studies, we focus on the multipartite graph networks which consisted of multi-layers non-identical chaotic oscillators. The synchronization is defined as the hierarchical synchronization, which means that nodes in the same layer achieve the same synchronization state while nodes in different layers reach different synchronization state.

This paper is organized as follows. In Section 2, the network model of our research and mathematical preliminaries are introduced. The linear coupling and adaptive coupling criteria for the hierarchical synchronization are derived respectively in Section 3. In Section 4, the numerical simulations are provided to verify the theoretical analysis. Finally, the further discussion and a brief summary are given in Section 5 and 6.



Fig.1. Schematic diagram of a multipartite graph network

### 2. Model Description

Let's consider a multipartite graph complex dynamical network with M-layers of nodes, each layer contains a number of nodes with identical chaotic oscillators, the individual dynamical system and its dimension in one layer can differ from that in

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other layers. Suppose that the K-th layer contains  $N_K$  nodes which is  $n_K$  dimensional chaotic dynamical system. The multipartite graph network (with M-layers) is described by

$$\begin{cases} \dot{x}_{I,i} = F_I(x_{I,i}) + \sum_{J=1,J\neq I}^{M} c_{IJ} \sum_{j=1}^{N_J} a_{IJ}(i,j) (\Gamma_{IJ} x_{J,j} - x_{I,i}) \\ i = 1, 2, \cdots, N_I, I = 1, 2, \cdots, M \end{cases}$$
(1)

where  $x_{I,i} \in \mathbb{R}^{nI}$  is state vector of the i-th node in the I-th layer.  $F_I$  is a smooth nonlinear function which describes the node's dynamics of I-th layer.  $c_{IJ} > 0$  is the external excitation intensity acting on I-th layer from J-th layer. The matrix  $A_{IJ} = (a_{IJ}(i, j)) \in \mathbb{R}^{N_I \times N_J}$  (where  $a_{IJ}(i, j) \ge 0, I, J = 1, \dots, M, I \ne J$ ) represents the external connection from the I-th to J-th layer which satisfies the "same input" condition(see Definition 1).  $\Gamma_{IJ} \in \mathbb{R}^{n_I \times n_J}$  is the dimension-transformation matrix whose form is E or [E, 0] or [E, 0]T (where E denotes identity matrix, 0 is a proper dimension zero matrix).

For model (1), we have following assumption and definitions.

Assumption 1. Suppose that there exists positive constant  $\mathcal{E}_{V}$  such that

$$\|F_{K}(z_{1}) - F_{K}(z_{2})\| \leq \varepsilon_{K} \|z_{1} - z_{2}\|.$$

where  $z_1$ ,  $z_2$  are time-varying vectors.

**Definition 1.** A matrix  $H = (h_{ij})_{m \times n}$  is said to satisfy condition SI, if its elements satisfy  $h_{ik} = h_{jk}$ ,  $i, j = 1, \dots, m, k = 1, \dots, n$ .

Moreover, if the external connection matrices  $A_{IJ}(I, J = 1, \dots, M, I \neq J)$  in model (1) satisfy the condition SI, then we say the network (1) satisfying "same input" condition.

Note that, if the model (1) satisfies "same input" condition, one can get  $a_{IJ}(i, j) = a_{IJ}(1, j)$  and

 $\sum_{j=1}^{N_J} a_{IJ}(i,j) \triangleq a_{IJ}, (i = 1, 2, \dots, N_I), \text{ where } a_{IJ} \text{ is a positive constant which does not depend on i.}$ 

Denote that 
$$s_I = \frac{1}{N_I} \sum_{i=1}^{N_I} xI$$
, *i*, and  $\tilde{F}_I(s_I) = \frac{1}{N_I} \sum_{i=1}^{N_I} F_I(x_{I,i})$ 

Then

$$\dot{s}_{I} = F_{I}(s_{I}) + \sum_{J=1,J\neq I}^{M} c_{IJ} \sum_{j=1}^{N_{I}} a_{IJ}(1,j) \Gamma_{IJ} x_{Jj} - \sum_{J=1,J\neq}^{M} c_{IJ} a_{IJ} s_{I}$$
(2)

Definition 2. The set

$$S = \{(x_{1,1}, \dots, x_{1,N_1}, \dots, x_{M,1}, \dots, x_{M,N_M}) : x_{I,i} = s_{I,i} = 1, \dots, N_I, I = 1, \dots, M\}$$
  
is called hierarchical synchronous manifold of the network (1).

The synchronous errors of the I-th layer oscillators can be denoted as  $\delta x_{I,i} = x_{I,i} - s_I (i = 1, \dots, N_I)$ . Combining (1) and (2), the hierarchical synchronous errors of the network (1) satisfy

$$\delta \dot{x}_{I,i} = F_I(x_{I,i}) - \tilde{F}_I(s_I) - \sum_{J=1,J\neq I}^M c_{IJ} a_{IJ} \delta x_{I,i} \qquad i = 1, 2, \cdots, N_I, I = 1, 2, \cdots, M.$$
(3)

Obviously, the stability of hierarchical synchronous manifold S in network (1) is equivalent to the stability of system (3) at zero. we try to derive the criteria for the coupling strength such that the network(1) achieves hierarchical synchronization ,that is,  $\lim_{t \to \infty} \left\| \delta x_{I,i} \right\| = 0$  ( $i = 1, \dots, N_I$ ,  $I = 1, \dots, M$ ), which implies that each layer reaches synchronization.

In this paper, we assume that the model (1) satisfies Assumption 1 and "same input" condition.

#### 3. Main Result

In this section, we derive hierarchical synchronization criteria for network (1) with linear and adaptive coupling, respectively.

**Theorem 1.** The hierarchical synchronous manifold S of the linearly coupled network (1) is globally exponentially stable if the linear coupling strength  $C_{IJ}$  satisfies

$$\sum_{I,J\neq I}^{M} c_{IJ} a_{IJ} > \varepsilon_{I}, \quad I = 1, \cdots, M.$$

$$\tag{4}$$

Proof: Consider the Lyapunov function

I =

$$V(t) = \frac{1}{2} \sum_{I=1}^{M} \sum_{i=1}^{N_{I}} (\delta x_{I,I})^{T} \, \delta x_{I,i}$$

Its time derivative along the trajectory of Eq.(3) is

$$\dot{V}(t) = \sum_{I=1}^{M} \left[ \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} (F_{I}(x_{I,i}) - F_{I}(s_{I})) + \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} (F_{I}(s_{I}) - \tilde{F}_{I}(s_{I})) - \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \sum_{J=1, J \neq I}^{M} c_{IJ} a_{IJ} \delta x_{I,i} \right],$$
Since
$$\sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} (F_{I}(s_{I}) - \tilde{F}_{I}(s_{I})) = (F_{I}(s_{I}) - \tilde{F}_{I}(s_{I})) \sum_{i=1}^{N_{I}} (x_{I,i} - s_{I})^{T} = 0$$
There has
$$\sum_{i=1}^{M} (N_{I})^{N_{I}} (S_{I}) = (S_{I})^{N_{I}} (S_{I})^{N_{I}} (S_{I})^{N_{I}} (S_{I})^{N_{I}} (S_{I})^{N_{I}} = 0$$

$$\begin{split} \dot{V}(t) &= \sum_{I=1}^{M} \left[ \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} (F_{I}(x_{I,i}) - F_{I}(s_{I})) - \sum_{J=1,J\neq I}^{M} c_{JJ} a_{JJ} \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i} \right] \\ &\leq \sum_{I=1}^{M} \left[ \varepsilon_{I} \sum_{I=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i} - \sum_{J=1,J\neq I}^{M} c_{JJ} a_{JJ} \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i} \right] \\ &\leq \sum_{I=1}^{M} \left( \varepsilon_{I} - \sum_{J=1,J\neq I}^{M} c_{JJ} a_{JJ} \right) \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i} \leq -2\theta V(t) \end{split}$$

where  $\theta = \min\{|\varepsilon_k - \sum_{I=1}^{M} c_{II} a_{II}| : 1 < I \le M\}$ . Then there has  $V(t) \le V(0)e^{-2\theta t}$ 

Notice that  $V(t) \ge \frac{1}{2} \left\| \delta x_{I,i} \right\|^2$ , so  $\left\| \delta x_{I,i} \right\| \le \sqrt{2V(0)} e^{-\theta} \to 0$ , i.e.,  $\lim_{t \to +\infty} \left\| \delta x_{I,i} \right\| = 0$   $(i = 1, \dots, N_I, I = 1, \dots, M)$ . It implies

that the hierarchical synchronization manifold S of network (1) is globally exponentially stable. Now the proof is completed.

According to Theorem 1, the hierarchical synchronization can be achieved if there exists a J such that  $c_{II}a_{II} > \varepsilon_{I}(I = 1, \dots, M)$ . It implies there only need an external coupling drive for each layer to achieved the hierarchical synchronization. So we can get the following Corollary 1.

**Corollary 1.** In network (1), if for each I there exists constants J such that

$$c_{IJ}a_{IJ} > \varepsilon_I \quad I = 1, \cdots, M. \tag{5}$$

Then the hierarchical synchronous manifold S of the linearly coupled network (1) is globally exponentially stable. Note that, according to the criteria (4) and (5), one need to know the Lipschitz constant  $\mathcal{E}_{I}$  However, it is often difficult to obtain the precise values of  $\mathcal{E}_{I}$  in some practical systems. To overcome this drawback, we design the coupling strength  $c_{IJ}$  in network (1) as adaptive variable, and present an adaptive coupling scheme to realize hierarchical synchronization as following.

**Theorem 2.** For the network (1), take the coupling strength  $c_{II}$  Jas adaptive variable  $c_{II}(t)$ , then the hierarchical synchronous manifold S is globally asymptotically stable under following adaptive laws

$$\begin{cases} c_{IJ}(t) = \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i}, \\ c_{IJ}(0) > 0, I, J = 1, \cdots, M, J \neq I. \end{cases}$$
(6)

Proof: Consider the Lyapunov function

$$W(t) = \frac{1}{2} \sum_{I=1}^{M} \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i} + \sum_{I=1}^{M} \sum_{I=1,J\neq I}^{M} \frac{1}{2a_{IJ}} (a_{IJ} c_{IJ} (t) - \frac{1}{M} \varepsilon_{I} - \frac{1}{M})^{2}$$

Its time derivative along the trajectory of Eq.(3) is

$$\begin{split} \dot{W}(t) &= \sum_{I=1}^{M} \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta \dot{x}_{I,i} + \sum_{I=1}^{M} \sum_{I=1,J\neq I}^{M} (a_{IJ} c_{IJ} (t) - \frac{1}{M} \varepsilon_{I} - \frac{1}{M}) \dot{c}_{IJ}(t) \\ &= \sum_{I=1}^{M} [\sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} (F_{I} (x_{I,i}) - \tilde{F}_{I} (s_{I})) - \sum_{J=1,J\neq I}^{M} c_{IJ} a_{IJ} \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i}] \\ &+ \sum_{I=1}^{M} (\sum_{I=1,J\neq I}^{M} c_{IJ} a_{IJ} - \varepsilon_{I} - 1) \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i} \\ &\leq \sum_{I=1}^{M} [\varepsilon_{I} \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i} - (\varepsilon_{I} + 1) \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \delta x_{I,i}] \end{split}$$

$$= -\sum_{I=1}^{M} \sum_{i=1}^{N_{I}} (\delta x_{I,i})^{T} \, \delta x_{I,i} \leq 0.$$

By the LaSalle-Yoshizawa theorem[18,19], there has,  $\lim_{t \to 0} \left\| \delta x_{I,i} \right\| = 0 (i = 1, \dots, N_I, I = 1, \dots, M)$ 

# which means that the hierarchical synchronous manifold S is globally asymptotically stable. Now the proof is completed.

# 4. Numerical Example

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In order to verify the above theoretical results, an illustrative example is provided in this section. Let us consider a triple-graph network consisting 3-layers of non-identical nodes, where x-layer, y-layer and z-layer contain 30 Lorenz chaotic oscillators[20], 20 hyperchaotic  $L\ddot{u}$  oscillators[21], and 50 Chen oscillators[22], respectively. The model as a numerical example of network (1) is described by

$$\begin{cases} \dot{x}_{i} = F_{1}(x_{i}) + c_{1} \sum_{j=1}^{20} b_{ij} (\Gamma_{1} y_{j} - x_{i}), 1 \le i \le 30 \\ \dot{y}_{i} = F_{2}(y_{i}) + c_{2} \sum_{j=1}^{50} \overline{b}_{ij} (\Gamma_{2} z_{j} - y_{i}), 1 \le i \le 20 \\ \dot{z}_{i} = F_{3}(z_{i}) + c_{3} \sum_{j=1}^{30} \widehat{b}_{ij} (\Gamma_{3} x_{j} - z_{i}), 1 \le i \le 50 \end{cases}$$

$$(7)$$

where the matrixes  $B_1 = (b_{ij}), B_2 = (\overline{b}_{ij}), and B_3 = (\widehat{b}_{ij})$  and  $\Gamma_1, \Gamma_2, \Gamma_3$  satisfy the conditions of model (1).

$$F_{1}(x_{i}) = (10(x_{i2} - x_{i1}), 28x_{i1} - x_{i2} - x_{i1}x_{i3}, x_{i1}x_{i2} - 8/3x_{i3})^{T},$$
  

$$F_{2}(y_{i}) = (36(y_{i2} - y_{i1}) + y_{i4}, 20y_{i2} - y_{i1}y_{i3}, y_{i1}y_{i2} - 3y_{i3}, y_{i1}y_{i3} + y_{i4})^{T},$$
  

$$F_{3}(z_{i}) = (35(z_{i2} - z_{i1}), -7z_{i1} + 28z_{i2} - z_{i1}z_{i3}, z_{i1}z_{i2} - 8/3z_{i3})^{T}.$$

In the simulation, we take initial values  $x_i(0) = (-5 + 0.5i, 0.5i, 5 + 0.5i)^T (1 \le i \le 30)$ ,

 $y_i(0) = (-5 + 0.5i, 0.5i, 5 + 0.5i)^T (1 \le i \le 20), \quad z_i(0) = (-5 + 0.5i, 0.5i, 5 + 0.5i, 10 + 0.5i)^T (1 \le i \le 50), \text{ and } c_1(0) = c_2(0) = c_3(0) = 1.$ 



Fig.2. Hierarchical synchronization in the linearly coupled network (7)( $c_i$ = 1, i = 1, 2, 3).(a) The synchrony states  $x_i(1 \le i \le 30)$  of x-layer oscillators; (b) The synchrony states  $y_i(1 \le i \le 20)$  of y-layer oscillators; (c) The synchrony states  $z_i(1 \le i \le 50)$  of z-layer oscillators.





Fig.3. Hierarchical synchronization of the adaptively coupled network (7) (the adaptive laws of variables c1, c2, c3 follow (6)). (a) The synchrony states  $xi(1 \le i \le 30)$  of x-layer oscillators; (b) The synchrony states  $yi(1 \le i \le 20)$  of y-layer oscillators; (c) The synchrony states  $zi(1 \le i \le 50)$  of z-layer oscillators;(d) The adaptive coupling strength c1, c2, c3.

Fig.2 and Fig.3 display the numerical simulation results of network (7) with linear coupling and adaptive oupling, respectively. They show that a set of nodes belonging to each layer synchrony to the same behavior, that is, the hierarchical synchronization has been achieved.

### 5. Further Discussion

In the above section, we have derived the global asymptotically stable synchronous criteria for linearly or adaptively coupled networks under the "same input" condition. However, the synchronous conditions are sufficient but not necessary. The Fig.2 and 3 show that the chaotic oscillators reach almost complete synchronization which implied the sufficient conditions are too strong. If we try to relax the theoretical conditions, what would happen?

First, we reduce part of the linear coupling strength in network (1). The numerical simulation results show that there has been partial hierarchical synchronization. Fig.4 shows that only x-layer reaches synchronization while other layers can not when we reduce the coupling strength of y-layer and z-layer. Next, we reduce part of the adaptive coupling variables in network (1). There also appeared partial hierarchical synchronization. Fig.5 shows only y-layer reaches synchronization while other layers can not by taking part of external adaptive coupling. In the same way, we can selectively implement any one layer-synchrony while other layers not so as to save the physical cost.



Fig.4. Partly hierarchical synchronization in the linearly coupled network (7)(c1= 1, c2= c3= 0.1). (a)The synchrony states  $xi(1 \le i \le 30)$  of x-layer oscillators; (b) The asynchrony states  $yi(1 \le i \le 20)$  of y-layer oscillators; (c) The asynchrony states  $zi(1 \le i \le 50)$  of z-layer oscillators.



Fig.5. Partly hierarchical synchronization in the partly adaptively coupled network (7)(c1=c3=0.1, c2is an adaptive variable). (a) The asynchrony states  $xi(1 \le i \le 30)$  of x-layer oscillators; (b) The ynchrony states  $yi(1 \le i \le 20)$  of y-layer oscillators; (c) The asynchrony states  $zi(1 \le i \le 50)$  of z-layer oscillators; (d) The adaptive coupling strength c2.

## 6. Conclusion

This paper has presented linear and adaptive coupling schemes and the corresponding criteria in order to achieve hierarchical synchronization in multipartite graph networks. The research shows that the global asymptotical stability of the hierarchical synchronization can be guaranteed by increasing the linear coupling strength or taking adaptive coupling scheme. Moreover, one can apply this method to selectively implement partial hierarchical synchronization so as to save the physical cost.

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