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Finite – Time Ruin Probability
In a Generalized Risk Processes under Interest Force

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Abstract

The aim of this paper is to build an exact formula for ruin probability of generalized risk processes under interest force with assumption that claims and premiums are assumed to be positive-valued random variables and interests are assumed to be non - negative- valued random variables (claims, premiums and interests are assumed to be independent). This situation is quite realistic for many situations. An exact formula for ruin (non-ruin) probabilities is derived in this paper. A numerical example is given to illustrate results. Our results is to extend models which is an exact formula derived by Claude Lefèvre and Stéphane Loisel [6].

Mathematics Subject Classifications. 62P05,60G40, 12E05

Key words. Ruin probability, Non- Ruin probability.

1. Introduction

For over a century, there has been a major interest in actuarial science. Since a large portion of the surplus of insurance business from investment income, actuaries have been studying ruin problems under risk models with rates of interest. For example, Teugels and Sundt [20], [21] studied the effects of constant rate on the ruin probability under the compound Poisson risk model. Yang [23] established both exponential and non – exponential upper bounds for ruin probabilities in a risk model with constant interest force and independent premiums and claims. Cai [3], [4] investigated the ruin probabilities in two risk models, with independent premiums

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and claims and used a first-order autoregressive process to model the rates of in interest. Cai and Dickson [5] obtained Lundberg inequalities for ruin probabilities in two discrete-time risk process with a Markov chain interest model and independent premiums and claims. However, those results is only given upper bounds for finite-time probabilities and ultimate ruin probability that they did not provide an exact formula for finite-time probabilities.

Claude Lefèvre and Stéphane Loisel [6] studied the problem of ruin in the classical compound binomial and compound Poisson risk models. Their primary purpose is to extend those models which is an exact formula derived by Pircard and Lefèvre [7] for the probability of (non-ruin) ruin within finite time.

However, Claude Lefèvre and Stéphane Loisel [6] did not provide an exact formula for ruin probability of generalized risk processes under interest force with surplus process $\{U_t\}_{t\geq 1}$ written as

$$U_t = U_{t-1}(1+I_t) + X_t - Y_t; t = 1,2,... \tag{1.1}$$

or

$$U_t = (U_{t-1} + X_t)(1+I_t) - Y_t; t = 1,2,... \tag{1.2}$$

where $U_o = u$ is initial surplus, u and t are positive integer numbers, $X = \{X_i\}_{i\geq 1}$ and $Y = \{Y_j\}_{j\geq 1}$ take values in a finite set of positive numbers; $I = \{I_k\}_{k\geq 1}$ take values in a finite set of non – negative numbers. X , Y and I are assumed to be independent.

The aim of this paper is to build an exact formula for finte time ruin (non-ruin) probability of model (1.1) and (1.2) with these asumptions. We establish an exact formula for ruin (non-ruin) probability of model (1.1) and (1.2) whose exact formula for finite time ruin (non-ruin) probability are derived.

The paper is organized as follows; in Section 2, we build an exact formula for ruin (non-ruin) probability for model (1.1) and (1.2) with $X = \{X_i\}_{i\geq 1}$ and $Y = \{Y_j\}_{j\geq 1}$ are independent and identically distributed positive-valued random variables; $I = \{I_k\}_{k\geq 1}$ are independent identically distributed non-negative-valued random variables, X, Y and I are assumed to be independent. An extended result in Section 2 for X, Y and I being non identically distributed random variables is given in Section 3. A numerical example is given to illustrate these results in Section 4. Finally, we conclude our paper in Section 5.

2. Finite – Time Ruin Probability in a Generalized Risk Processes under Interest Force with sequences of independent and identically distributed random variables

Let model (1.1). We assume that:

Assumption 2.1. u, t are possitive integer numbers.

Assumption 2.2. $X = \{X_n\}_{n\geq 1}$ is a sequence of independent and identically distributed random variables, X_n take values in a finite set of positive numbers

$E_X = \{x_1, x_2, ..., x_M\} (0 < x_1 < x_2 < ... < x_M)$ with $p_k = P(X_1 = x_k)(x_k \in E_X)$, $0 \leq p_k \leq 1, \sum_{k=1}^{M} p_k = 1$.

Assumption 2.3. $Y = \{Y_n\}_{n\geq 1}$ is a sequence of independent and identically distributed random variables, Y_n take values in a finite set of positive numbers

$E_Y = \{y_1, y_2, ..., y_N\} (0 < y_1 < y_2 < ... < y_N)$ with $q_k = P(Y_1 = y_k)(y_k \in E_Y)$, $0 \leq q_k \leq 1, \sum_{k=1}^{N} q_k = 1$.

Assumption 2.4. $I = \{I_n\}_{n\geq 1}$ is a sequence of independent and identically distributed random variables, I_n take values in a finite set of non - negative numbers

$E_I = \{i_1, i_2, ..., i_R\} (0 \leq i_1 < i_2 < ... < i_R)$ with $r_k = P(I_1 = i_k)(i_k \in E_I)$, $0 \leq r_k \leq 1, \sum_{k=1}^{R} r_k = 1$.

Asumption 2.5. The sequences $\{X_n\}_{n\geq 1}$, $\{Y_n\}_{n\geq 1}$ and $\{I_n\}_{n\geq 1}$ are assumed to be independent.

From (1.1), we have:

$$U_t = u.\prod_{k=1}^{t}(1 + I_k) + \sum_{k=1}^{t-1}\left((X_k - Y_k)\prod_{j=k+1}^{t}(1 + I_j)\right) + X_t - Y_t . \tag{2.1}$$

where throughout this paper, we denote $\prod_{t=a}^{b} x_t = 1$ and $\sum_{t=a}^{b} x_t = 0$ if $a > b$

and $A \overset{as}{=} B$ if $P(A\Delta B) = 0$ with $A\Delta B = (A \setminus B) \cup (B \setminus A)$.

Supposing that the ruin time is defined by $T_u = inf\{j : U_j < 0\}$, where $\inf \phi = \infty$.

We define the finite time ruin (non-ruin) probabilities of model (1.1) with assumption 2.1 to assumption 2.4, respectively, by

$$\psi_t^{(1)}(u) = P(T_u \leq t) = P\left(\bigcup_{j=1}^{t}(U_j < 0)\right), \tag{2.2}$$

$$\phi_t^{(1)}(u) = 1 - \psi_t^{(1)}(u) = P(T_u \geq t+1) = P\left(\bigcap_{j=1}^{t}(U_j \geq 0)\right). \tag{2.3}$$

To establish a fomula for $\psi_t^{(1)}(u), \varphi_t^{(1)}(u)$, we first proof the following Lemma.

Lemma 2.1. Let u, $\{x_i\}_{i=1}^t, \{y_i\}_{i=1}^t$ be positive numbers, $\{i_k\}_{k=1}^t$ be non - negative numbers. If p is a positive integer number and $1 \leq p \leq t-1$ satisfies:

$$y_p \leq u\prod_{k=1}^{p}(1+i_k) + \sum_{k=1}^{p-1}(x_k - y_k)\prod_{j=k+1}^{p}(1+i_j) + x_p, \tag{2.4}$$

then, we have

$$u\prod_{k=1}^{p+1}(1+i_k) + \sum_{k=1}^{p}(x_k - y_k)\prod_{j=k+1}^{p+1}(1+i_j) + x_{p+1} > 0. \tag{2.5}$$

Proof.

From (2.4), we have

$$y_p \leq u\prod_{k=1}^{p}(1+i_k) + \sum_{k=1}^{p-1}(x_k - y_k)\prod_{j=k+1}^{p}(1+i_j) + x_p.$$

The above inequality is equivalent to

$$x_p - y_p \geq -u\prod_{k=1}^{p}(1+i_k) - \sum_{k=1}^{p-1}(x_k - y_k)\prod_{j=k+1}^{p}(1+i_j).$$

This inequality imlpies that

$$u\prod_{k=1}^{p+1}(1+i_k) + \sum_{k=1}^{p}(x_k - y_k)\prod_{j=k+1}^{p+1}(1+i_j) + x_{p+1}$$

$$= u\prod_{k=1}^{p+1}(1+i_k) + \sum_{k=1}^{p-1}(x_k - y_k)\prod_{j=k+1}^{p+1}(1+i_j) + (x_p - y_p)(1+i_{p+1}) + x_{p+1}$$

$$\geq u\prod_{k=1}^{p+1}(1+i_k) + \sum_{k=1}^{p-1}(x_k - y_k)\prod_{j=k+1}^{p+1}(1+i_j) + \left[-u\prod_{k=1}^{p}(1+i_k) - \sum_{k=1}^{p-1}(x_k - y_k)\prod_{j=k+1}^{p}(1+i_j)\right](1+i_{p+1}) + x_{p+1}$$

$$= x_{p+1} > 0.$$

Hence (2.5) holds.

This completes the proof of the Lemma 2.1.□

Now, we give an exact formula for finite time ruin (non-ruin) probability of model (1.1).

Theorem 2.1. If model (1.1) satisfies assumptions 2.1 to 2.5, then finite time non-ruin probability of model (1.1) is defined by

$$\varphi_t^{(1)}(u)=\sum_{c_1,c_2,..,c_t=1}^{R}\sum_{m_1,m_2,...,m_t=1}^{M} r_{c_1}r_{c_2}...r_{c_t}p_{m_1}p_{m_2}...p_{m_t}\left(\sum_{1\le n_1\le g_1}\sum_{1\le n_2\le g_2}...\sum_{1\le n_t\le g_t} q_{n_1}q_{n_2}...q_{n_t}\right), \quad (2.6)$$

where

$$g_1=\max\left\{n_1 : y_{n_1}\le \min\left\{u\prod_{k=1}^{1}(1+i_{c_k})+x_{m_1}, y_N\right\}\right\},$$

$$g_2=max\left\{n_2 : y_{n_2}\le \min\left\{u\prod_{k=1}^{2}(1+i_{c_k})+\sum_{k=1}^{1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{2}(1+i_{c_j})+x_{m_2}, y_N\right\}\right\},$$

...

$$g_t=max\left\{n_t : y_{n_t}\le \min\left\{u\prod_{k=1}^{t}(1+i_{c_k})+\sum_{k=1}^{t-1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{t}(1+i_{c_j})+x_{m_t}, y_N\right\}\right\}.$$

Proof.

Firstly, we have

$$A:=\bigcap_{j=1}^{t}(U_j\ge 0)$$

$$=\left(Y_1\le u\prod_{k=1}^{1}(1+I_k)+X_1\right)\cap\left(Y_2\le u\prod_{k=1}^{2}(1+I_k)+\sum_{k=1}^{1}(X_k-Y_k)\prod_{j=k+1}^{2}(1+I_j)+X_2\right)\cap$$

$$\left(Y_3\le u\prod_{k=1}^{3}(1+I_k)+\sum_{k=1}^{2}(X_k-Y_k)\prod_{j=k+1}^{3}(1+I_j)+X_3\right)\cap...$$

$$...\cap\left(Y_t\le u\prod_{k=1}^{t}(1+I_k)+\sum_{k=1}^{t-1}(X_k-Y_k)\prod_{j=k+1}^{t}(1+I_j)+X_t\right). \quad (2.7)$$

By assumption 2.4, we put $I_1=i_{c_1}, I_2=i_{c_2},...,I_t=i_{c_t}$ with $i_{c_1}, i_{c_2},...,i_{c_t}$ being non - negative numbers and statisfy condition: $0\le i_{c_1}, i_{c_2},...,i_{c_t}\le i_R$.

Let $A_{i_{c_1}i_{c_2}...i_{c_t}}=\left(I_1=i_{c_1}\right)\cap\left(I_2=i_{c_2}\right)\cap...\cap\left(I_t=i_{c_t}\right)$.

Since $I=\{I_n\}_{n\ge 1}$ is a sequence of independent random variables then

$$P(A_{i_{c_1}i_{c_2}...i_{c_t}})=P\left[\left(I_1=i_{c_1}\right)\cap\left(I_2=i_{c_2}\right)\cap...\cap\left(I_t=i_{c_t}\right)\right]$$

$$=P\left(I_1=i_{c_1}\right).P\left(I_2=i_{c_2}\right)...P\left(I_t=i_{c_t}\right)=r_{c_1}r_{c_2}...r_{c_t}. \quad (2.8)$$

By Assumption 2.2, we put $X_1=x_{m_1}, X_2=x_{m_2},...,X_t=x_{m_t}$ with $x_{m_1}, x_{m_2},...,x_{m_t}$ being positive numbers and satisfy condition: $0<x_{m_1}, x_{m_2},...,x_{m_t}\le x_M$.

Let $A_{x_{m_1} x_{m_2} \dots x_{m_t}} = \left(X_1 = x_{m_1} \right) \cap \left(X_2 = x_{m_2} \right) \cap \dots \cap \left(X_t = x_{m_t} \right)$.

Since $X = \{X_n\}_{n\geq 1}$ is a sequence of independent random variables then

$$P(A_{x_{m_1} x_{m_2} \dots x_{m_t}}) = P\left[\left(X_1 = x_{m_1} \right) \cap \left(X_2 = x_{m_2} \right) \cap \dots \cap \left(X_t = x_{m_t} \right) \right]$$

$$= P\left(X_1 = x_{m_1} \right).P\left(X_2 = x_{m_2} \right) \dots P\left(X_t = x_{m_t} \right) = p_{m_1} p_{m_2} \dots p_{m_t} . \tag{2.9}$$

Firsly, we consider $I_1 = i_{c_1} (c_1 = \overline{1, R})$ then (2.7) is given

$$A \overset{as}{=} \bigcup_{c_1=1}^{R} \left(I_1 = i_{c_1} \right) \cap \left(\left(Y_1 \leq u \prod_{c_1=1}^{1} (1 + i_{c_1}) + X_1 \right) \cap \right.$$

$$\left(Y_2 \leq u(1 + i_{c_1}) \prod_{k=2}^{2} (1 + I_k) + \sum_{k=1}^{1} (X_k - Y_k) \prod_{j=k+1}^{2} (1 + I_j) + X_2 \right) \cap$$

$$\left(Y_3 \leq u(1 + i_{c_1}) \prod_{k=2}^{3} (1 + I_k) + \sum_{k=1}^{2} (X_k - Y_k) \prod_{j=k+1}^{3} (1 + I_j) + X_3 \right) \cap \dots$$

$$\left. \dots \cap \left(Y_t \leq u(1 + i_{c_1}) \prod_{k=2}^{t} (1 + I_k) + \sum_{k=1}^{t-1} (X_k - Y_k) \prod_{j=k+1}^{t} (1 + I_j) + X_t \right) \right)$$

Similarly, we consider $I_2 = i_{c_2}, \dots, I_t = i_{c_t} (c_2, \dots, c_t = \overline{1, R})$, (2.7) can be written as

$$A \overset{as}{=} \bigcup_{c_1, c_2, \dots, c_t = 1}^{R} \left\{ \left(I_1 = i_{c_1} \right) \cap \left(I_2 = i_{c_2} \right) \cap \dots \cap \left(I_t = i_{c_t} \right) \right\} \cap \left(\left(Y_1 \leq u \prod_{k=1}^{1} (1 + i_{c_k}) + X_1 \right) \cap \right.$$

$$\left(Y_2 \leq u \prod_{k=1}^{2} (1 + i_{c_k}) + \sum_{k=1}^{1} (X_k - Y_k) \prod_{j=k+1}^{2} (1 + i_{c_j}) + X_2 \right) \cap$$

$$\left(Y_3 \leq u \prod_{k=1}^{3} (1 + i_{c_k}) + \sum_{k=1}^{2} (X_k - Y_k) \prod_{j=k+1}^{3} (1 + i_{c_j}) + X_3 \right) \cap \dots$$

$$\left. \dots \cap \left(Y_t \leq u \prod_{k=1}^{t} (1 + i_{c_k}) + \sum_{k=1}^{t-1} (X_k - Y_k) \prod_{j=k+1}^{t} (1 + i_{c_j}) + X_t \right) \right).$$

Next, we consider $X_1 = x_{m_1} (m_1 = \overline{1, M})$, then

$$A \overset{as}{=} \bigcup_{c_1, c_2, \dots, c_t = 1}^{R} \left(\left\{ \left(I_1 = i_{c_1} \right) \cap \left(I_2 = i_{c_2} \right) \cap \dots \cap \left(I_t = i_{c_t} \right) \right\} \cap \left(\bigcup_{m_1=1}^{M} (X_1 = x_{m_1}) \cap \left(\left(Y_1 \leq u \prod_{k=1}^{1} (1 + i_{c_k}) + x_{m_1} \right) \cap \right. \right. \right.$$

$$\left(Y_2 \leq u \prod_{k=1}^{2} (1 + i_{c_k}) + \sum_{k=1}^{1} (x_{m_k} - Y_k) \prod_{j=k+1}^{2} (1 + i_{c_j}) + X_2 \right) \cap$$

$$\left(Y_3 \le u\prod_{k=1}^{3}(1+i_{c_k}) + \left[(x_{m_1}-Y_1)+\sum_{k=2}^{2}(X_k - Y_k)\right]\prod_{j=k+1}^{3}(1+i_{c_j}) + X_3\right)\cap ...$$

$$...\cap\left(Y_t \le u\prod_{k=1}^{t}(1+i_{c_k}) + \left[(x_{m_1}-Y_1)+\sum_{k=2}^{t-1}(X_k - Y_k)\right]\prod_{j=k+1}^{t}(1+i_{c_j}) + X_t\right)\Bigg)\Bigg)\Bigg).$$

Similarly, we consider $X_2 = x_{m_2},..., X_t = x_{m_t}\ (m_2,...,m_t = \overline{1,M})$, (2.7) can be rearranged as

$$A \overset{as}{=} \bigcup_{c_1,c_2,...,c_t=1}^{R}\Bigg(\Big\{\Big(I_1 = i_{c_1}\Big)\cap\Big(I_2 = i_{c_2}\Big)\cap...\cap\Big(I_t = i_{c_t}\Big)\Big\}\cap\Bigg(\bigcup_{m_1,m_2,...,m_t=1}^{M}\Big\{(X_1 = x_{m_1})\cap(X_2 = x_{m_2})\cap...\cap(X_t = x_{m_t})\Big\}\cap$$

$$\Bigg(\Bigg(Y_1 \le u\prod_{k=1}^{1}(1+i_{c_k}) + x_{m_1}\Bigg)\cap$$

$$\left(Y_2 \le u\prod_{k=1}^{2}(1+i_{c_k}) + \sum_{k=1}^{1}(x_{m_k}-Y_k)\prod_{j=k+1}^{2}(1+i_{c_j}) + x_{m_2}\right)\cap$$

$$\left(Y_3 \le u\prod_{k=1}^{3}(1+i_{c_k}) + \sum_{k=1}^{2}(x_{m_k}-Y_k)\prod_{j=k+1}^{3}(1+i_{c_j}) + x_{m_3}\right)\cap ...$$

$$...\cap\left(Y_t \le u\prod_{k=1}^{t}(1+i_{c_k}) + \sum_{k=1}^{t-1}(x_{m_k}-Y_k)\prod_{j=k+1}^{t}(1+i_{c_j}) + x_{m_t}\right)\Bigg)\Bigg)\Bigg)$$

$$\overset{as}{=} \bigcup_{c_1,c_2,...,c_t=1}^{R}\left(\Big\{(I_1 = i_{c_1})\cap(I_2 = i_{c_2})\cap...\cap(I_t = i_{c_t})\Big\}\cap\bigcup_{m_1,m_2,...,m_t=1}^{M}\left(\Big\{(X_1 = x_{m_1})\cap(X_2 = x_{m_2})\cap...\cap(X_t = x_{m_t})\Big\}\cap C_{i_{c_1}i_{c_2}...i_{c_t}}^{x_{m_1}x_{m_2}...x_{m_t}}\right)\right)$$

$$\overset{as}{=} \bigcup_{c_1,c_2,...,c_t=1}^{R}\left(\bigcup_{m_1,m_2,...,m_t=1}^{M}\left\{A_{i_{c_1}i_{c_2}...i_{c_t}} \cap B_{x_{m_1}x_{m_2}...x_{m_t}} \cap C_{i_{c_1}i_{c_2}...i_{c_t}}^{x_{m_1}x_{m_2}...x_{m_t}}\right\}\right), \qquad (2.10)$$

where

$$C_{i_{c_1}i_{c_2}...i_{c_t}}^{x_{m_1}x_{m_2}...x_{m_t}} \overset{as}{=} \left(Y_1 \le u\prod_{k=1}^{1}(1+i_{c_k}) + x_{m_1}\right)\cap\left(Y_2 \le u\prod_{k=1}^{2}(1+i_{c_k}) + \sum_{k=1}^{1}(x_{m_k}-Y_k)\prod_{j=k+1}^{2}(1+i_{c_j}) + x_{m_2}\right)\cap$$

$$\left(Y_3 \le u\prod_{k=1}^{3}(1+i_{c_k}) + \sum_{k=1}^{2}(x_{m_k}-Y_k)\prod_{j=k+1}^{3}(1+i_{c_j}) + x_{m_3}\right)\cap ...$$

$$...\cap\left(Y_t \le u\prod_{k=1}^{t}(1+i_{c_k}) + \sum_{k=1}^{t-1}(x_{m_k}-Y_k)\prod_{j=k+1}^{t}(1+i_{c_j}) + x_{m_t}\right). \qquad (2.11)$$

By assumption 2.3, we put $Y_1 = y_{n_1}, Y_2 = y_{n_2},..., Y_{t-1} = y_{n_{t-1}}$ with $y_{n_1}, y_{n_2},..., y_{n_{t-1}}$ being positive numbers and satisfy condition: $0 < y_{n_1}, y_{n_2},..., y_{n_{t-1}} \le y_N$.

Thus, (2.11) can be written as

$$C^{x_1x_2\ldots x_t}_{i_{m_1}i_{m_2}\ldots i_{m_t}} \overset{as}{=} \bigcup_{y_{n_1}\le u\prod_{k=1}^{1}(1+i_{c_k})+x_{m_1}} \left(Y_1=y_{n_1}\right)\cap\left(\left(Y_2\le u\prod_{k=1}^{2}(1+i_{c_k})+\sum_{k=1}^{1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{2}(1+i_{c_j})+x_{m_2}\right)\cap\right.$$

$$\left(Y_3\le u\prod_{k=1}^{3}(1+i_{c_k})+\left[(x_{m_1}-y_{n_1})+\sum_{k=2}^{2}(x_{m_k}-y_{n_k})\right]\prod_{j=k+1}^{3}(1+i_{c_j})+x_{m_3}\right)\cap\ldots$$

$$\left.\ldots\cap\left(Y_t\le u\prod_{k=1}^{t}(1+i_{c_k})+\left[(x_{m_1}-y_{n_1})+\sum_{k=2}^{t-1}(x_{m_k}-y_{n_k})\right]\prod_{j=k+1}^{t}(1+i_{c_j})+x_{m_t}\right)\right)$$

$$\overset{as}{=} \bigcup_{y_{n_1}\le\left(u\prod_{k=1}^{1}(1+i_{c_k})+x_{m_1}\right)} (Y_1=y_{n_1})\cap\left(\bigcup_{y_{n_2}\le\left(u\prod_{k=1}^{2}(1+i_{c_k})+\sum_{k=1}^{1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{2}(1+i_{x_j})+x_{m_2}\right)} (Y_2=y_{n_2})\cap\right.$$

$$\left(\bigcup_{y_{n_3}\le\left(u\prod_{k=1}^{3}(1+i_{c_k})+\sum_{k=1}^{2}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{3}(1+i_{x_j})+x_{m_3}\right)} (Y_3=y_{n_3})\cap\ldots\right.$$

$$\left.\left.\ldots\cap\left(Y_t\le u\prod_{k=1}^{t}(1+i_{c_k})+\sum_{k=1}^{t-1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{t}(1+i_{c_j})+x_{m_t}\right)\ldots\right)\right). \tag{2.12}$$

Using by assumption 2.3, we put $Y_t = y_{n_t}$ with y_{n_t} being positive number and statisfy condition $0 < y_{n_t} \le y_N$ then (2.11) can be rearranged as

$$C^{x_1x_2\ldots x_t}_{i_{m_1}i_{m_2}\ldots i_{m_t}} \overset{as}{=} \bigcup_{y_{n_1}\le\left(u\prod_{k=1}^{1}(1+i_{c_k})+x_{m_1}\right)}\left(\bigcup_{y_{n_2}\le\left(u\prod_{k=1}^{2}(1+i_{c_k})+\sum_{k=1}^{1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{2}(1+i_{c_j})+x_{m_2}\right)}\left(\bigcup_{y_{n_3}\le\left(u\prod_{k=1}^{3}(1+i_{c_k})+\sum_{k=1}^{2}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{3}(1+i_{c_j})+x_{m_3}\right)}\ldots\right.\right.$$

$$\left.\left.\ldots\left(\bigcup_{y_{n_t}\le\left(u\prod_{k=1}^{t}(1+i_{c_k})+\sum_{k=1}^{t-1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{t}(1+i_{c_j})+x_{m_t}\right)}\left\{(Y_1=y_{n_1})\cap\ldots\cap(Y_t=y_{n_t})\right\}\ldots\right)\right)\right). \tag{2.13}$$

By using Lemma 2.1, $u\prod_{k=1}^{1}(1+i_{c_k})+x_{m_1}$,

$u\prod_{k=1}^{2}(1+i_{c_k})+\sum_{k=1}^{1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{2}(1+i_{c_j})+x_{m_2}$, …,

$u\prod_{k=1}^{t}(1+i_{c_k})+\sum_{k=1}^{t}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{t}(1+i_{c_j})+x_{m_t}$ are positive numbers and

$0<y_{n_1},y_{n_2},...,y_{n_t}\le y_N$ then, we define

$$g_1=\max\left\{n_1:y_{n_1}\le\min\left\{u\prod_{k=1}^{1}(1+i_{c_k})+x_{m_1},y_N\right\}\right\},$$

$$g_2=\max\left\{n_2:y_{n_2}\le\min\left\{u\prod_{k=1}^{2}(1+i_{c_k})+\sum_{k=1}^{1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{2}(1+i_{c_j})+x_{m_2},y_N\right\}\right\},$$

...

$$g_t=\max\left\{n_t:y_{n_t}\le\min\left\{u\prod_{k=1}^{t}(1+i_{c_k})+\sum_{k=1}^{t-1}(x_{m_k}-y_{n_k})\prod_{j=k+1}^{t}(1+i_{c_j})+x_{m_t},y_N\right\}\right\}.$$

Thus, (2.13) can be rearranged as

$$C_{i_{m_1}i_{m_2}...i_{m_t}}^{x_1x_2...x_t}\overset{as}{=}\bigcup_{1\le n_1\le g_1}\bigcup_{1\le n_2\le g_2}...\bigcup_{1\le n_t\le g_t}\left\{\left(Y_1=y_{n_1}\right)\cap\left(Y_2=y_{n_2}\right)\cap...\cap\left(Y_t=y_{n_t}\right)\right\}. \qquad (2.14)$$

Because $Y=\{Y_n\}_{n\ge1}$ is a sequence of independent independent random variables then

$$P\left[\left(Y_1=y_{n_1}\right)\cap\left(Y_2=y_{n_2}\right)\cap...\cap\left(Y_t=y_{n_t}\right)\right]=P\left(Y_1=y_{n_1}\right).P\left(Y_2=y_{n_2}\right)...P\left(Y_t=y_{n_t}\right)=q_{n_1}q_{n_2}...q_{n_t}$$

In the other hand, system of events $\left\{\left(Y_1=y_{n_1}\right)\cap\left(Y_2=y_{n_2}\right)\cap...\cap\left(Y_t=y_{n_t}\right)\right\}_{1\le n_j\le g_j\,(j=\overline{1,t})}$ in

(2.14) be incompatible then

$$P(B_{x_{m_1}x_{m_2}...x_{m_t}})=\sum_{1\le n_1\le g_1}\sum_{1\le n_2\le g_2}...\sum_{1\le n_t\le g_t}q_{n_1}q_{n_2}...q_{n_t}. \qquad (2.15)$$

By X,Y,I are assumed to be independent, with $c_1,c_2,...,c_t$ and $m_1,m_2,...,m_t$ hold then

$A_{i_{c_1}i_{c_2}...i_{c_t}}, B_{x_{m_1}x_{m_2}...x_{m_t}}, C_{i_{c_1}i_{c_2}...i_{c_t}}^{x_{m_1}x_{m_2}...x_{m_t}}$ are independent events.

In addition, system of events $\left\{A_{i_{c_1}i_{c_2}...i_{c_t}}\cap B_{x_{m_1}x_{m_2}...x_{m_t}}\cap C_{i_{c_1}i_{c_2}...i_{c_t}}^{x_{m_1}x_{m_2}...x_{m_t}}\right\}_{c_j=\overline{1,R};\,m_j=\overline{1,M}\,(\overline{j=1,t})}$ in (2.10) is

incompatible.

Therefore, combining (2.8), (2.9) and (2.15), we have

$$\varphi_t^{(1)}(u) = P(A) = \sum_{c_1,c_2,..,c_t=1}^{R} \left(\sum_{m_1,m_2,...,m_t=1}^{M} P\left\{ A_{i_{c_1} i_{c_2} ... i_{c_t}} \cap B_{x_{m_1} x_{m_2} ... x_{m_t}} \cap C_{i_{c_1} i_{c_2} ... i_{c_t}}^{x_{m_1} x_{m_2} ... x_{m_t}} \right\} \right)$$

$$= \sum_{c_1,c_2,...,c_t=1}^{R} \left(\sum_{m_1,m_2,...,m_t=1}^{M} P\left(A_{i_{c_1} i_{c_2} ... i_{c_t}} \right).P\left(B_{x_{m_1} x_{m_2} ... x_{m_t}} \right).P\left(C_{i_{c_1} i_{c_2} ... i_{c_t}}^{x_{m_1} x_{m_2} ... x_{m_t}} \right) \right)$$

$$= \sum_{c_1,c_2,..,c_t=1}^{R} \sum_{x_{m_1},x_{m_2},...,x_{m_t}=1}^{M} P\left(A_{i_{c_1} i_{c_2} ... i_{c_t}} \right).P\left(B_{x_{m_1} x_{m_2} ... x_{m_t}} \right) \left(\sum_{1 \le n_1 \le g_1} \sum_{1 \le n_2 \le g_2} .... \sum_{1 \le n_t \le g_t} q_{n_1} q_{n_2} ... q_{n_t} \right)$$

$$= \sum_{c_1,c_2,..,c_t=1}^{R} \sum_{m_1,m_2,...,m_t=1}^{M} r_{c_1} r_{c_2} ... r_{c_t} p_{m_1} p_{m_2} ... p_{m_t} \left(\sum_{1 \le n_1 \le g_1} \sum_{1 \le n_2 \le g_2} ... \sum_{1 \le n_t \le g_t} q_{n_1} q_{n_2} ... q_{n_t} \right). \qquad (2.16)$$

This completes the proof of the Theorem 2.1.□

Corollary 2.1. If model (1.1) satisfies assumptions 2.1 to 2.4, then finite time ruin probability of model (1.1) is defined by

$$\psi_t^{(1)}(u) = 1 - \varphi_t^{(1)}(u)$$

$$= 1 - \sum_{c_1,c_2,..,c_t=1}^{R} \sum_{m_1,m_2,...,m_t=1}^{M} r_{c_1} r_{c_2} ... r_{c_t} p_{m_1} p_{m_2} ... p_{m_t} \left(\sum_{1 \le n_1 \le g_1} \sum_{1 \le n_2 \le g_2} ... \sum_{1 \le n_t \le g_t} q_{n_1} q_{n_2} ... q_{n_t} \right). \qquad (2.17)$$

Remark 2.1. Fomula (2.6) (or (2.17)) gives a method to compute axactly finite time non-ruin (ruin) probability of model (1.1) which $X = \{X_n\}_{n \ge 1}$ and $Y = \{Y_n\}_{n \ge 1}$ are sequences of independent and identically distributed random variables, they take values in a finite set of positive numbers and $I = \{I_n\}_{n \ge 1}$ is a sequence of independent and identically distributed random variables, and they take values in a finite set of non- negative numbers.

Let model (1.2) satisfy assumptions 2.1 to 2.5.

From (1.2), we have:

$$U_t = u.\prod_{k=1}^{t}(1+I_k) + \sum_{k=1}^{t-1}\left((X_k(1+I_k) - Y_k) \prod_{j=k+1}^{t}(1+I_j) \right) + X_t - Y_t. \qquad (2.18)$$

Supposing that the ruin time of model (1.2) is defined by $T_u = inf\{j : U_j < 0\}$, where $\inf \phi = \infty$.

We define the finite time ruin (non-ruin) probabilities of model (1.2) with ssumptions 2.1 to 2.5, respectively, by

$$\psi_t^{(2)}(u) = P(T_u \le t) = P\left(\bigcup_{k=1}^{t} (U_k < 0) \right), \qquad (2.19)$$

$$\varphi_t^{(2)}(u) = 1 - \psi_t^{(2)}(u) = P(T_u \geq t+1) = P\left(\bigcap_{k=1}^{t}(U_k \geq 0)\right). \tag{2.20}$$

To establish an fomula for $\psi_t^{(2)}(u), \varphi_t^{(2)}(u)$, we have the following Lemma.

Lemma 2.2. Let u, $\{x_i\}_{i=1}^t, \{y_i\}_{i=1}^t$ are positive numbers and $\{i_k\}_{k=1}^t$ are non - negative numbers. If p is a positive integer number and $1 \leq p \leq t-1$ satisfies:

$$y_p \leq u\prod_{k=1}^{p}(1+i_k) + \sum_{k=1}^{p-1}(x_k(1+i_k) - y_k)\prod_{j=k+1}^{p}(1+i_j) + x_p(1+i_p), \tag{2.21}$$

then, we have

$$u\prod_{k=1}^{p+1}(1+i_k) + \sum_{k=1}^{p}(x_k(1+i_k) - y_k)\prod_{j=k+1}^{p}(1+i_j) + x_{p+1}(1+i_{p+1}) > 0. \tag{2.22}$$

Proof.

We proof similarly as Lemma 2.1. □

Next, we give an exact formula for finite time ruin (non ruin) probability of model (1.1).

Theorem 2.2. If model (1.2) satisfies assumptions 2.1 to 2.5, then finite time non-ruin probability of model (1.2) is defined by

$$\varphi_t^{(2)}(u) = \sum_{c_1,c_2,..,c_t=1}^{R}\sum_{m_1,m_2,...,m_t=1}^{M} r_{c_1}r_{c_2}...r_{c_t}p_{m_1}p_{m_2}...p_{m_t}\left(\sum_{1\leq n_1\leq g_1}\sum_{1\leq n_2\leq g_2}...\sum_{1\leq n_t\leq g_t} q_{n_1}q_{n_2}...q_{n_t}\right), \tag{2.23}$$

where

$$g_1 = \max\left\{n_1 : y_{n_1} \leq \min\left\{u\prod_{k=1}^{1}(1+i_{c_k}) + x_{m_1}(1+i_{c_1}), y_N\right\}\right\},$$

$$g_2 = \max\left\{n_2 : y_{n_2} \leq \min\left\{u\prod_{k=1}^{2}(1+i_{c_k}) + \sum_{k=1}^{1}(x_{m_k}(1+i_{c_k}) - y_{n_k})\prod_{j=k+1}^{2}(1+i_{c_j}) + x_{m_2}(1+i_{c_2}), y_N\right\}\right\},$$

...

$$g_t = \max\left\{n_t : y_{n_t} \leq \min\left\{u\prod_{k=1}^{t}(1+i_{c_k}) + \sum_{k=1}^{t-1}(x_{m_k}(1+i_{c_k}) - y_{n_k})\prod_{j=k+1}^{t}(1+i_{c_j}) + x_{m_t}(1+i_{c_t}), y_N\right\}\right\}.$$

Proof.

We proof similarly as Theorem 2.1.□

Corollary 2.2. If model (1.2) satisfies assumptions 2.1 to 2.5, then finite time ruin probability of model (1.2) is defined by

$$\psi_t^{(2)}(u) = 1 - \varphi_t^{(2)}(u)$$

$$=1-\sum_{c_1,c_2,\dots,c_t=1}^{R}\ \sum_{m_1,m_2,\dots,m_t=1}^{M}(r_{c_1}r_{c_2}\dots r_{c_t}).(p_{m_1}p_{m_2}\dots p_{m_t})\left(\sum_{1\le n_1\le g_1}\ \sum_{1\le n_2\le g_2}\dots\sum_{1\le n_t\le g_t} q_{n_1}q_{n_2}\dots q_{n_t}\right). \qquad (2.24)$$

Remark 2.2. Fomula (2.23) (or (2.24)) give a method to compute exact finite time non-ruin (ruin) probability of model (1.2) which $X=\{X_n\}_{n\ge 1}$ and $Y=\{Y_n\}_{n\ge 1}$ are sequences of independent and identically distributed random variables and they take values in a finite set of positive numbers. In addition, $I=\{I_n\}_{n\ge 1}$ is also a sequence of independent and identically distributed random variables, and they take values in a finite set of non- negative numbers.

3. Finite – Time Ruin Probability in a Generalized Risk Processes under Interest Force with sequences of independent and non identically distributed random variables

Let model (1.1). We assume that:

Assumption 3.1. u, t are positive integer numbers.

Assumption 3.2. $X=\{X_n\}_{n\ge 1}$ is a sequence of independent and non identically distributed random variables , X_n takes values in a finite set of positive numbers $E_X=\{x_1,x_2,\dots,x_M\}(0<x_1<x_2<\dots<x_M)$ and X_n has a distribution:

$$p_k^{(n)}=P(X_n=x_k)\,(x_k\in E_X,n\in N^*),\ 0\le p_k^{(n)}\le 1,\sum_{k=1}^{M}p_k^{(n)}=1(n\in N^*).$$

Assumption 3.3. $Y=\{Y_n\}_{n\ge 1}$ is a sequence of independent and non identically distributed random variables, Y_n takes values in a finite set of positive integer numbers $E_Y=\{y_1,y_2,\dots,y_N\}(0<y_1<y_2<\dots<y_N)$ and Y_n has a distribution:

$$q_k^{(n)}=P(Y_n=y_k)\,(y_k\in E_Y,n\in N^*),\ 0\le q_k^{(n)}\le 1,\sum_{k=1}^{N}q_k^{(n)}=1(n\in N^*).$$

Assumption 3.4. $I=\{I_n\}_{n\ge 1}$ is a sequence of independent and non identically distributed random variables, I_n takes values in a finite set of non-negative numbers $E_I=\{i_1,i_2,\dots,i_R\}(0\le i_1<i_2<\dots<i_R)$ and I_n has a distribution:

$$r_k^{(n)}=P(I_n=r_k)\,(r_k\in E_I,n\in N^*),\ 0\le r_k^{(n)}\le 1,\sum_{k=1}^{R}r_k^{(n)}=1(n\in N^*).$$

Assumption 3.5. The sequences $\{X_n\}_{n\ge 1}$, $\{Y_n\}_{n\ge 1}$ and $\{I_n\}_{n\ge 1}$ are assumed to be independent.

Supposing that the ruin time of model (1.1) is defined by $T_u = inf\{j : U_j < 0\}$ where $\inf \phi = \infty$.

We define the finite time ruin (non-ruin) probabilities of model (1.1) with assumptions 3.1 to 3.5, respectively, by

$$\psi_t^{(3)}(u) = P(T_u \le t) = P\left(\bigcup_{k=1}^{t}(U_k < 0)\right), \tag{3.1}$$

$$\varphi_t^{(3)}(u) = 1 - \psi_t^{(3)}(u) = P(T_u \ge t+1) = P\left(\bigcap_{k=1}^{t}(U_k \ge 0)\right). \tag{3.2}$$

Similar to Themrem 2.1, we have

Theorem 3.1. If model (1.1) satisfies assumptions 3.1 to 3.5, then finite time non-ruin probability of model (1.1) is defined by

$$\varphi_t^{(3)}(u) = \sum_{c_1,c_2,..,c_t=1}^{R} \sum_{m_1,m_2,...,m_t=1}^{M} r_{c_1}^{(1)} r_{c_2}^{(2)} ... r_{c_t}^{(t)} p_{m_1}^{(1)} p_{m_2}^{(2)} ... p_{m_t}^{(t)} \left(\sum_{1\le n_1 \le g_1} \sum_{1\le n_2 \le g_2} ... \sum_{1\le n_t \le g_t} q_{n_1}^{(1)} q_{n_2}^{(2)} ... q_{n_t}^{(t)} \right), \tag{3.3}$$

where, $g_1, g_2, ..., g_t$ is defined in the same way with Theorem 2.1.

Proof.

We proof similarly as Theorem 2.1, where

(2.8) substitued by

$$P(A_{i_{c_1} i_{c_2} ... i_{c_t}}) = P\left[\left(I_1 = i_{c_1}\right) \cap \left(I_2 = i_{c_2}\right) \cap ... \cap \left(I_t = i_{c_t}\right)\right]$$
$$= P\left(I_1 = i_{c_1}\right).P\left(I_2 = i_{c_2}\right)...P\left(I_t = i_{c_t}\right) = r_{c_1}^{(1)} r_{c_2}^{(2)} ... r_{c_t}^{(t)},$$

In addition (2.9) replaced by

$$P(B_{x_{m_1} x_{m_2} ... x_{m_t}}) = P\left[\left(X_1 = x_{m_1}\right) \cap \left(X_2 = x_{m_2}\right) \cap ... \cap \left(X_t = x_{m_t}\right)\right]$$
$$= P\left(X_1 = x_{m_1}\right).P\left(X_2 = x_{m_2}\right)...P\left(X_t = x_{m_t}\right) = p_{m_1}^{(1)} p_{m_2}^{(2)} ... p_{m_t}^{(t)},$$

and (2.15) substituted by

$$P\left(C_{i_{c_1} i_{c_2} ... i_{c_t}}^{x_{m_1} x_{m_2} ... x_{m_t}}\right) = \sum_{1\le n_1 \le g_1} \sum_{1\le n_2 \le g_2} ... \sum_{1\le n_t \le g_t} q_{n_1}^{(1)} q_{n_2}^{(2)} ... q_{n_t}^{(t)}.$$

By using the same method to prove Theorem 2.1, we have formula (3.3).

This completes the proof of the Theorem 3.1.□

Corollary 3.1. If model (1.1) satisfies assumptions 3.1 to 3.5, then finite time ruin probability of model (1.1) is defined by

$$\psi_t^{(3)}(u) = 1 - \varphi_t^{(3)}(u)$$

$$=1-\sum_{c_1,c_2,..,c_t=1}^{R}\sum_{m_1,m_2,...,m_t=1}^{M} r_{c_1}^{(1)}r_{c_2}^{(2)}...r_{c_t}^{(t)}p_{m_1}^{(1)}p_{m_2}^{(2)}...p_{m_t}^{(t)}\left(\sum_{1\le n_1\le g_1}\sum_{1\le n_2\le g_2}...\sum_{1\le n_t\le g_t} q_{n_1}^{(1)}q_{n_2}^{(2)}...q_{n_t}^{(t)}\right), \quad (3.4)$$

Remark 3.1. Fomula (3.3) (or (3.4)) gives a method to compute axactly finited time non-ruin (ruin) probability of model (1.1) which $X=\{X_n\}_{n\ge 1}$ and $Y=\{Y_n\}_{n\ge 1}$ are sequences of independent and non identically distributed random variables, they take values in a finite set of positive numbers. In addition, $I=\{I_n\}_{n\ge 1}$ is a sequence of independent and non identically distributed random variables, and they take values in a finite set of non- negative numbers.

Similarly, we consider model (1.2) satisfy assumptions 3.1 to 3.5.
Supposing that the ruin time of model (1.2) is defined by $T_u = inf\{j : U_j < 0\}$ where $\inf \phi = \infty$.
We define the finite time ruin (non-ruin) probabilities of model (1.2) with assumptions 3.1 to 3.5, respectively, by

$$\psi_t^{(4)}(u) = P(T_u \le t) = P\left(\bigcup_{k=1}^{t}(U_k < 0)\right), \quad (3.5)$$

$$\varphi_t^{(4)}(u) = 1-\psi_t^{(4)}(u) = P(T_u \ge t+1) = P\left(\bigcap_{k=1}^{t}(U_k \ge 0)\right). \quad (3.6)$$

Similar to Theorem 2.2, we have
Theorem 3.2. If model (1.2) satisfies assumptions 3.1 to 3.5, then finite time non-ruin probability of model (1.2) is defined by

$$\varphi_t^{(4)}(u) = \sum_{c_1,c_2,..,c_t=1}^{R}\sum_{m_1,m_2,...,m_t=1}^{M} (r_{c_1}^{(1)}r_{c_2}^{(2)}...r_{c_t}^{(t)}).(p_{m_1}^{(1)}p_{m_2}^{(2)}...p_{m_t}^{(t)})\left(\sum_{1\le n_1\le g_1}\sum_{1\le n_2\le g_2}...\sum_{1\le n_t\le g_t} q_{n_1}^{(1)}q_{n_2}^{(2)}...q_{n_t}^{(t)}\right), \quad (3.7)$$

where, $g_1, g_2,..., g_t$ is defined in the same way with Theorem 2.2.

Proof.

We proof similarly as Theorem 3.1.□

Corollary 3.2. If model (1.2) satisfies assumptions 3.1 to 3.5, then finite time ruin probability of model (1.2) is defined by

$$\psi_t^{(4)}(u) = 1-\varphi_t^{(4)}(u)$$

$$=1-\sum_{c_1,c_2,..,c_t=1}^{R}\sum_{m_1,m_2,...,m_t=1}^{M} (r_{c_1}^{(1)}r_{c_2}^{(2)}...r_{c_t}^{(t)}).(p_{m_1}^{(1)}p_{m_2}^{(2)}...p_{m_t}^{(t)})\left(\sum_{1\le n_1\le g_1}\sum_{1\le n_2\le g_2}...\sum_{1\le n_t\le g_t} q_{n_1}^{(1)}q_{n_2}^{(2)}...q_{n_t}^{(t)}\right). \quad (3.8)$$

Remark 3.2. Fomula (3.7) (or (3.8)) gives a method to compute axactly finite time non-ruin (ruin) probability of model (1.2) which $X = \{X_n\}_{n\geq 1}$ and $Y = \{Y_n\}_{n\geq 1}$ are sequences of independent and non identically distributed random variables, they take values in a finite set of positive numbers. In addition, $I = \{I_n\}_{n\geq 1}$ is a sequence of independent and non identically distributed random variables, and they take values in a finite set of non- negative numbers.

4. A numerical Illustration

4.1. A numerical Illustration for $\psi_t^{(1)}(u)$

Let $X = \{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables, X_n takes values in a finite set of positive integer numbers $E_X = \{1, 2, 3, 4\}$ with X_1 having a distribution:

X_1	1	2	3	4
P	0,475112	0,176783	0,153448	0,194657

Let $Y = \{Y_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables, Y_n take values in a finite set of possitive integer numbers $E_Y = \{1, 2, 3, 4\}$ with Y_1 having a distribution:

Y_1	1	2	3	4
P	0,910703	0,009639	0,026892	0,052766

Let $I = \{I_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables, I_n take values in a finite set of possitive integer numbers $E_I = \{0,1; 0,11; 0,12; 0,13\}$ with I_1 having a distribution:

I_1	0,10	0,11	0,12	0,13
P	0,758171	0,228950	0,002498	0,010380

By using the C program, the $\psi_t^{(1)}(u)$ is calculated with the assumptions above of random variables X_1, Y_1, I_1 . Table 4.1 shows $\psi_t^{(1)}(u)$ for a range of value of u

u	t		
	t = 3	t = 4	t =5

1,5	0,136250	0,207778	0,274130
2,5	0,037408	0,065189	0,099821
3,5	0,010500	0,020001	0,033349
4,5	0,001619	0,004698	0,009572
5,5	0,000279	0,000911	0,002280
6,5	0,000058	0,000201	0,000531
7,5	0,000001	0,000029	0,000109

Table 4.1.Ruin probabilities (1.1) with Assumption 2.1- Assumption 2.5.

4.2. A numerical Illustration for $\psi_t^{(2)}(u)$

Let $X = \{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables, X_n take values in a finite set of positive integer numbers $E_X = \{1,2,3,4\}$ with X_1 having a distribution:

X_1	1	2	3	4
P	0,910367	0,042479	0,045050	0,002104

Let $Y = \{Y_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables, Y_n take values in a finite set of positive integer numbers $E_Y = \{1,2,3,4\}$ with Y_1 having a distribution:

Y_1	1	2	3	4
P	0,326243	0,184154	0,115890	0,373713

Let $I = \{I_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables, I_n take values in a finite set of positive integer numbers $E_I = \{0,1;0,11;0,12;0,13\}$ with I_1 having a distribution:

I_1	0,1	0,11	0,12	0,13
P	0,481185	0,103107	0,261119	0,154588

By using the C program, the $\psi_t^{(2)}(u)$ is calculated with the assumptions above of random variables X_1, Y_1, I_1 .

Table 4.2 shows $\psi_t^{(2)}(u)$ for a range of value of u

u	t		
	t = 3	t = 4	t = 5
1,5	0,293167	0,327225	0,352079
2,5	0,155001	0,188188	0,213372
3,5	0,070132	0,097067	0,118840
4,5	0,032686	0,050891	0,067123
5,5	0,011821	0,023018	0,034128
6,5	0,003710	0,009619	0,016400
7,5	0,000996	0,003650	0,007374

Table 4.2.Ruin probabilities (1.2) with Assumption 2.1- Assumption 2.5.

5. Conclusion

Using technique of classical probability with u, t, claims, premiums which all are positive numbers and interests are non – negative numbers, this paper constructed an exact formula for ruin (non-ruin) probability for model (1.1) and model (1.2) where sequences of claims, premiums and interests are independent (non) identically distributed random variables. Our main results in this paper are not only Theorem 2.1, Theorem 2.2, Theorem 3.1 and Theorem 3.2. In addition, numerical examples are given to illustrate for Theorem 2.1 and Theorem 2.2. These results proof for the suitability of theoretical result and practical examples. It also means that:

- When initial u is increasing then $\psi_t^{(1)}(u), \psi_t^{(2)}(u)$ are decreasing,
- With u being unchanged, when t is increasing then $\psi_t^{(1)}(u), \psi_t^{(2)}(u)$ are increasing.

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