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Analytical Hierarchy Process approach – An application of engineering education

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Abstract

This paper aims at giving an application of Analytical Hierarchy Process (AHP, a Multi Criteria Decision Making method) . Here AHP is applied for selection of a student from an Engineering college who is eligible for *All Round Excellence Award* for the year 2004-05 by taking subjective judgments of decision maker into consideration. Seven criteria were identified for getting this award and the alternatives are the five Branches of an Engineering college, in the state of Andhra Pradesh, INDIA. It is observed that a student of ECE (Electronics and communications engineering) branch has received the award.

Key words: AHP, Multi Criteria Decision Making, consistency, priorities, Eigen vector method

1 Introduction

The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach and was introduced by Saaty .The AHP has attracted the interest of many researchers mainly due to the nice mathematical properties of the method and the fact that the required input data are rather easy to obtain. The AHP is a decision support tool which can be used to solve complex decision problems. It uses a multi-level hierarchical structure of objectives, criteria, sub criteria, and alternatives. The data are derived by using a set of pair wise comparisons. These comparisons are used to obtain the weights of importance of the decision criteria, and the relative performance measures of the alternatives in terms of each individual decision criterion. If the comparisons are not perfectly consistent, then it provides a mechanism for improving consistency.

The Analytical Hierarchy Process or AHP was first developed by Professor Thomas L. Saaty in the 1970's and since that time has received wide application in a variety of areas. AHP has been applied for a vast number of areas, but it was not applied for the problem that has been taken up in this paper. Thomas.L.Saaty [4,5] has explained about Hierarchies, multiple objectives and Fuzzy sets . Patric T.Harker [1] explained AHP in detail as an art of Science and Decision-making. R.Ramanathan and L.S.Ganesh [2] used group preference aggregation methods employed in AHP by deriving members' weightages. AHP has been applied to many areas but problem chosen presently was not taken into consideration earlier in the literature.

2 Methodology

A. Establishment of a structural Hierarchy

A complex decision is to be structured in to a hierarchy descending from an overall objective to various criteria, sub criteria till the lowest level. The overall goal of the decision is represented at the top level of the hierarchy. The criteria and the sub criteria, which contribute to the decision, are represented at the intermediate levels. Finally the decision alternatives are laid down at the last level of the hierarchy. According to Saaty (2000), a hierarchy can be constructed by creative thinking, recollection and using people's perspectives.

B. Establishment of comparative judgments

Once the hierarchy has been structured, the next step is to determine the priorities of elements at each level. A set of comparison matrices of all elements in a level with to respect to an element of the immediately higher level are constructed. The pair wise comparisons are given in terms of how much element A is more important than element B. The preferences are quantified using a nine – point scale that is shown inTable1.

C. Synthesis of priorities and measurement of consistency

The pair wise comparisons generate the matrix of rankings for each level of the hierarchy after all matrices are developed and all pair wise comparisons are obtained, Eigen vectors (relative weights) are obtained.

Eigen Vector Method: Suppose we wish to compare a set of 'n' objects in pairs according to their relative weights. Denote the objects by $A_1, A_2, \ldots A_n$ and their weights by $w_1, w_2, \ldots w_n$. The pair wise comparisons may be represented by a matrix as given in Table 1a).

Table 1a) Matrix containing weights

	A_1	A_2	…	A_n
A_1	w_1/w_1	w_1/w_2		w_1/w_n
A_2	w_2/w_1	w_2/w_1		w_2/w_1
. . .				
A_n	w_n/w_1	w_n/w_1		w_n/w_1

The matrix shown in Table 1a) has positive entries everywhere and satisfies the reciprocal property $a_{ji} = 1/a_{ij}$. It is called a reciprocal matrix. If we multiply this matrix by the transpose of the vector $w^T = (w_1,w_2,.....w_n)$ we obtain the vector nw.

Table1: Saaty's Ratio scale for pair wise comparison of importance of weights of criteria/ alternatives

Intensity of Importance	**Definition**	**Explanation**
1	Equal importance	Two elements contribute equally to the property
3	Moderate importance of one over another	Experience and judgment slightly favor one over the other
5	Essential or strong importance	Experience and judgment strongly favor one over another
7	Very strong importance	An element is strongly favored and its dominance is demonstrated in practice.
9	Extreme importance	The evidence favoring one element over another is one of the highest possible order of affirmation
2,4,6,8	Intermediate values between two adjacent judgments	Comprise is needed between two judgments
Reciprocals	When activity i compared to j is assigned one of the above numbers, the activity j compared to i is assigned its reciprocal	
Rational	Ratios arising from forcing consistency of judgments	

Our problem takes the form Aw= nw. We started with the assumption that w was given.But if we only had A and wanted to recover w, we would have to solve the system (A- nI) w = 0 in the unknown w. This has a nonzero solution if n is an eigenvalue of A, i.e., it is a root of the characteristic equation of A. But A has unit rank since every row is a constant multiple of the first row. Thus all the eigenvalue λ_i ,i=1,2,.....n of A are zero except one. Also it is known that

$\sum_{i=1}^{n} \lambda_i$ = tr (A) = n, and $\lambda_i = 0$, $\lambda_i \neq \lambda_{max}$. The solution w of this problem is any column of A. These solutions differ by a multiplicative constant. However, this solution is normalized so that its components sum to unity. The result is unique solution no matter which column is used. The matrix A satisfies the cardinal consistency property

The consistency ratio is calculated as per the following steps i) Calculate the Eigen vector or the relative weights and λ_{max} for each matrix of order n.ii) Compute the consistency index for each matrix of order n by the formulae CI = $(\lambda_{max} - n) / (n - 1)$ iii) The consistency ratio is then calculated using the formulae CR = CI / RI ,where RI is a known random consistency index obtained from a large number of simulation runs and varies depending upon the order of the matrix .

Table 2: Average random index (RI) based on matrix size (adapted from Saaty, 2000)

Size of matrix (n)	***Random consistency index (RI)***
1	0
2	0
3	0.52
4	0.89
5	1.11
6	1.25
7	1.35
8	1.40
9	1.45
10	1.49

The acceptable CR range varies according to the size of the matrix i.e. 0.05 for a 3 by 3 matrix, 0.08 for a 4 by 4 matrix and 0.1 for all larger matrices, for $n \geq 5$ (Saaty, 2000) if the value of CR is equal to, or less than that value it implies that the evaluation within the matrix is acceptable or indicates a good level of consistency in the comparative judgments represented in that matrix. If CR is more than that acceptable value, inconsistency of the judgments within the matrix has occurred and the evaluation process should be reviewed.

2.1. The Use of Pairwise Comparisons

One of the most crucial steps in many decision-making methods is the accurate estimation of the pertinent data. This is a problem not bound in the AHP method only, but it is crucial in many other methods which need to elicit qualitative information from the decision-maker. Very often qualitative data cannot be known in terms of absolute values. For instance, *"With respect to Academics Criterion, what is the relative performance of EEE over ECE? "* Although information about questions like the previous one are vital in making the correct decision, it is very difficult, if not impossible, to quantify them correctly. Therefore, many decision-making methods attempt to determine the **relative** importance, or weight, of the alternatives in terms of each criterion involved in a given decision-making problem. Pairwise comparisons are used to determine the relative importance of each alternative in terms of each criterion. In this approach the decision-maker has to express his opinion about the value of one single pairwise comparison at a time. Usually, the decision-maker has to choose his answer among 10-17 discrete choices. Each choice is a linguistic phrase. Some examples of such linguistic phrases are: *"A is more important than B"*, or *"A is of the same importance as B"*, or *"A is a little more important than B"*, and so on .The main problem with the pairwise comparisons is **how to quantify** the linguistic choices selected by the decision maker during their evaluation. All the methods which use the pairwise comparisons approach eventually express the qualitative answers of a decision maker into some numbers which, most of the time, are ratios of integers. Since pairwise comparisons are the keystone of these decision-making processes, correctly quantifying them is the most crucial step in multi-criteria decision-making methods which use qualitative data. Pairwise comparisons are quantified by using a **scale**. Such a scale is a one-to-one mapping between the set of discrete linguistic choices available to the decision maker and a discrete set of numbers which represent the importance, or weight, of the previous linguistic choices. The scale proposed by Saaty is depicted in table 1. Other scales have also been proposed by others.

3 Physical Significance of all the Criteria

Attendance(C1): The students' attendance has to be above 75% throughout four-year period in all the semesters.

Academics(C2): The students' academic record should be consistently above 70% in all the Semesters throughout four-year period.
Co-curricular activities(C3): A student has to participate in co-curricular activities like Paper presentation, debates, Group Discussions or quizzes etc, either in inter college or intra- college and need to win some prizes.
Extra curricular activities(C4): A student has to participate in extra-curricular activities like Indoor games, Outdoor games which are held in intra college or inter college and need to win some prizes.
Cultural activities(C5): A student has to participate in cultural activities like Singing or choreography which are held in Intra College or Inter College and need to win some prizes.
General behavior(C6): A student is required to be honest and need to maintain good relationship with his / her peers and with teachers.
Departmental activities(C7): A student need to participate in the activities conducted by the department and need to posses certain managerial skills and need to coordinate different activities/events held in the department.
The matrix of pair wise comparisons of the criteria as given by the decision maker is shown in Table 3, along with the resulting vector of priorities. The vector of priorities is the principal Eigen vector of the matrix .It gives the relative priority of the criteria measured on a ratio scale given in Table 1.Next we move to pair wise comparisons of the lower level and lastly to the pair wise comparisons of the lowest level .The elements to be compared pair wise are the engineering branches with respect to how much better one is than the other in satisfying each criterion in level 2.Thus there will be fifteen 5 x 5 matrices of judgments. To understand these judgments, a brief description of the engineering branches is follows.

EEE: This branch consists of students who are good at academics, attendance and Co-Curricular activities. Their participation is comparatively less in Extra Curricular activities when compared to other branch students.

ECE: The students of this branch are highly motivated and hence have good academic records and attendance. Their general behavior is good. The departmental activities are conducted well. Though their participation in Extra curricular activities and Cultural activities is less, compared to other branch students, they are good at Co Curricular activities.

ICE: This branch consists of students who are less motivated and hence poor in academics and attendance. Their relation ship with teachers and peers is not good when compared with other branch students. They are good at Extra curricular activities , cultural activities and are able to manage events well in their departments.

CSE: The students of this branch are good in academics and attendance as students of EEE.The relationship with peers is not good. They are good in Extra Curricular activities and Co curricular activities. They manage events well as students of EEE and ECE.

MECH: The students of this branch are less motivated and hence are not good in academics and attendance. They are good at Extra Curricular activities and Cultural activities, but not good at Co Curricular activities. Their relationship with teachers and peers is not good.

Table 3 gives the opinions of the decision maker regarding the alternatives with respect to each of the criteria/ sub criteria. The opinions given are converted to numbers using Saaty's ratio scale. The consistency index and Consistency ratio of each matrix are calculated using the formulae given in section II. Table 4 gives the weights of the criteria calculated using Eigen vector method shown in section II. Table 5 shows the local and global priorities of the alternatives. Figure 1 shows the hierarchical decomposition of criteria, sub criteria and alternatives. Level 0 shows the overall goal of "All round Excellence award "in the zeroth level, shown in blue color. The next level, namely level 1 shows the criteria as its elements, which are shown in orange colored cells. Its next level namely level 2 shows the sub criteria in the lavender colored cells. It can be observed that not all the criteria have sub criteria. The criteria like Academics and attendance do not have any criteria as identified by the decision maker. The next level namely level 3 is the highest level given by alternatives shown in green colored cells.

Figures 2 – 16 shows the weights of alternatives with respect to each of the criteria or sub criteria. Table 6 shows the weights and ranks of alternatives.

Figure 17 shows the ranks of alternatives through bar diagrams.

4 Conclusions & Scope:

The above problem of selecting a student for all round Excellence award is an application of Analytic hierarchy Process, as this uses qualitative opinions. AHP has been used for a wide number of areas, but AHP waste not applied to the problem studied here. The student of ECE gets the All Round Excellence Award as he/she is good in academics and general behavior which are highest priority criteria, had the largest priority from figure 17 and Table 6. The student of CSE branch is equivalently good who performed better than EEE students. It can be observed that the student of ICE branch is relatively very poor in everything.

The Fuzzy AHP method could be applied to the above and the results could be compared. Sensitivity analysis could be applied to check how sensitive the alternatives and criteria weights are, to small changes.

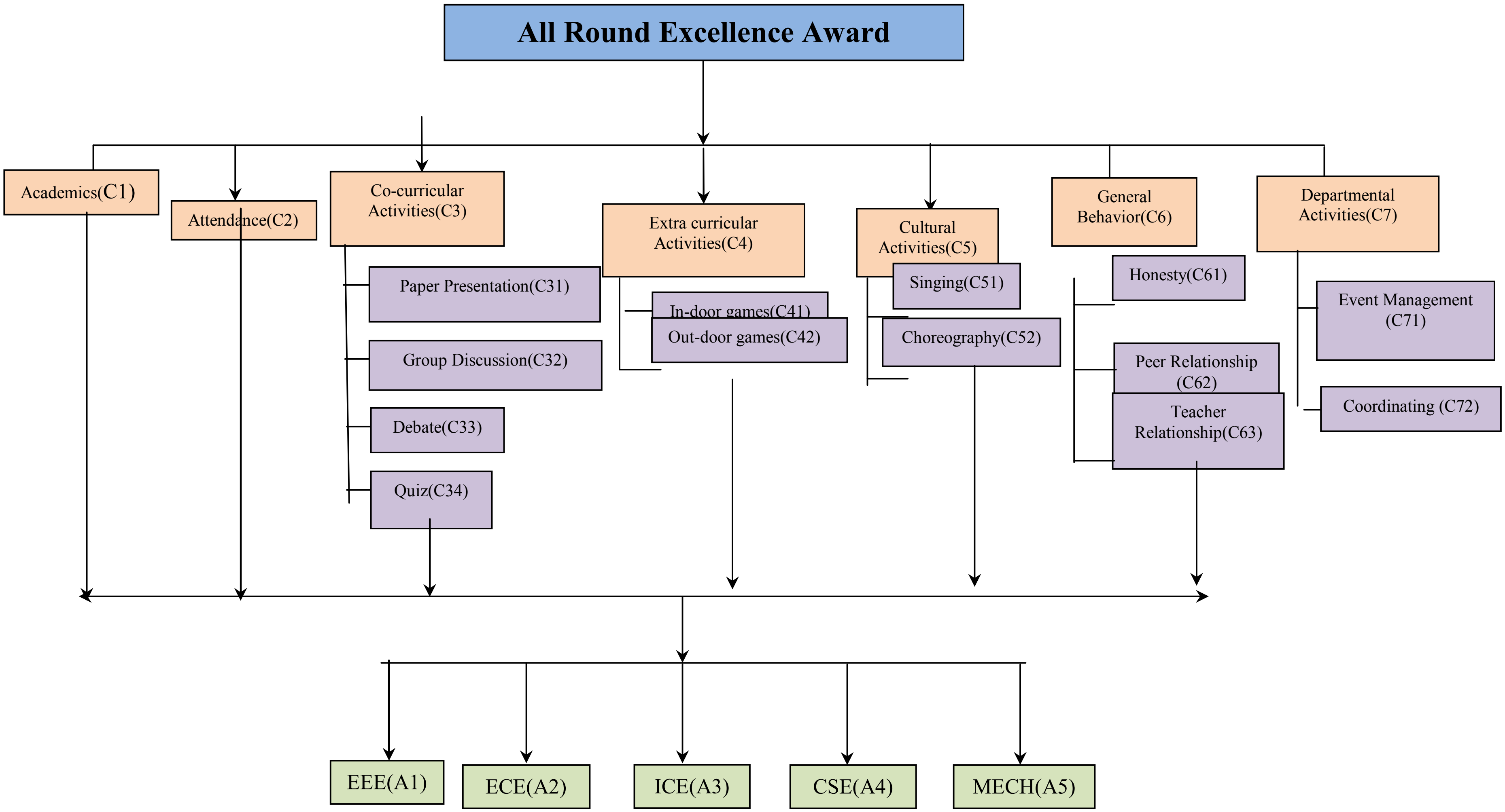

Figure: 1 Hierarchical decomposition of criteria, sub criteria and alternatives

Table 3 :Pair wise comparison matrices for level 2

Academics

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	0.33	5	3	5	0.28428
ECE	3	1	5	3	5	0.44204
ICE	0.2	0.2	1	0.33	1	0.0634
CSE	0.33	0.33	3	1	3	0.15004
MECH	0.2	0.2	1	0.3	1	0.0622

λ_{max} = 5.1892, C.I =0.0473,C.R=0.0426

Attendance

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	0.33	0.2	0.33	5	0.1032
ECE	3	1	5	3	7	0.508
ICE	5	0.2	1	1	1	0.1608
CSE	3	0.33	1	1	3	0.1999
MECH	0.2	0.143	1	0.33	1	0.0633

λ_{max} = 5.9761, C.I =0.244,C.R=0.2198

Paper presentation

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	0.33	5	1	5	0.23164
ECE	3	1	5	3	5	0.4487
ICE	0.2	0.2	1	0.33	0.33	0.05121
CSE	1	0.33	3	1	3	0.18883
MECH	0.2	0.2	3	0.33	1	0.07963

λ_{max} = 5.2914, C.I =0.0729,C.R=0.0656

Debate

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	1	3	1	3	0.2818
ECE	1	1	1	3	3	0.2818
ICE	0.33	1	1	0.33	0.33	0.0934
CSE	1	0.33	3	1	3	0.2258
MECH	0.33	0.33	3	0.33	1	0.1163

λ_{max} = 5.6874, C.I =0.1719,C.R=0.1548

Group discussion

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	0.33	3	1	3	0.19448
ECE	3	1	5	3	5	0.46212
ICE	0.33	0.2	1	0.33	0.33	0.05829
CSE	1	0.33	3	1	3	0.19448
MECH	0.33	0.2	3	0.33	1	0.09065

λ_{max} =5.1952, C.I =0.0488,C.R=0.044

Quiz

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	1	3	1	0.33	0.1776
ECE	1	1	3	3	5	0.381
ICE	0.33	0.33	1	0.33	1	0.0915
CSE	1	0.33	3	1	3	0.2212
MECH	3	0.2	1	0.33	1	0.1287

λ_{max} =5.9064, C.I =0.2266,C.R=0.2041

Indoor games

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	1	3	0.33	0.33	0.13526
ECE	1	1	0.33	0.33	0.33	0.08698
ICE	0.33	3	1	0.33	0.33	0.10836
CSE	3	3	3	1	0.33	0.262
MECH	3	3	3	3	1	0.40741

λ_{max} =5.547, C.I =0.1368,C.R=0.1232

Outdoor games

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	3	1	1	0.33	0.183
ECE	0.33	1	0.33	0.33	0.33	0.0755
ICE	1	3	1	1	1	0.2284
CSE	1	3	1	1	1	0.2284
MECH	3	3	1	1	1	0.2846

λ_{max} =5.1416, C.I =0.0354,C.R=0.0319

Singing

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	3	3	1	1	0.27188
ECE	0.33	1	3	0.33	0.33	0.11222

Choreography

	EEE	ECE	ICE	CSE	MECH	priority vector
EEE	1	0.33	3	1	1	0.1811
ECE	3	1	3	0.33	0.33	0.1808

ICE	0.33	0.33	1	0.33	0.33	0.07217
CSE	1	3	3	1	1	0.27188
MECH	1	3	3	1	1	0.27188
λ_{max}=5.1372, C.I =0.0343,C.R=0.0309						

ICE	0.33	0.33	1	0.33	0.33	0.0748
CSE	1	3	3	1	1	0.2817
MECH	1	3	3	1	1	0.2817
λ_{max}=5.5737, C.I =0.1434,C.R=0.1292						

Honesty

	EEE	**ECE**	**ICE**	**CSE**	**MECH**	**priority vector**
EEE	1	1	3	1	3	0.27303
ECE	1	1	3	1	3	0.27303
ICE	0.33	0.33	1	0.33	1	0.09046
CSE	1	1	3	1	3	0.27303
MECH	0.33	0.33	1	0.33	1	0.09046
λ_{max}=4.988, C.I =-0.003,C.R=-0.0027						

Peer relationship

	EEE	**ECE**	**ICE**	**CSE**	**MECH**	**priority vector**
EEE	1	1	1	1	1	0.2
ECE	1	1	1	1	1	0.2
ICE	1	1	1	1	1	0.2
CSE	1	1	1	1	1	0.2
MECH	1	1	1	1	1	0.2
λ_{max}=5.000, C.I =0.0,C.R=0.0						

Teacher relationship

	EEE	**ECE**	**ICE**	**CSE**	**MECH**	**priority vector**
EEE	1	1	3	1	3	0.27303
ECE	1	1	3	1	3	0.27303
ICE	0.33	0.33	1	0.33	1	0.09046
CSE	1	1	3	1	3	0.27303
MECH	0.33	0.33	1	0.33	1	0.09046
λ_{max}=4.988, C.I =-0.003,C.R=-0.0027						

Event management

	EEE	**ECE**	**ICE**	**CSE**	**MECH**	**priority vector**
EEE	1	1	3	1	1	0.2284
ECE	1	1	3	1	3	0.2846
ICE	0.33	0.33	1	0.33	0.33	0.0755
CSE	1	1	3	1	1	0.2284
MECH	1	0.33	3	1	1	0.183
λ_{max}=5.1416, C.I =0.0354,C.R=0.0319						

Coordinating

	EEE	**ECE**	**ICE**	**CSE**	**MECH**	**priority vector**
EEE	1	3	3	1	1	0.29227
ECE	0.33	1	3	1	1	0.18796
ICE	0.33	0.33	1	0.33	1	0.09684
CSE	1	1	3	1	1	0.23461
MECH	1	1	1	1	1	0.18833
λ_{max}=5.2931, C.I =0.0733,C.R=0.066						

<u>Table: 4 weight vector of criteria</u>

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**priority vector**
C1	1	5	3	5	5	1	3	0.316851
C2	0.2	1	0.33	0.33	0.33	0.2	1	0.045595
C3	0.33	3	1	3	3	1	1	0.158735
C4	0.2	3	0.33	1	1	0.33	0.33	0.067133
C5	0.2	3	0.33	1	1	0.33	1	0.078653
C6	1	5	1	3	3	1	3	0.234051
C7	0.33	1	1	3	1	0.33	1	0.098985
λ_{max} = 7.4524, C.I = 0.0754,C.R=0.0559								

Table: 5 Local and global priorities

	C1 (0.3)	**C2 (0.04)**	**C3 (0.2)**	**C4 (0.2)**	**C5 (0.2)**	**C6 (0.2)**	**C7 (0.5)**	**C8 (0.5)**	**C9 (0.5)**	**C10 (0.5)**	**C11 (0.33)**	**C12 (0.33)**	**C13 (0.33)**	**C14 (0.5)**	**C15 (0.5)**	
A1	0.28	0.1	0.23	0.28	0.19	0.18	0.14	0.18	0.272	0.181	0.273	0.2	0.27	0.23	0.29	0.225
A2	0.442	0.51	0.45	0.28	0.46	0.38	0.09	0.08	0.112	0.181	0.273	0.2	0.27	0.28	0.19	0.236
A3	0.063	0.01	0.05	0.09	0.06	0.09	0.11	0.23	0.072	0.075	0.09	0.2	0.09	0.08	0.1	0.102
A4	0.15	0.02	0.19	0.22	0.19	0.22	0.26	0.23	0.272	0.282	0.273	0.2	0.27	0.23	0.23	0.234
A5	0.06	0.01	0.0	0.11	0.09	0.1	0.4	0.2	0.27	0.28	0.09	0.2	0.09	0.18	0.1	0.197

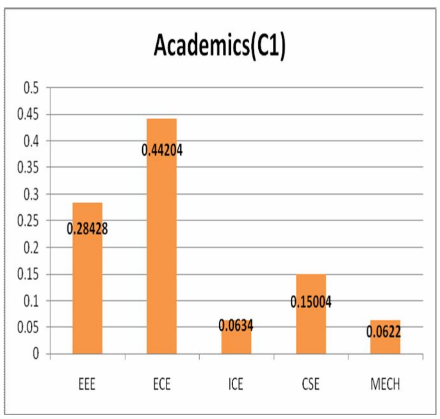

Figure2: weights of alternatives w.r.t Academics

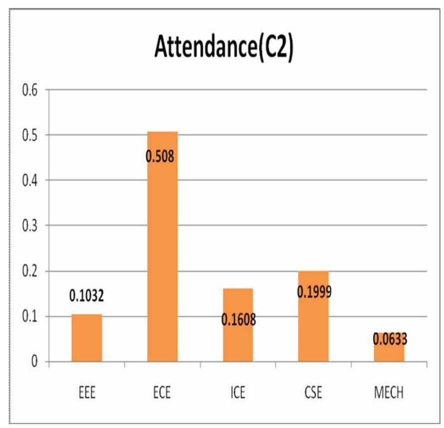

Figure 3: Weights of alternatives w.r.t Attendance

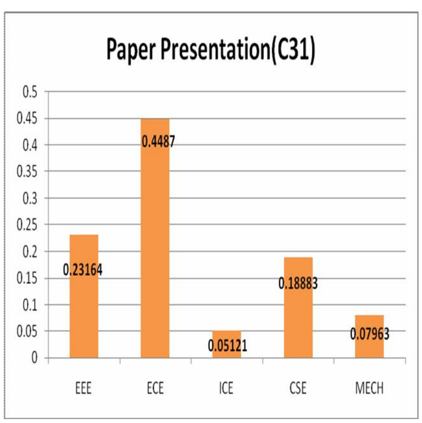

Figure 4: weights of alternatives w.r.t Paper Presentation

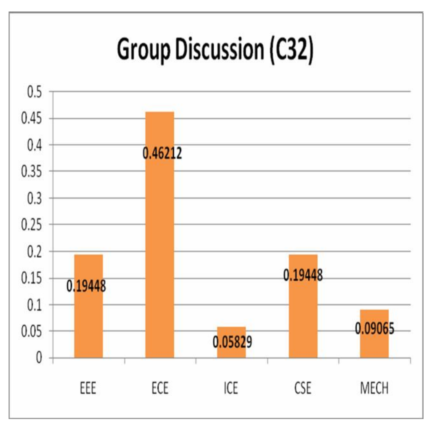

Figure 5: Weights of alternatives w.r.t Group Discussion

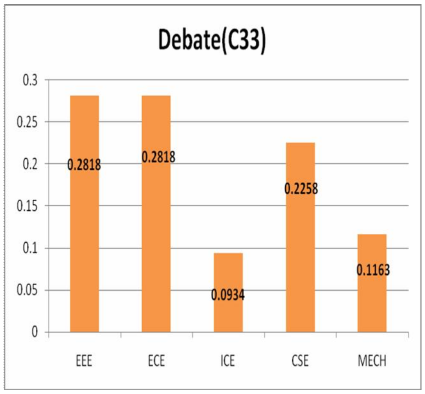

Figure 6: weights of alternatives w.r.t Debate

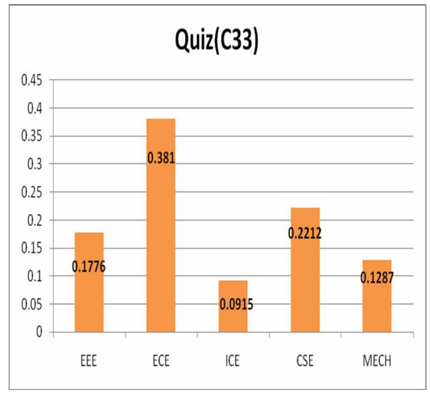

Figure 7: Weights of alternatives w.r.t Quiz

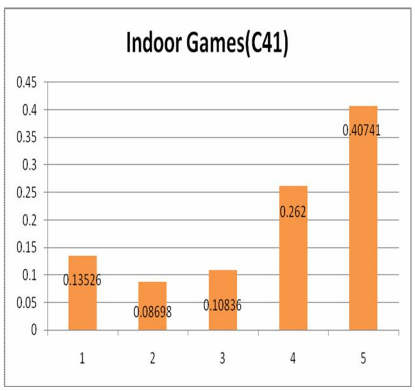

Figure 8: weights of alternatives w.r.t Indoor Games

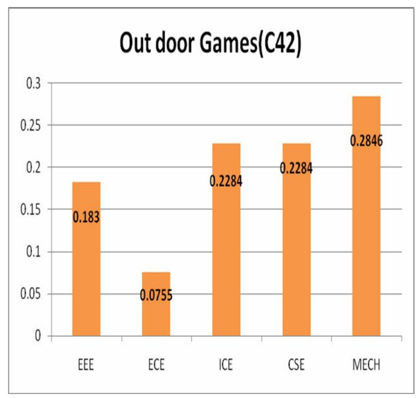

Figure 9: Weights of alternatives w.r.t Outdoor Games

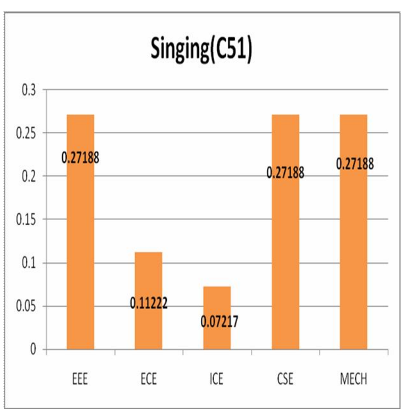

Figure 10: weights of alternatives w.r.t Singing

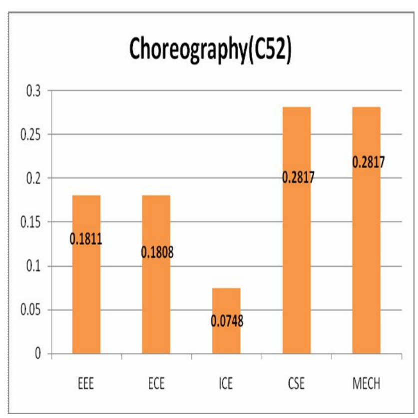

Figure 11: Weights of alternatives w.r.t Choreography

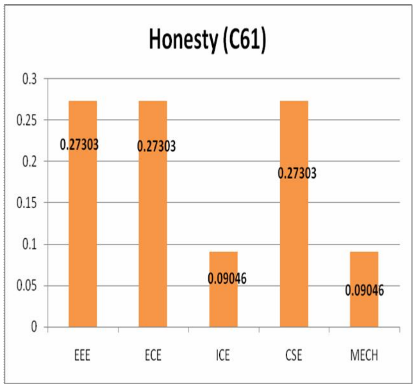

Figure12: weights of alternatives w.r.t Honesty

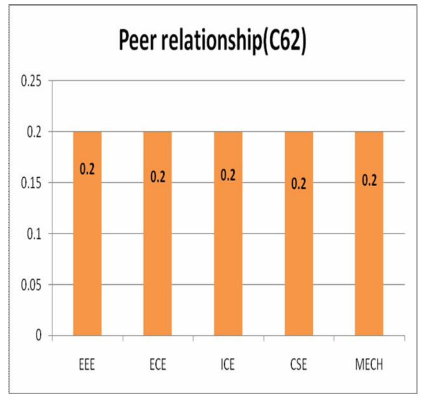

Figure 13: Weights of alternatives w.r.t Peer relationship

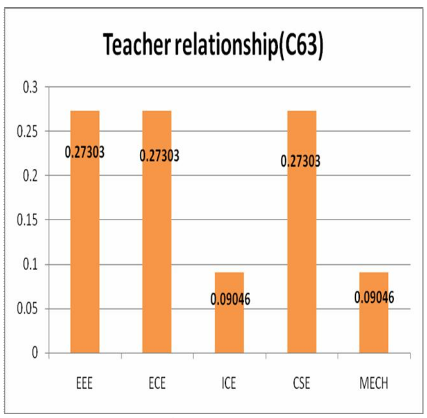

Figure 14: weights of alternatives w.r.t Teacher relationship

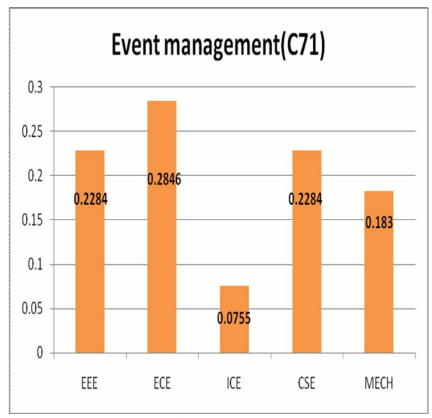

Figure 15: Weights of alternatives w.r.t Event management

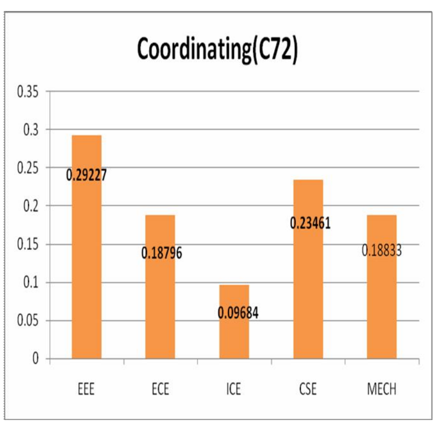

Figure 16: weights of alternatives w.r.t Coordinating

Table:6 Weights and Ranks of alternatives

S.No	Alternatives	Weights of alternatives	Ranks of alternatives
1.	EEE	0.225	3
2.	ECE	0.236	1
3.	ICE	0.102	5
4.	CSE	0.234	2
5.	MECH	0.197	4

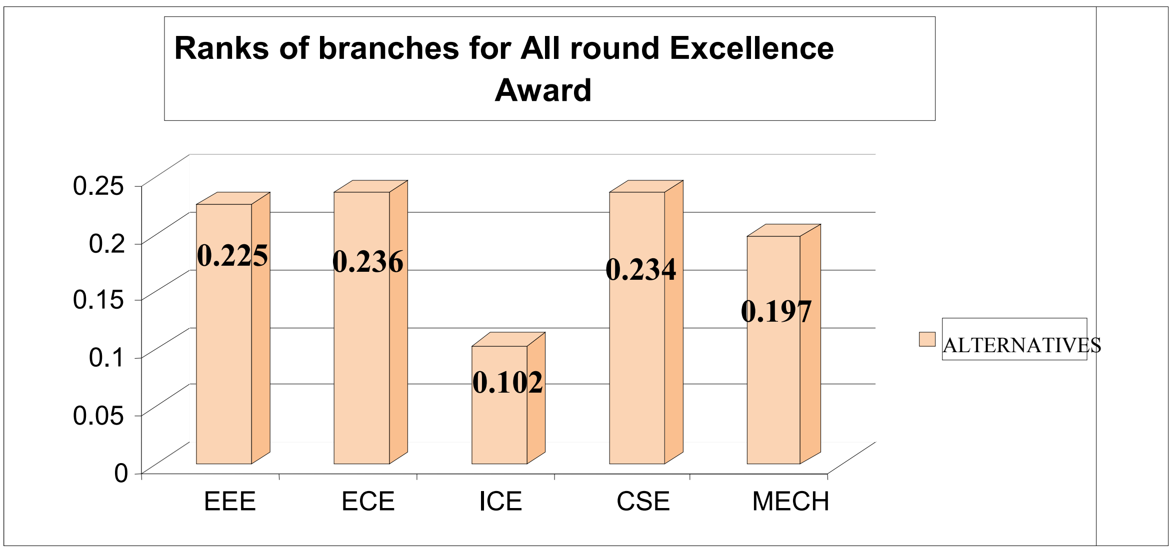

Figure: 17 Ranks of alternatives (five engineering branches)

References:

[1] Patric T.Harker, *"The art of Science and Decision-making: The Analytic Hierarchy Process"*, Springer Verlag, 1989.

[2] R.Ramanathan and L.S.Ganesh, *"Group preference aggregation methods employed in AHP: An evaluation and an intrinsic process for deriving members' weightages"*, European Journal of Operational Research 79(1994)249-265.

[3] T.Ramsha Prabhu and K.Vizaya Kumar, *“Fuzzy Hierarchical Decision Making (FHDM): A Methodology for Technology choice”* (1996), International Journal of Computer Applications in Technology, Vol.9 No.5 &6.

[4] Thomas.L.Saaty *“Exploring interface between Hierarchies, multiple objectives and Fuzzy sets ”*, Fuzzy sets and systems 1(1978), 57-68.

[5] Thomas, L.Saaty (1980), *“The Analytical Hierarchy Process”*, Tata McGraw Hill, New York.

Proceedings of Conferences:

[6] *“Analytical Hierarchy Process for selection of a student for All Round Excellence Award”*, National conference on Mathematics and computational sciences, Adikavi Nannaya University, Andhra Pradesh, India(2012).

[7] *“Selection of a student for All Round Excellence Award – A mathematical model of a Multi-criteria Decision Making approach”*, 4th International conference, Bengal Institute of Technology and Management, West Bengal (2012).

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