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Flow of a viscous fluid through a porous circular pipe in presence of magnetic field

T.S.Chauhan*, I.S.Chauhan** and Shikha***
Department of Mathematics
*Bareilly College, Bareilly
** GanjdundwaraP.G.College, Ganjdundwara ,Kanshiram Nagar
***Research Scholar, Bareilly College, Bareilly

Abstract
In this paper we have developed a model for two dimensional motion of a steady MHD flow through circular pipe which is bounded by a porous medium in presence of applied magnetic field. We have considered Brinkman equation for the flow through porous media using approximate boundary conditions the governing equation have been solved and solution for velocity in both the cases with porous media have been obtained. The results for various values of magnetic field in both the cases on flow characteristic have been obtained and discussed through graphs.

Key Words
magnetic field, viscous fluid, porous medium and magneto hydrodynamic flows.

1. Introduction
The requirement of modern technology has stimulated the interest in fluid flow studies, which involve the interaction of several phenomenon. One such study is presented, when a viscous fluid flows over a porous surface, because of its importance in many engineering problems such as flow of liquid in a porous bearing (Joseph and Tao [1]), in the field of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification process. Several geophysical applications of fluid flow in porous media have been reported by Cunningham and Williams [2].

The mathematical theory of the flow of fluid through a porous medium was initiated by Darcy [3]. Later on Brinkman [4] proposed modification of the Darcy's law for porous medium. Several authors have studied problems of flow of fluid in two regions: in region I fluid is free to flow and in region II the fluid flows through porous media. Such types of coupled flow with different geometries, and several kinds of matching conditions at the interface have been examined [5-9].

The study of magneto hydrodynamic flows through porous medium has been studied by several authors [10-13]. Effects of magnetic field on blood flows have been studied by suri and suri [18], sud et.al.[19] and yamamato et.al.[16]. Sud and sekhan[16] have developed a model of blood flow through a human arterial system subject to a steady magnetic field. Srivastavaet. al.[8] have developed a model for flow of viscous fluid through a circular pipe and its surrounding porous medium bounded by a rigid cylinder and the unbounded regions.

In the present paper, we have studied the flow through a circular pipe which is bounded by a porous medium fully saturated with the viscous fluid in the presence of applied magnetic field. Now, we have considered two cases:

I : When porous medium is unbounded
II: When it is bounded by another by coaxial impervious circular cylinder. It is assumed that the flow inside the pipe, in which a clear fluid is there, is governed by Navier-Stokes equation and in porous medium outside the cylinder the flow is given by Brinkman equation[4]. Ochoa-Tapia and Whitaker[7] suggested the matching condition at the interface of two flows.

2. Mathematical Formulation of the Problem

μ = Viscosity of the fluid in the first zone
μ_e= Viscosity of the fluid in the second zone
v_1^* = Velocity of the fluid in the z^* direction in first zone
v_2^* = Velocity of the fluid in the z^* direction in second zone
k = Permeability of the porous medium
j = Magnetic flux
E = Electric field
B = Magnetic field
σ = Electrical conductivity
μ_p = Permeability of the medium
H_o = Magnetic field intensity
β = Constant (which takes both positive and negative)
A_1 ,D_1, E_1, F_1 = Constants
Referring to the figure-I, zone-I inside the pipe in which clear fluid is flowing and the zone-II outside the pipe $r^* \geq a$ which is occupied by the porous medium fully saturated with the viscous fluid is flowing and both the regions are subjected to a constant applied magnetic field in the direction normal to the common axis. The velocity component in the direction of z^* is denoted by $v_1^*(r)$ and $v_2^*(r)$ in the zone I and zone II respectively. Both v_1^* and v_2^* are functions of r^* only. We further assume that the flows in zone I and II are respectively governed by Navier-Stokes equation and Brinkman equation[4]. Pressures in both the zones are functions of z^* alone and in both the zones are same.
Equation of motion in z^* direction in zone I is given by:

(2.1) $$\frac{dp}{dz^*} = \mu\left[\frac{d^2 v_1^*}{dr^{*2}} + \frac{1}{r^*}\frac{dv_1^*}{dr^*}\right] + J^* \times B^*, \qquad o \leq r^* < a$$

In zone II, the equation of motion in z^* diretion is given by

(2.2) $$\frac{dp}{dz^*} = \mu_e\left[\frac{d^2 v_2^*}{dr^{*2}} + \frac{1}{r^*}\frac{dv_2^*}{dr^*}\right] - \frac{\mu}{k}v_2^* + J^* \times B^*, \qquad r^* > a$$

(2.3) $$J^* = \sigma\,(E^* + v_1^* \times B^*)$$

The value of

(2.4) $J^* \times B^* = -\sigma\mu_p^2 H_0^2 U_0 v_1^*$for zone I

(2.5) $J^* \times B^* = -\sigma\mu_p^2 H_0^2 U_0 v_2^*$for zone II

Boundary and Matching Conditions

(2.6) $v_1^*(r^*) = v_2^*\,(r^*)atr^* = a$

(2.7) $\mu_e\frac{dv_2^*}{dr^*} - \mu\,\frac{dv_1^*}{dr^*} = \frac{\mu\beta}{\sqrt{k}}v_2^*atr^* = a$

and(2.8) v_1^* is finite at $r^* = 0$

(2.9) $v_2^*$is finite at $r^* = \infty$

We write

(2.10) $\frac{dp}{dz^*} = -\mu A$

Non Dimensional Scheme

$v_1^* = A\,a^2 v_1,\; v_2^* = A\,a^2 v_2,\;\; r^* = ar$

for zone-1 non dimensional equation are given by

(2.11) $\frac{d^2 v_1}{dr^2} + \frac{1}{r}\frac{dv_1}{dr} - M_0^2 v_1 = -1 \qquad 0 \le r < 1$

And for zone-II non dimensional equation are given by

(2.12) $\frac{d^2 v_2}{dr^2} + \frac{1}{r}\frac{dv_2}{dr} - Q^2 v_1 = -\frac{1}{\gamma^2} \qquad r > 1$

Where $M = \frac{\sigma \mu_p^2 H_0^2 r_0^2}{\mu}$, $N = \frac{\sigma \mu_p^2 H_0^2 r_0^2}{\mu_e}$, $Q^2 = \alpha^2 + N^2, \gamma^2 = \frac{\mu_e}{\mu}$

$\sigma = \frac{r_0}{\sqrt{k}}$ and $\alpha = \frac{\sigma}{\gamma}$

Non dimensional boundary and matching condition

(2.13) $v_1 = v_2$ at $r = 1$

(2.14) $\gamma^2 \frac{dv_2}{dr} - \frac{dv_1}{dr} = \beta\sigma v_2$ at $r = 1$

(2.15) v_1 is finite at $r = 0$

(2.16) v_2 is finite at $r = \infty$

3. Solution for the problem

Solution equations have been obtained by using the method of variation using boundary conditions. We shall discuss the following two cases:

Case-1

In zone-I

(3.1) $v(r) = \frac{1}{M_0^2} + A_1 I_0(M_0 r) \qquad 0 \le r < 1$

And zone-II

(3.2) $v(r) = \frac{1}{Q^2\gamma^2} + D_1 K_0(Qr) \qquad r > 1$

Where A_1 and D_1 are constants to be determined using the matching conditions at the interface and boundary conditions

(3.3) $$A_1 = \frac{(M_0^2 - Q^2\gamma^2)k_1(Q) + Q\,\beta\,\sigma\,k_0(Q)}{M_0^2 Q[\{Q\gamma^2 k_1(Q) - \beta\sigma k_0(Q)\}I_0(M_0) - M_0 I_1(M_0)k_0(Q)}$$

(3.4) $$D_1 = \frac{(M_0^2 - Q^2\gamma^2)I_1(Q) + M_0\,\beta\,\sigma\,I_0(Q)}{M_0^2 Q^2\gamma^2[\{Q\gamma^2 k_1(Q) - \beta\sigma k_0(Q)\}I_0(M_0) - M_0 I_1(M_0)k_0(Q)}$$

Case-2

The porous medium fills the annular region between $r = 1$ and $r = \lambda$. The expression for $v(r)$ in zone-I is given by

(3.5) $v\,(\,r\,) = \dfrac{1}{M_0^2} + A_1 I_0\,(\,M_0 r\,)$

And $v\,(\,r\,)$ in zone-II is given by

(3.6) $v\,(\,r\,) = \dfrac{1}{Q^2\gamma^2} + E_1 I_0(\,Qr\,) + F_1 k_0(\,Qr\,)$

As $r = \lambda$ is impervious cylinder $v\,(\,r\,)$ has to satisfy the following conditions:

(3.7) $v\,(\,\lambda\,) = 0$

The solution for the second case subject to the boundary and matching conditions are obtained with the following relations for the constants involved:

(3.8) $E_1 I_0(\,Qr\,) + F_1 k_0(\,Qr\,) + \dfrac{1}{Q\gamma^2} = 0$

Where

(3.9) $$E_1 = \frac{\begin{array}{c} M_0^2 I_1(M_0)k_0(\,Q\,) - M_0\,Q\,\gamma^2 I_0\,(\,M_0)\,k_1(\,Q\,) + \\ M_0\,\beta\,\sigma\,I_0(M_0)k_0(\,Q\,) + Q^2\gamma^2 I_1(M_0)k_0\,(\,Q\lambda\,) \\ - M_0^2 I_1(M_0)k_0(\,Q\lambda\,) - M_0\,\beta\sigma\,I_0(M_0)k_0\,(\,Q\lambda\,) \end{array}}{\begin{array}{c} M_0 Q^2\gamma^2\,[\,\{M_0 I_1(M_0) + \beta\,\sigma\,I_0(M_0)\} \\ \{I_0(\,Q\,)k_0(\,Q\lambda\,) - k_0(\,Q\,)I_0(\,Q\lambda\,)\} + Q\,\gamma^2 I_0(M_0) \\ \{\,k_1(\,Q\,)I_0(\,Q\lambda\,) - I_1(\,Q\,)k_0(\,Q\lambda\,)\}] \end{array}}$$

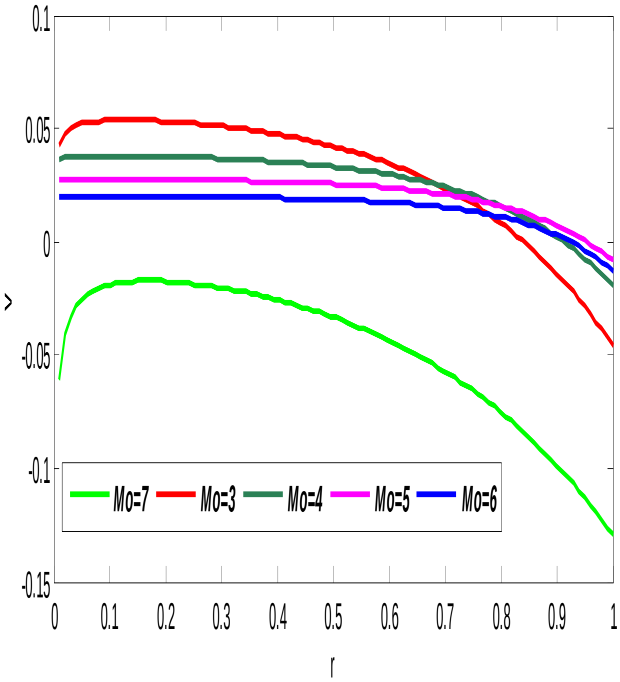
(3.10) $$F_1 = \frac{\begin{array}{c} M_0\{M_0 I_1(M_0) + \beta\sigma\,I_0(M_0)\}\{I_o(\,Q\lambda\,) - I_0(\,Q\,)\} \\ + Q\,\gamma^2\,\{\,M_0 I_0(M_0)I_1(\,Q\,) - Q\,I_1(M_0)I_0(\,Q\lambda\,)\} \end{array}}{\begin{array}{c} M_0 Q^2\gamma^2\,[\,\{M_0 I_1(M_0) + \beta\,\sigma\,I_0(M_0)\} \\ \{I_0(\,Q\,)k_0(\,Q\lambda\,) - k_0(\,Q\,)I_0(\,Q\lambda\,)\} + Q\,\gamma^2 I_0(M_0) \\ \{\,k_1(\,Q\,)I_0(\,Q\lambda\,) - I_1(\,Q\,)k_0(\,Q\lambda\,)\}] \end{array}}$$

And

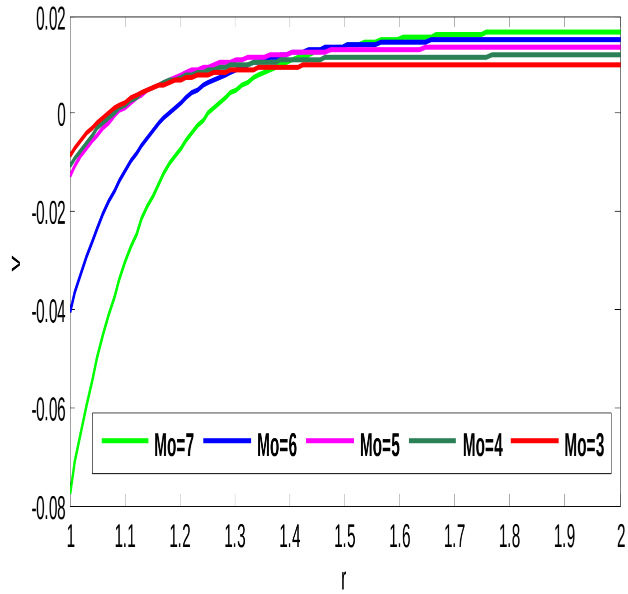
(3.11) $$A_1 = \frac{\begin{array}{c} M_0^2\{I_1(\,Q\,)k_0(\,Q\,) - I_0(\,Q\,)k_1(\,Q\,)\} \\ - Q\,\beta\,\sigma\,\{I_0(\,Q\,)K_0(\,Q\lambda\,) - k_0(\,Q\,)I_0(\,Q\lambda\,)\} \\ + \{M_0^2 - Q^2\gamma^2\}\{\,k_1(\,Q\,)I_0(\,Q\lambda\,) - I_1(\,Q\,)k_0(\,Q\lambda\,)\} \end{array}}{\begin{array}{c} M_0^2\,Q\,[\,\{M_0 I_1(M_0) + \beta\,\sigma\,I_0(M_0)\} \\ \{I_0(\,Q\,)k_0(\,Q\lambda\,) - k_0(\,Q\,)I_0(\,Q\lambda\,)\} + Q\,\gamma^2 I_0(M_0) \\ \{\,k_1(\,Q\,)I_0(\,Q\lambda\,) - I_1(\,Q\,)k_0(\,Q\lambda\,)\}] \end{array}}$$

4. Results and discussion:

In accordance with our analysis we observed the effect of applied magnetic field with various parameters. By extending is the earlier work of Srivastava et.al. [8], we have introduced more realistic results as below:

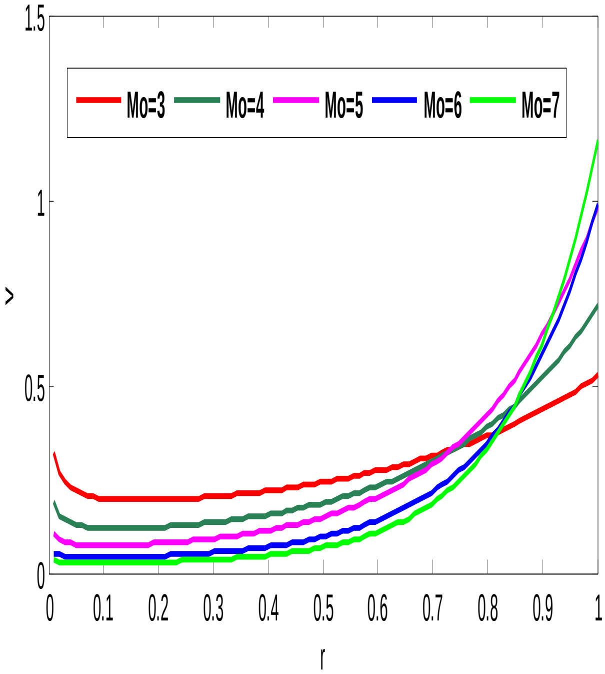


Variation of velocity with magnetic field 2(a)

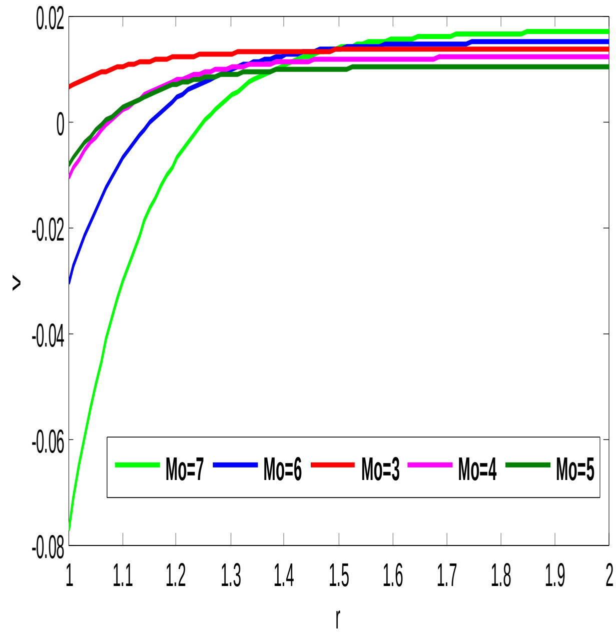


Variation of velocity with magnetic field2(b)

The effect of magnetic field may be observed from the fig-2(a) and 2(b). One may observe that increasing value of magnetic field parameter M_0 decreases the velocity profile in both the region.



Variation of velocity with magnetic field3(a)



Variation of velocity with magnetic field3(b)

In fig-3(a) and 3(b) the application of magnetic field on the two regions flow is to decreases the values of M_0 and there is no change in porous medium.

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