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FUZZY TOPSIS ALGORITHM FOR MULTI CRITERIA DECISION MAKING WITH APPLICATION OF MARKETING MIX UNDER α-CUT

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Abstract
The aim of this paper is to express the importance of a fuzzy TOPSIS (Technique for Ordering Preference by Similarity to Ideal Solution) model with the components of Marketing Mix for Multi Criteria Decision Making (MCDM) problem. In this paper the weight of each alternative and the weight of each criterion are described by linguistic in positive trapezoid fuzzy numbers. A α-cut method is proposed to calculate a closeness coefficient for the rating of all alternatives from fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). Finally, a numerical method demonstrates the possibility of the proposed method.

Keywords: *Fuzzy TOPSIS; Marketing Mix; MCDM; α-Cut.*

1. Introduction
Decision making is a procedure for finding the alternative among a series of available alternatives. When we consider several criterions in decision making problems, they can be referred to as multi criteria decision making (MCDM). MCDM, has been one of the research areas in management operations and sciences, that due to the various applied need, has developed rapidly during the recent decade.

A MCDM problem can be concisely expressed in matrix format as:

$$G = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ . & . & & . \\ . & . & \cdots & . \\ . & . & & . \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \end{array}, W = [W_1, W_2 \ldots W_n]$$

Where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to be chosen , C_1 , $C_2,\ldots,C_n$ are criteria with which alternative performance are measured, G_{ij} is the rating of alternative A_i with respect to criterion C_j, W_j is weight of criterion C_j. MCDM problems can be divided into two types of problems. One of them is classic MCDM problems, in which the rates and the weights of the criteria are measured precisely [7, 8, 13]. The other decision making is fuzzy multi criteria (FMCDM) in which rates and weights are appraised in uncertain and vague form and usually are stated in linguistic variables and fuzzy numbers [1, 17].

TOPSIS technique is one of the known technique for classical MCDM which was first introduced by Yoon and Hwang [8]. The main concept of TOPSIS algorithm is the definition of positive and negative ideal solution. The ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria. The optimal alternative is the one which has the shortest distance from the positive ideal solution (PIS) and the farthest from negative ideal solution (NIS).

Fuzzy TOPSIS is to assign the importance of criteria and the performance of alternatives by using fuzzy number instead of crisp numbers. According to the concept of TOPSIS, we define the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS). Finally, a closeness coefficient of each alternative is defined to determine the ranking order of all alternatives.

The alternative is close to the (FPIS), and far from the (FNIS), and then this alternative will get a high ranking order.
In this paper the concept of "Marketing Mix", Neil Borden reconstructed the history of the term Marketing Mix [2, 3]. The Marketer E.Jerome Mc Carthy proposed a four Ps classification in 1960, which has since been used by marketers throughout the world [11]. It is a term used to described the combination of tactics used by a business to achieve its objectives by marketing its products or services effectively to a particular target customer group. It is also referred to as the '4PS' product, price, promotion and place.

Product is seen as an item that satisfies what a consumer demands. You need to be sure that your products and services continue to meet your customers' needs. The amount a customer pays for the product. The price is very important as it determines the company's profit and hence survive.

Adjusting the price has a profound impact on the marketing strategy, and depending on the price elasticity of the product, often will affect the demand and sales as well.

Place (distribution) refers to providing at a place which is convenient for consumers to access.

Promotion refers to all of the communicational methods that a marketer may use to provide information to different parties about the product. In this paper, at first, basic definition is started. In the next part the fuzzy TOPSIS decision making method is introduced for suitable problem solving of alternatives and then the suggested method is explained by the use of Marketing Mix and an example. Finally, the paper will be finished by a conclusion.

2. Preliminaries

Zadeh (1965) [19] first introduced the fuzzy sets theory to resolve the vagueness, ambiguity of human judgment. In this section, we present some basic definitions related to the fuzzy sets theory.

A. Fuzzy set and fuzzy numbers

Definition 1. Let X be the universe of discourse. A fuzzy set $\tilde{Q}$ of X is characterized by a membership function $\mu_{\tilde{Q}}$(x): X → [0,1] which indicates the degree of $x \in X\ in\ Q.$

Definition 2. The fuzzy number Q is a triangular fuzzy number if its membership function f_Q is given by van Larrhoven & Pedrycz [14, 6, 10] :

$$f_Q(x) = \begin{cases} 0, & x < a, \\ (x-a)/(b-a), & a \le x \le b, \\ (x-c)/(b-c), & b \le x \le c, \\ 0, & x > c, \end{cases} \qquad (1)$$

Where a , b and c are real numbers. For convenience, Q can be defined as a triplet (a, b, c).

Definition 3. The membership function f_Q of the fuzzy number Q can also be expressed as:

$$f_Q(x) = \begin{cases} f_Q^L(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ f_Q^R(x), & c \le x \le d, \\ 0, & otherwise \end{cases} \qquad (2)$$

Where $f_Q^L(x)$ and $f_Q^R(x)$ are the left and right membership functions of fuzzy number Q, respectively [6].

It is assumed that Q is convex, normal and bounded, i.e. $-\infty < a, d < \infty$. For convenience, the fuzzy number in definition 3 can be denoted by Q=[a, b, c, d].

Definition 4. The α-cut of fuzzy number Q can be defined as (Kauffman & Gupta [9]):

$$Q^\alpha = \{x \mid f_Q(x) \ge \alpha,\}, \quad where\ x \in R, \alpha \in [0,1].$$

Q^α is a non-empty bounded closed interval contained in R and can be denoted by $Q^\alpha = [Q_L^\alpha, Q_u^\alpha], where\ Q_L^\alpha\ and\ Q_u^\alpha$ are its lower and upper bounds, respectively [6]. For example, if trapezoid fuzzy number Q=[a, b, c, d], then the α -cut of Q is expressed as:

$$Q^\alpha = [(b-a)\alpha + a, (c-d)\alpha + d] \qquad (3)$$

B. Arithmetic operations of fuzzy numbers

Given fuzzy numbers Q and K, Q, k $\in R^+$, the α -cuts of Q and K are $Q^\alpha = [Q_L^\alpha, Q_u^\alpha]\ and\ K^\alpha = [K_L^\alpha, K_u^\alpha]$, respecti-vely. By the interval arithmetic, some operations of Q and K can be expressed as follows (Kauffman & Gupta, [9]):

$$(Q \oplus K)^\alpha = [Q_L^\alpha + K_L^\alpha, Q_u^\alpha + K_u^\alpha], \qquad (4)$$

$$(Q \Theta K)^\alpha = [Q_L^\alpha - K_u^\alpha, Q_u^\alpha - K_L^\alpha], \qquad (5)$$

$$(Q \otimes K)^\alpha = [Q_L^\alpha . K_L^\alpha, Q_u^\alpha . K_u^\alpha], \qquad (6)$$

$$(Q \phi K)^\alpha = \left[\frac{Q_L^\alpha}{K_u^\alpha}, \frac{Q_u^\alpha}{K_L^\alpha} \right], \qquad (7)$$

$$(Q \otimes r)^\alpha = [Q_L^\alpha . r, Q_u^\alpha . r], \quad r \in R^+. \qquad (8)$$

3. Ranking Fuzzy Numbers

There are various methods for ranking fuzzy numbers. The articles [4, 15, 16] are a comprehensive overview of existing approaches which referred to as below:

3.1. Ranking fuzzy numbers by mean of removals

The mean of removals, by Kauffman & Gupta (1991) is applied to consider a fuzzy number Q = [a, b, c, d] . The left removal of Q , denoted by Q_L, and the right removal of Q, denoted by Q_R, are defined as follows [6]:

$$Q_L = b - \int_a^b f_Q^L(x)dx, \qquad (9)$$

$$Q_R = c + \int_c^d f_Q^R(x)dx, \qquad (10)$$

The mean removal of the Q_L and Q_R is then defined as:

$$M(Q) = \frac{1}{2}(Q_L + Q_R) \qquad (11)$$

$$= \frac{1}{2}(b+c) + \frac{1}{2}\left(\int_c^d f_Q^R(x)dx - \int_a^d f_Q^L(x)dx \right)$$

Herein, M(Q) is used to compare fuzzy numbers. The larger the M(Q), the larger the fuzzy number Q. Therefore, for any two fuzzy numbers Q_i and Q_j, if $M(Q_i) > M(Q_j)$, then $Q_i > Q_j$. if $M(Q_i) = M(Q_j)$, then $Q_i = Q_j$. Finally, if $M(Q_i) < M(Q_j)$, then $Q_i < Q_j$.

3.2. Ranking based on the distance concept

Saade and Schwarzlander [12], by the use of the distance concept, have made the contrast fuzzy numbers. They have used only unnegative values for arranging the fuzzy numbers. Yao and Wu [18] have considered the distance of d^* an R which is $d^*(a,0)=a$, $d^*(a,b)=a-b$ for all a, $b \in R$. They have defined the distance for $\tilde{A}, \tilde{B} \in$(H(R) is fuzzy numbers' family) as below:

$$d(\tilde{A}, \tilde{B}) = \frac{1}{2}\int_0^1 \left([\tilde{A}]_\alpha^L + [\tilde{A}]_\alpha^u - [\tilde{B}]_\alpha^L - [\tilde{B}]_\alpha^u \right) d\alpha \qquad (12)$$

Where $[\tilde{A}]_\alpha^L, [\tilde{A}]_\alpha^u, [\tilde{B}]_\alpha^L, [\tilde{B}]_\alpha^u$ are lower and upper bounds of $\tilde{A}, \tilde{B}$, respectively. From (12) let:

$$\tilde{B} < \tilde{A} \ if \ d(\tilde{A}, \tilde{B}) > 0 \ ,$$

$$\tilde{B} > \tilde{A} \ if \ d(\tilde{A}, \tilde{B}) < 0 \ ,$$

$$\tilde{B} \approx \tilde{A} \ if \ d(\tilde{A}, \tilde{B}) = 0$$

3.3. Vertex method

Let [5] $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) be two triangular fuzzy numbers, then the vertex method is defined to measure the distance between them as:

$$d(\tilde{A}, \tilde{B}) = \sqrt{1/3[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \qquad (13)$$

3.4. Fuzzy Euclidean distance

Let $\tilde{A}$ = (a_1, a_2, a_3, a_4) and $\tilde{B}$ = (b_1, b_2, b_3, b_4) be two trapezoidal fuzzy numbers, then:

$$d(\tilde{A}, \tilde{B}) = \sqrt{1/4[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2]} \qquad (14)$$

4. The proposed fuzzy TOPSIS algorithm

The concept of linguistic variables is used for providing approximate characterization, when conventional quantitative terms are complex or ill defined.

These linguistic variables can be expressed in positive trapezoid fuzzy numbers as table I and table II.

TABLE I. Linguistic variables for the rating

Linguistic variable	fuzzy number
Very Poor (VP)	(0,1,1.67,3)
Poor (P)	(1,5.5,7,8)
Medium Poor (MP)	(2,7.67,8,9)
Fair (F)	(5,7,7.67,9)
Medium Good (MG)	(4,7.67,8,9)
Good (G)	(3,8.33,9.33,10)
Very Good (VG)	(6,8.67,10,10)

TABLE II. Linguistic variables for importance weight of each criterion

Linguistic variable	Corresponding trapezoid fuzzy number
Very Low (VL)	(0,0,1,2)
Low (L)	(1,2,2,3)
Medium Low (ML)	(2,3,4,5)
Medium (M)	(4,5,5,6)
Medium High (MH)	(5,6,7,8)
High (H)	(7,8,8,9)
Very High (VH)	(8,9,10,10)

Assume that x= $\{x_{ij} \mid i=1,2,\ldots,m\ ,\ j=1,2,\ldots,n\}$ is the rate alternatives with respect to various criteria (C_j, j=1…,n).

A decision maker D_K (k=1, 2,…, K) can express his membership function $\mu_{\tilde{R}k}(x)$ with the positive trapezoid fuzzy numbers $\tilde{R}_k\,(k=1,2,\ldots,K)$.

If decision maker's fuzzy rates be positive trapezoid fuzzy numbers $\tilde{R}_k=(r_k^a,r_k^b,r_k^c,r_k^d)$ then the combination of decision makers' fuzzy rates can be denoted by $\tilde{R}=(r^a,r^b,r^c,r^d)$.

Where:

$$r^a=\min_k\{r_k^a\},$$
$$r^b=\frac{1}{K}\sum_{k=1}^{K}r_k^b, \qquad (15)$$
$$r^c=\frac{1}{K}\sum_{k=1}^{K}r_k^c,\ r^d=\max_k\{r_k^d\}$$

Assume that rating of alternatives and the importance fuzzy weights of each criteria for the kth decision maker is:

$$\tilde{X}_{ijk}=(x_{ijk}^a,x_{ijk}^b,x_{ijk}^c,x_{ijk}^d),$$
$$\tilde{w}_{jk}=(w_{jk}^a,w_{jk}^b,w_{jk}^c,w_{jk}^d)$$

(i=1, 2, …, m , j=1, 2,…, n)

As it is state above, the combination of alternatives' fuzzy rates $\tilde{X}_{ij}$ can be described as:

$\tilde{X}_{ij} = (x_{ij}^{a}, x_{ij}^{b}, x_{ij}^{c}, x_{ij}^{d})$. Where:

$$x_{ij}^{a} = \min_{k}\{x_{ijk}^{a}\} ,$$

$$x_{ij}^{b} = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^{b},$$

$$x_{ij}^{c} = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^{c}, \qquad (16)$$

$$x_{ij}^{d} = \max_{k}\{x_{ijk}^{d}\}$$

For combining of each criteria of fuzzy weights $\tilde{w}_j$ can be denoted by:

$$\tilde{w}_j = (w_j^{a}, w_j^{b}, w_j^{c}, w_j^{d})$$

where:

$$w_j^{a} = \min_{k}\{w_{jk}^{a}\}, w_j^{b} = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^{b} \qquad (17)$$

$$w_j^{c} = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^{c}, w_j^{d} = \max_{k}\{w_{jk}^{d}\}$$

A fuzzy multi criteria decision making problem can be briefly expressed in matrix format as:

$$\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$$

$$\tilde{D} = \begin{bmatrix} \tilde{X}_{11} & \tilde{X}_{12} & \cdots & \tilde{X}_{1n} \\ \tilde{X}_{21} & \tilde{X}_{22} & \cdots & \tilde{X}_{2n} \\ \tilde{X}_{m1} & \tilde{X}_{m2} & \cdots & \tilde{X}_{mn} \end{bmatrix}$$

Therefore, we can obtain the normalized fuzzy decision matrix denoted by:

$$\tilde{R} = [\tilde{r}_{ij}]_{mxn} \qquad (18)$$

Where B and C are the set of benefit criteria and cost criteria, respectively, and

$$\tilde{r}_{ij} = x_{ij} / x_{ij}^{+} , \ x_{ij}^{+} = \max_{i}\{x_{ij}^{d}\} \text{ for each j } \in B$$

$$\tilde{r}_{ij} = x_{ij}^{-} / x_{ij} , \ x_{ij}^{-} = \min_{i}\{x_{ij}^{a}\} \text{ for each j } \in C \qquad (19)$$

The normalization method mentioned above is to preserve the property that the ranges of normalized trapezoid fuzzy numbers belong to [0,1].

The weighted normalized fuzzy decision matrix is denoted by:

$$\tilde{P} = [\tilde{p}_{ij}]_{m\times n} \quad (i = 1,2,\ldots,m,\ j = 1,2,\ldots,n) \qquad (20)$$

$$\text{where}: \tilde{P}_{ij} = \tilde{r}_{ij} (.) \tilde{w}_j$$

So we can define the fuzzy positive ideal solution (FPIS,O^+) and fuzzy negative ideal solution (FNIS,O^-) as:

$$O^{+} = (\tilde{F}_1^{*}, \tilde{F}_2^{*}, \ldots, \tilde{F}_n^{*})$$

$$(21)$$

$$O^{-} = (\tilde{F}_1^{-}, \tilde{F}_2^{-}, \ldots, \tilde{F}_n^{-})$$

Where $\tilde{F}_j^{*} = (f_j^{*}, f_j^{*}, f_j^{*}, f_j^{*}), \tilde{F}_j^{-} = (f_j^{-}, f_j^{-}, f_j^{-}, f_j^{-})\ \forall i, j$

$$f_j^* = \max_i \{p_{ij}^d\},\ f_j^- = \min_i \{p_{ij}^a\}$$

The distance of each alternative from O+, O- for $\alpha \in [0,1]$ can be calculated as:

$$d_i^*(\alpha) = \sum_{j=1}^{n} d(\tilde{F}_j^*, \tilde{p}_{ij}) = \sum_{j=1}^{n} (2f_j^* - p_{ij}^a - p_{ij}^d) +$$

$$+\left(\frac{\alpha}{2}\right)\sum_{j=1}^{n} (p_{ij}^a + p_{ij}^d - p_{ij}^b - p_{ij}^c) \quad , \quad \forall i \qquad (22)$$

$$d_i^-(\alpha) = \sum_{j=1}^{n} d(\tilde{p}_{ij}, \tilde{F}_j^-) = \sum_{j=1}^{n} (p_{ij}^a + p_{ij}^d - 2f_j^-) +$$

$$+\left(\frac{\alpha}{2}\right)\sum_{j=1}^{n} (p_{ij}^b + p_{ij}^c - p_{ij}^a - p_{ij}^d) \quad , \quad \forall i \qquad (23)$$

A closeness coefficient is defined to determine the ranking order for all alternatives: Once the $d_i^*(\alpha)$ *and* $d_i^-(\alpha), \alpha \in [0,1]$ of each alternative has been calculated, the closeness coefficient of each alternative is calculated as:

$$CC_i(\alpha) = \frac{d_i^-(\alpha)}{d_i^*(\alpha) + d_i^-(\alpha)} \qquad i = 1,2,\ldots,m \qquad (24)$$

Obviously, a large value of index $CC_i(\alpha)$ demonstrates that the alternative is closer to the FPIS (O^+) and farther from FNIS (O^-) , and thus alternative will approach to rank 1.

5. Numerical Example

Imagine that a tile factory with the help of some of its agencies all over the country want to rank the determined alternatives under Marketing Mix' components that are made the basement of a marketing system. After the early probe there remain 15 alternatives for more probs. 13 decision making committees have been elected for interviewing and choosing the best alternative.

Alternatives are considered as below:

Determining the suitable structure of **product** that includes:

(A_1) Variety
(A_2) Coloring
(A_3) Quality
(A_4) Services (Pre – within – post) the sale Determining the suitable **pricing** way that are:
(A_5) Installment saling
(A_6) Offering different kinds of discounts
(A_7) The balance between price and quality of manufactures

Determining the proper quality of **place** which include:

(A_8) Goods suitability
(A_9) On time goods delivery
(A_{10}) Sellers' characteristics (credit, …)
(A_{11}) Agencies' suitable location

Determining the suitable quality of **promotion** including:

(A_{12}) Advertising between contractors and mass producers
(A_{13}) Advertising in specialized journals
(A_{14}) Participation in fairs
(A_{15}) Presenting advertising gifts

Criterions are considered as below:

(C_1) Achieving sustainable competitive advantage
(C_2) Pioneering in designing fields and new models in internal markets as the same as the universal tile industry
(C_3) Production quality leadership
(C_4) Increasing customers satisfaction
(C_5) Maintaining profitability
(C_6) Increasing exports

And the results are calculated as Table III , IV, V.

TABLE III.

d_i^*					
α \ A	0	0.25	0.5	0.75	1
A_1	59.7000	59.1992	58.6983	58.1975	57.6966
A_2	59.3000	58.5158	57.7316	56.9474	56.1632
A_3	59.5000	58.9721	58.4442	57.9163	57.3884
A_4	59.4000	58.9321	58.4641	57.9962	57.5282
A_5	58.9000	58.1267	57.3534	56.5801	55.8069
A_6	59.1000	58.3000	57.5001	56.7001	55.9002
A_7	59.3000	58.7710	58.2420	57.7130	57.1840
A_8	58.7000	58.0041	57.3082	56.6123	55.9164
A_9	58.4000	57.5933	56.7867	55.9800	55.1733
A_{10}	58.6000	57.8299	57.0599	56.2898	55.5197
A_{11}	58.2000	57.6728	57.1456	56.6184	56.0912
A_{12}	59.3000	58.8861	58.4722	58.0583	57.6443
A_{13}	59.5000	59.1763	58.8526	58.5290	58.2053
A_{14}	59.3000	58.8951	58.4902	58.0854	57.6805
A_{15}	59.5000	59.2716	59.0432	58.8147	58.5863

TABLE IV.

d_i^-					
α \ A	0	0.25	0.5	0.75	1
A_1	81.0030	81.5038	82.0047	82.5055	83.0064
A_2	84.0000	84.7842	85.5684	86.3526	87.1368
A_3	82.0010	82.5289	83.0568	83.5847	84.1126
A_4	85.0010	85.4689	85.9369	86.4048	86.8728
A_5	86.0000	86.7733	87.5466	88.3199	89.0931
A_6	86.0010	86.8010	87.6009	88.4009	89.2008
A_7	83.0000	83.5290	84.0580	84.5870	85.1160
A_8	90.0000	90.6959	91.3918	92.0877	92.7836
A_9	91.0000	91.8067	92.6133	93.4200	94.2267
A_{10}	91.0000	91.7701	92.5401	93.3102	94.0803
A_{11}	88.0010	88.5282	89.0554	89.5826	90.1098
A_{12}	82.0010	82.4149	82.8288	83.2427	83.6567
A_{13}	82.0010	82.3247	82.6484	82.9720	83.2957
A_{14}	83.0000	83.4049	83.8098	84.2146	84.6195
A_{15}	82.0010	82.2294	82.4578	82.6863	82.9147

TABLE V. The Closeness Coefficient

CC						
α \ A	0	RANK	0.25	RANK	0.5	RANK
A_1	0.5757	12	0.5793	15	0.5828	14
A_2	0.5862	8	0.5917	8	0.5971	7
A_3	0.5795	11	0.5832	12	0.5870	11
A_4	0.5886	7	0.5919	7	0.5951	8
A_5	0.5935	5	0.5988	5	0.6042	5
A_6	0.5927	6	0.5982	6	0.6037	6
A_7	0.5833	9	0.5870	9	0.5907	9
A_8	0.6052	3	0.6099	3	0.6146	3
A_9	0.6091	1	0.6145	1	0.6199	1
A_{10}	0.6083	2	0.6134	2	0.6186	2
A_{11}	0.6019	4	0.6055	4	0.6091	4
A_{12}	0.5803	10	0.5833	11	0.5862	12
A_{13}	0.5795	11	0.5818	13	0.5841	13
A_{14}	0.5833	9	0.5861	10	0.5890	10
A_{15}	0.5795	11	0.5811	14	0.5827	15

TABLE VI. The Closeness Coefficient

CC				
α \ A	0.75	RANK	1	RANK
A_1	0.5864	13	0.5899	13
A_2	0.6026	7	0.6081	7
A_3	0.5907	11	0.5944	11
A_4	0.5984	8	0.6016	8
A_5	0.6095	5	0.6149	5
A_6	0.6092	6	0.6148	6
A_7	0.5944	9	0.5981	9
A_8	0.6193	3	0.6240	3
A_9	0.6253	1	0.6307	1
A_{10}	0.6237	2	0.6289	2
A_{11}	0.6127	4	0.6163	4
A_{12}	0.5891	12	0.5920	12
A_{13}	0.5864	13	0.5887	14
A_{14}	0.5918	10	0.5947	10
A_{15}	0.5844	14	0.5860	15

TABLE VII.

A	TOTAL	RANK
A_1	0.5863	14
A_2	0.6026	7
A_3	0.5906	11
A_4	0.5983	8
A_5	0.6095	5
A_6	0.6092	6
A_7	0.5944	9
A_8	0.6193	3
A_9	0.6253	1
A_{10}	0.6237	2
A_{11}	0.6127	4
A_{12}	0.5891	12
A_{13}	0.5864	13
A_{14}	0.5918	10
A_{15}	0.5843	15

$$\text{TOTAL} = \frac{1}{\sum_{\alpha}}[CC_i\big|_{\alpha=0} *(0) + CC_i\big|_{\alpha=0.25} *(0.25) + CC_i\big|_{\alpha=0.5} *(0.5) + CC_i\big|_{\alpha=0.75} *(0.75) +$$

$$+ CC_i\big|_{\alpha=1} *(1)] \qquad \alpha = (0, 0.25, 0.5, 0.75, 1),\ (i = 1,\dots,m)$$

For example from TABLE VII for A_1 the TOTAL is calculated as:

$$\frac{1}{2.5}[(0.5757*0) + (0.5793*0.25) + (0.5828*0.5) +$$

$$+ (0.5864*0.75) + (0.5899*1)] = 0.5863$$

6. Conclusion

In general, this paper has two purposes: the first one is to propose fuzzy TOPSIS algorithm for multi criteria decision making, and the second one is to examine the proposed algorithm by a case study of a tile factory in Iran. Here, the linguistic variables are applied instead of numerical values to solve the MCDM problem under α-cuts method. The distances of an alternative from the FPIS and the FNIS are considered. The alternative is closer to the FPIS and farther from FNIS, will get a high ranking order. The combining of Marketing Mix components can be considered with the proposed method. They can generally cause to make a successful method of marketing as table V, VI. We can see when the α is increased the CC_i (α) is increased too, for all alternatives.

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